An Introduction to
Mildly Context-Sensitive Grammar Formalisms

— *Minimalist Grammars* —

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Expressions over a feature set

$\text{Feat} = \text{Syn} \cup \text{NonSyn}$

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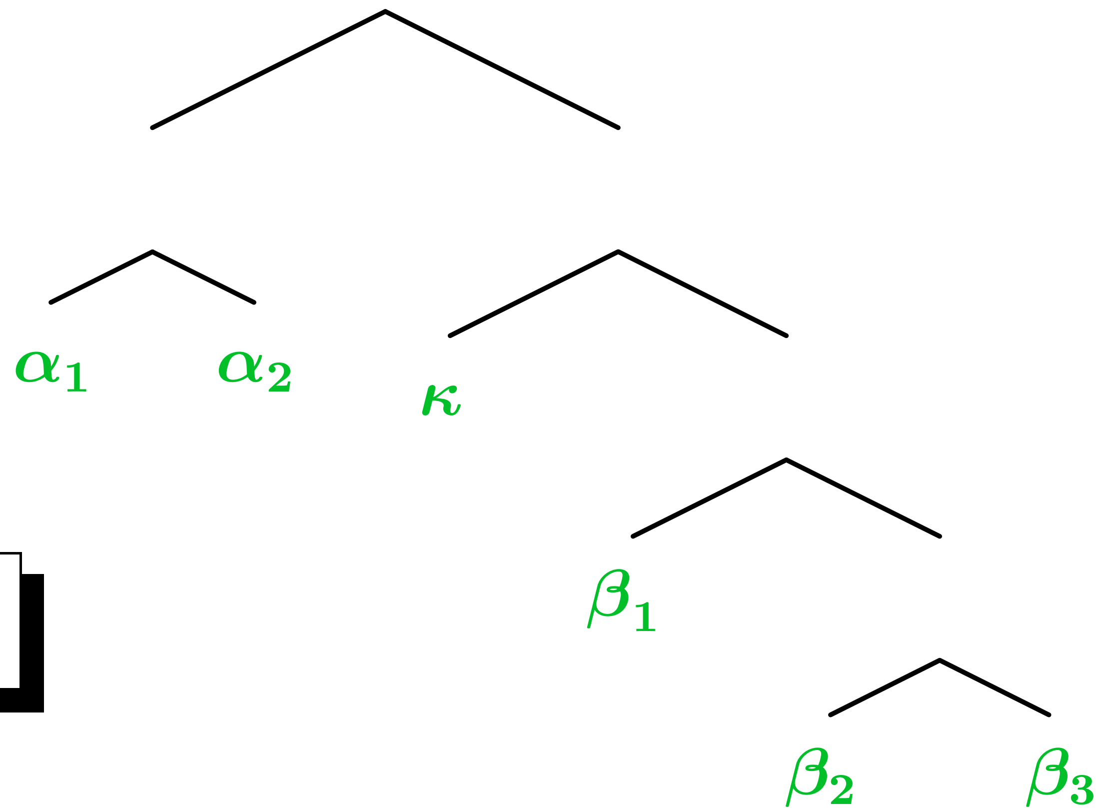
Feat $=$ Syn $\cup$ NonSyn



$$\tau = \langle \mathsf{N}_\tau , \lhd^*_\tau , \prec_\tau , \mathsf{label}_\tau \rangle$$

- $\langle \mathsf{N}_\tau , \lhd^*_\tau , \prec_\tau \rangle$ is a finite, binary tree
- $\mathsf{label}_\tau : \mathsf{Leaves}_\tau \longrightarrow$ Syn*NonSyn*

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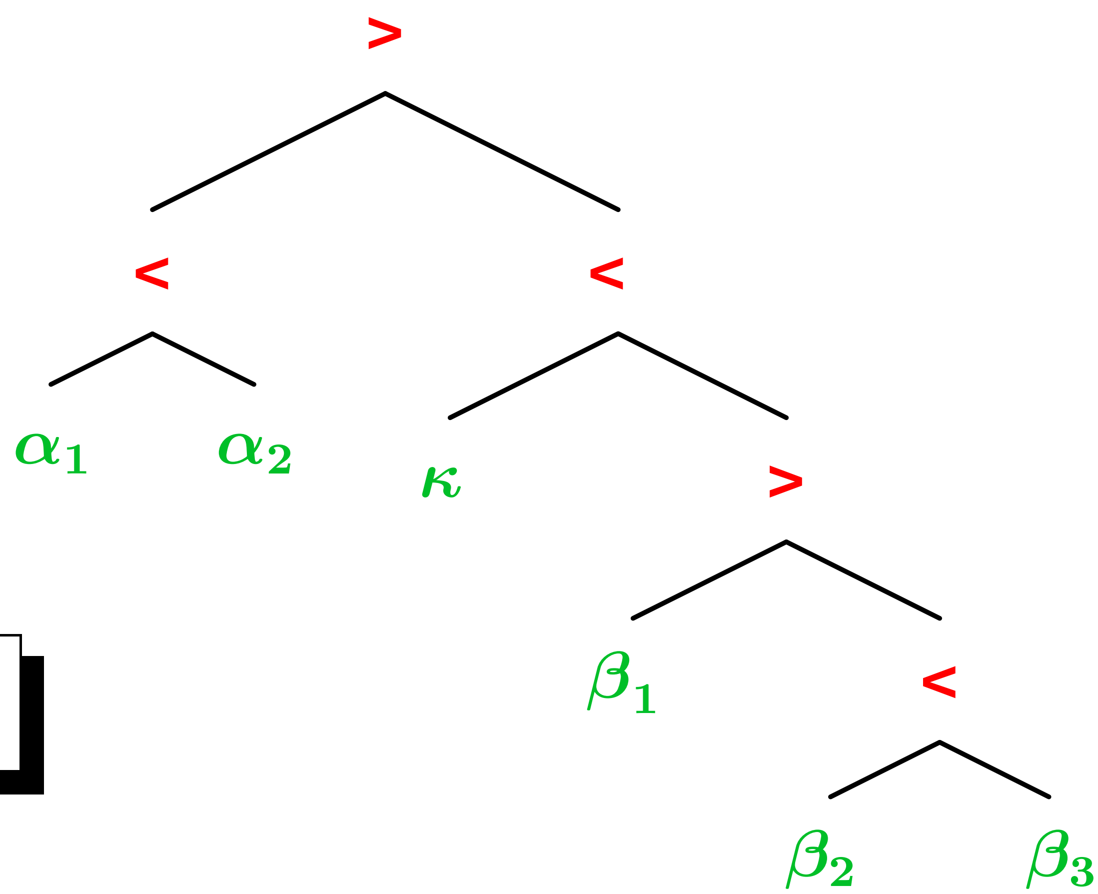

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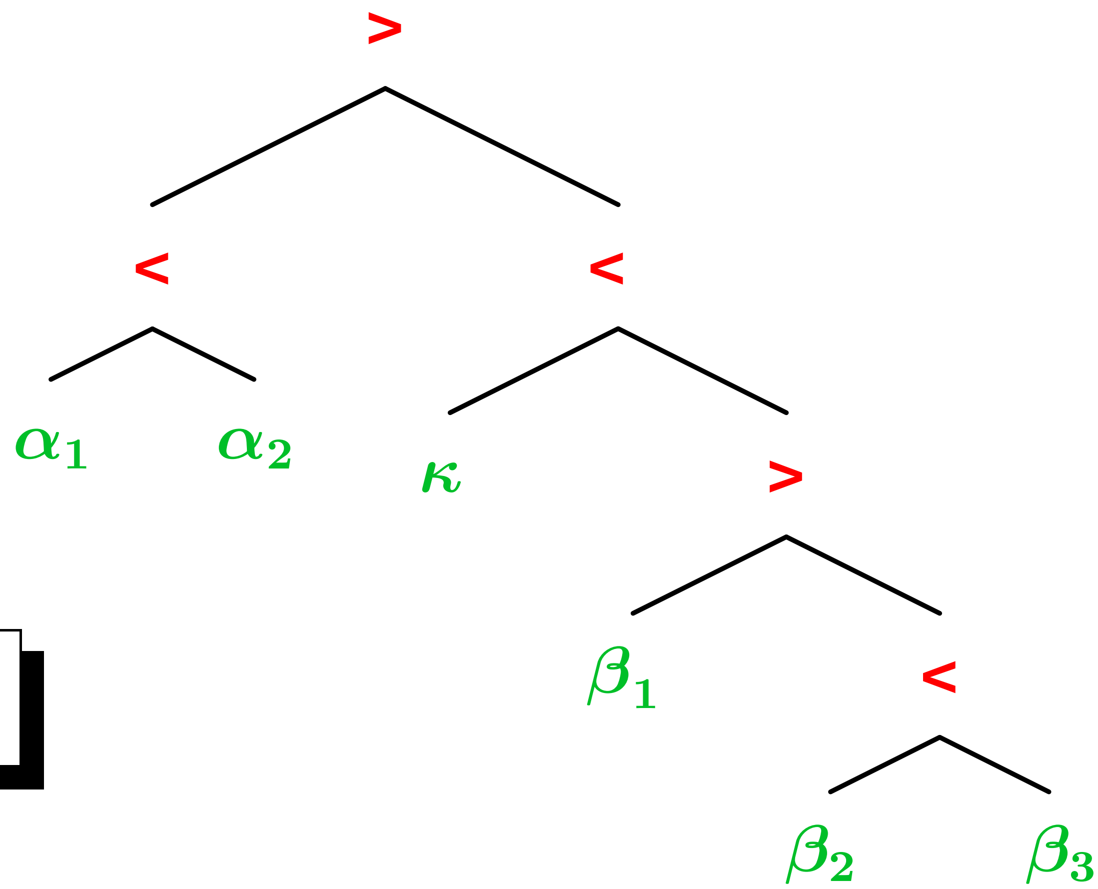

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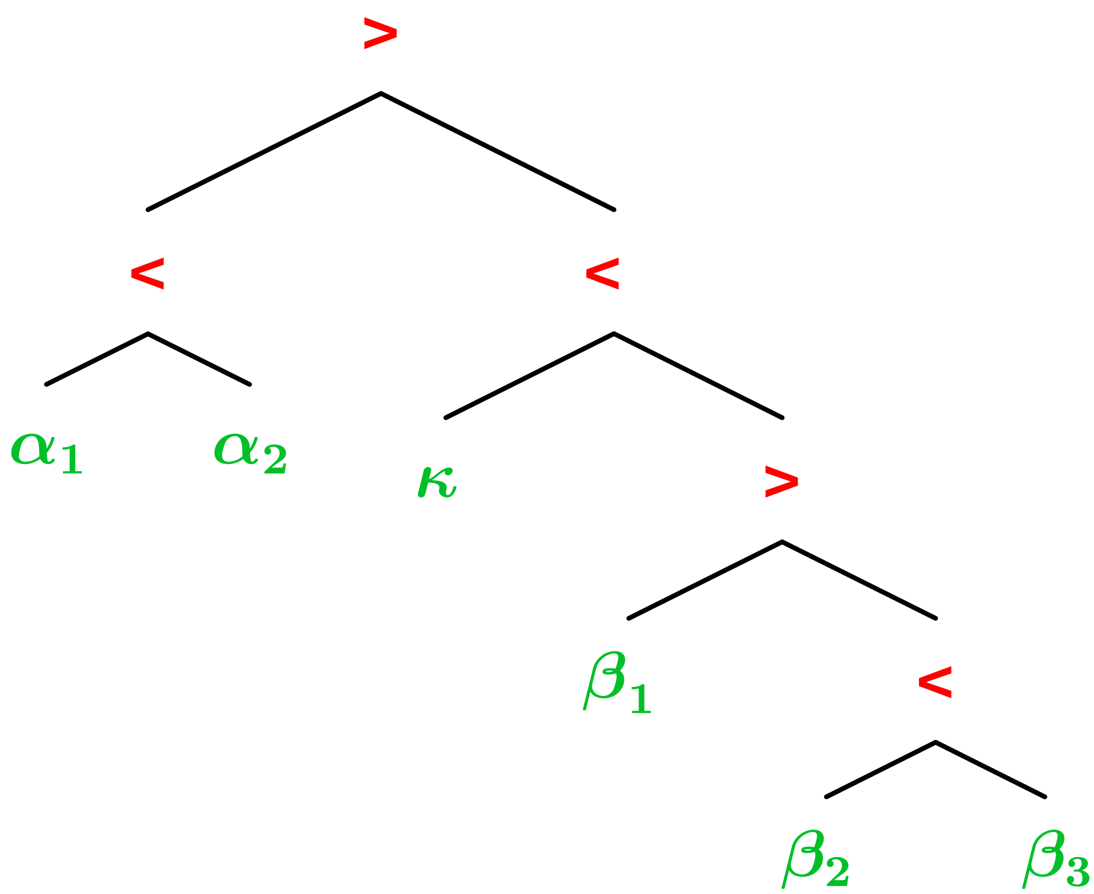
  $\mathsf{NonLeaves}_\tau \longrightarrow \{ <, > \}$ [“relation” of projection]

Expressions over a feature set

- **<** “left daughter projects”
- **>** “right daughter projects”



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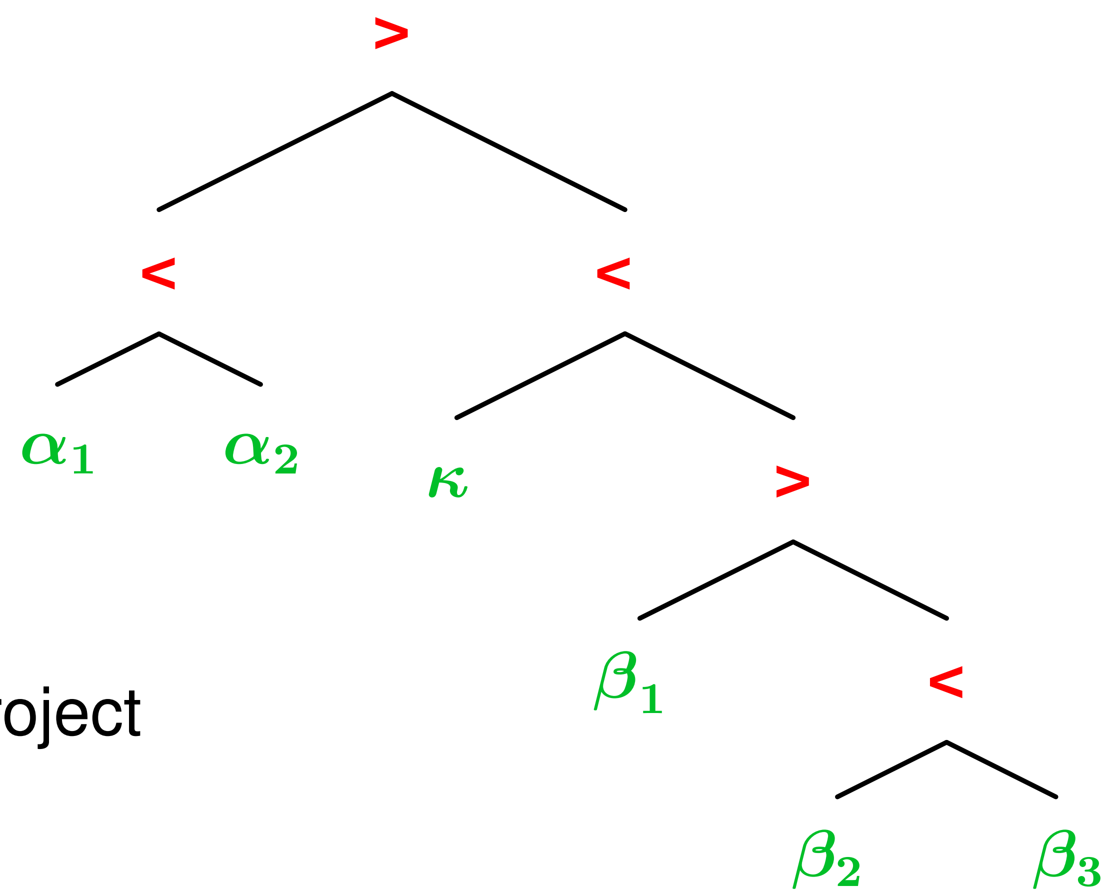

maximal projections :

subtrees whose roots don’t project

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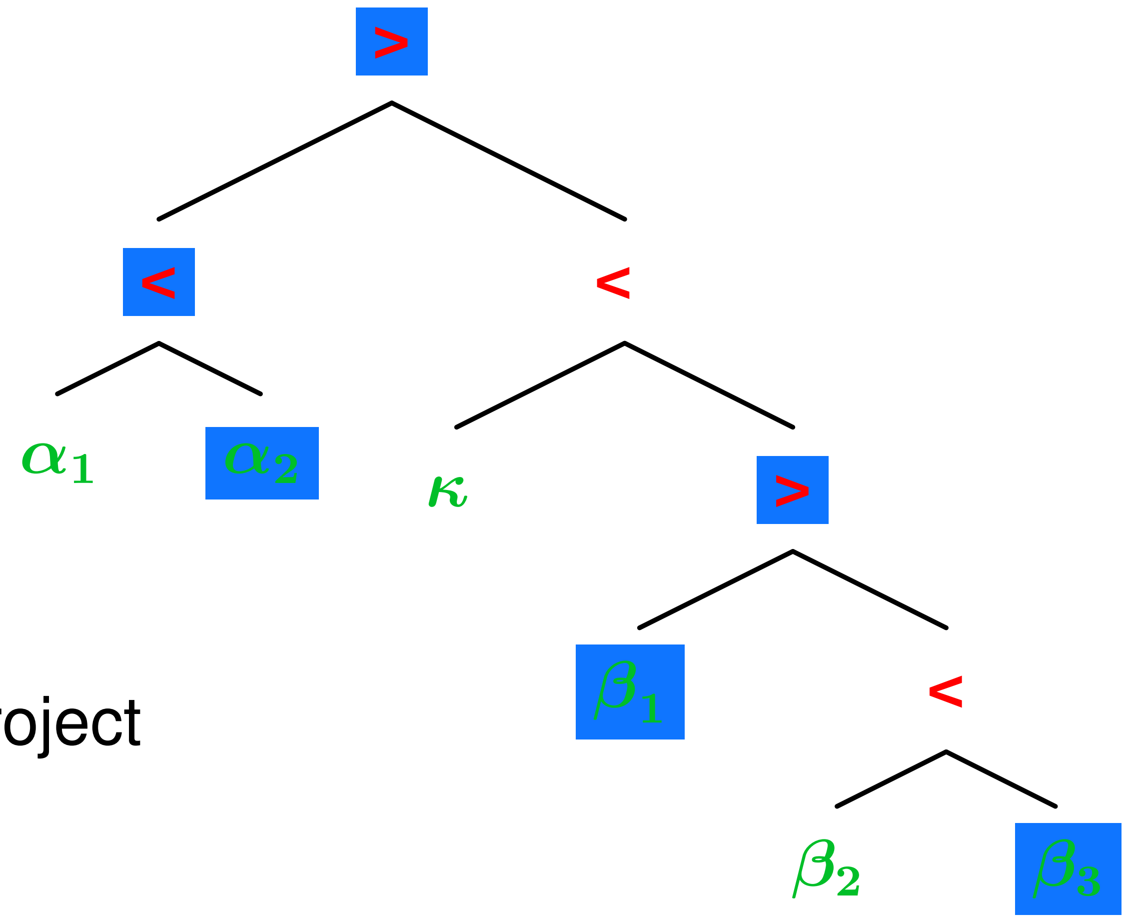

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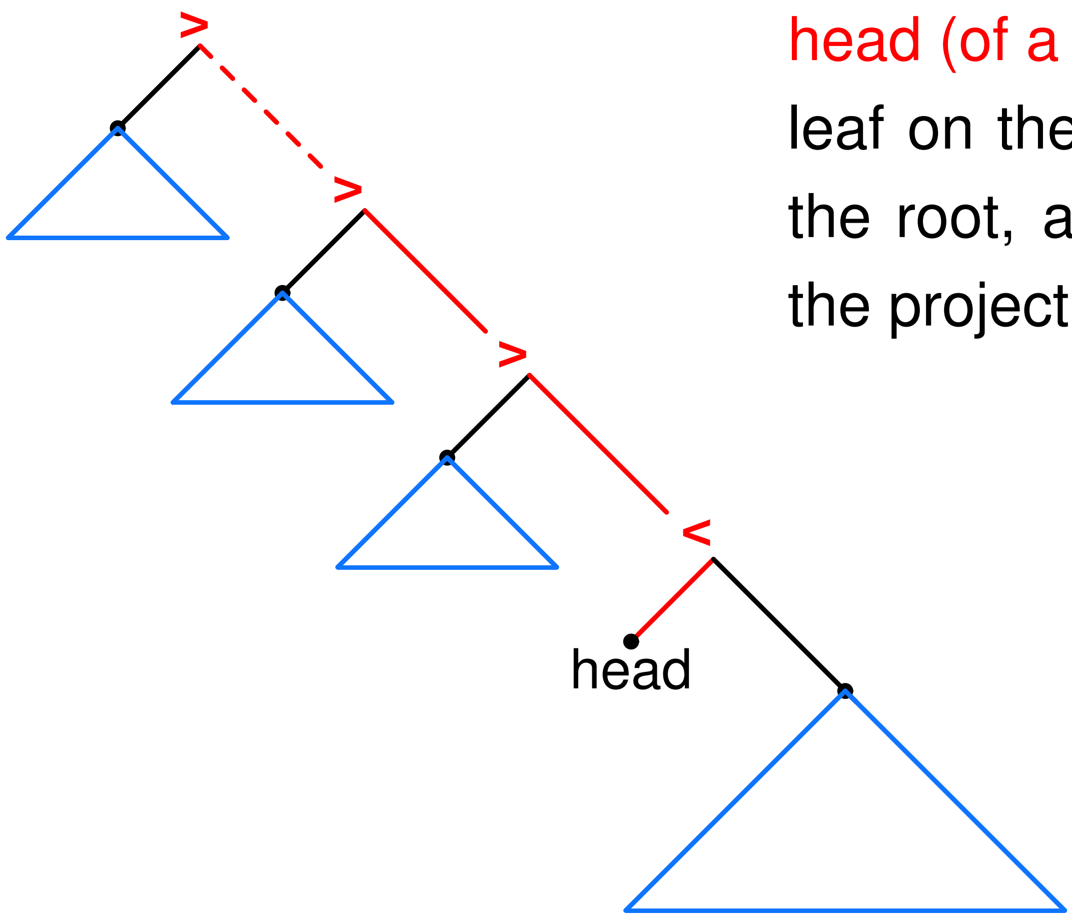
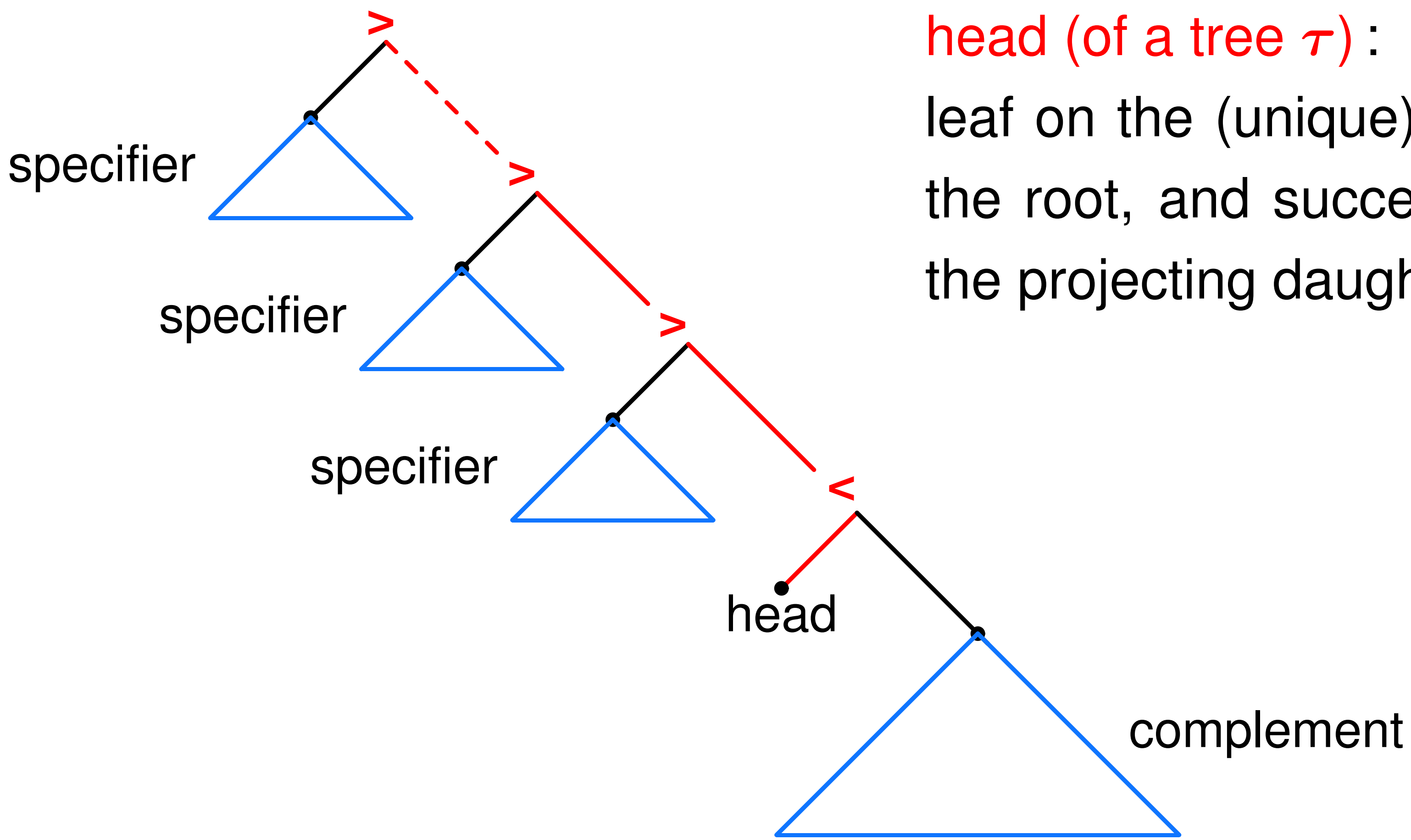


head (of a tree τ) :

leaf on the (unique) path starting at the root, and successively following the projecting daughters



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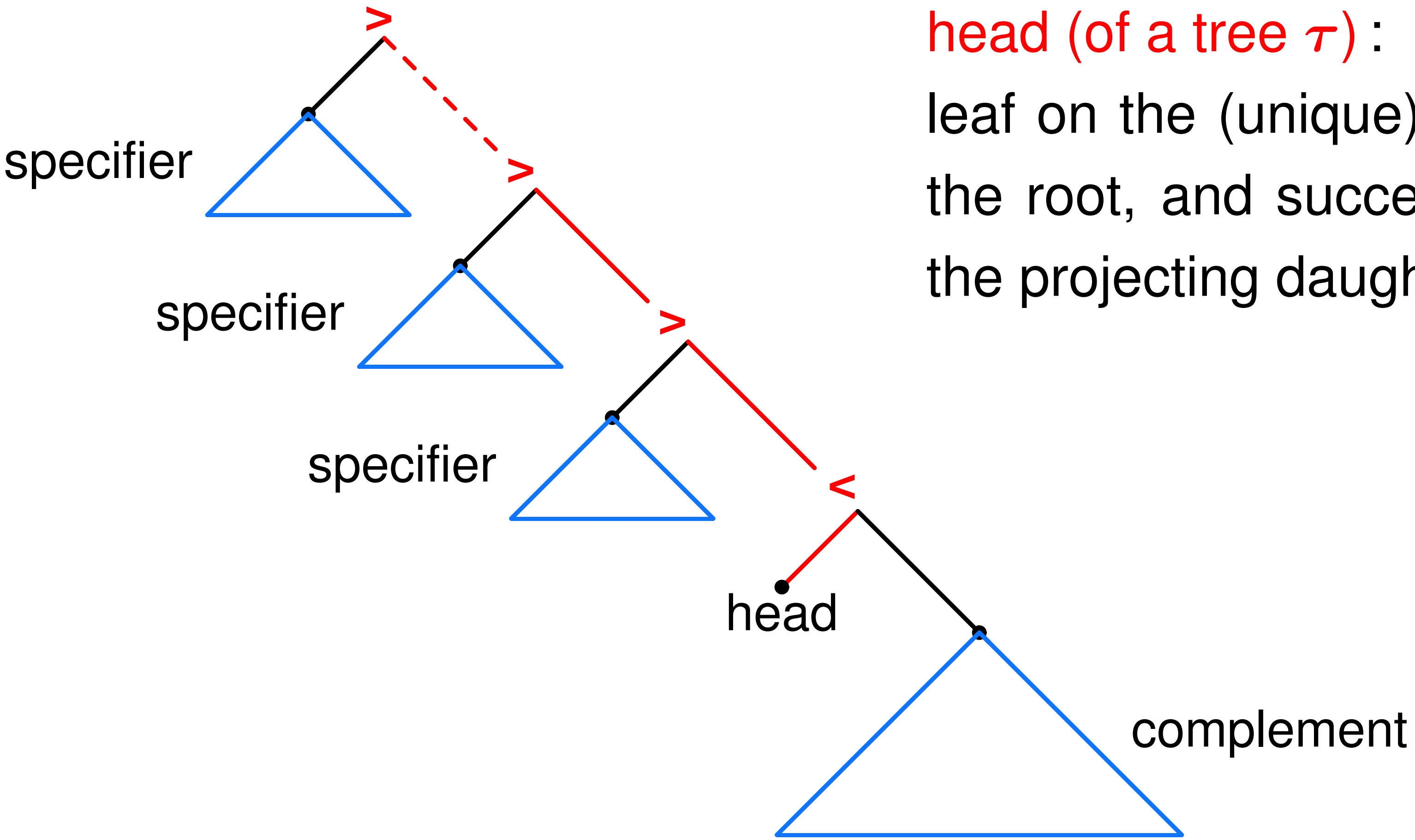

head (of a tree τ) :
leaf on the (unique) path starting at the root, and successively following the projecting daughters

τ displays feature f $:\Longleftrightarrow$ τ's head-label is of the form $f\alpha$

Building expressions

- Expressions can be built up from other expressions by applying structure building functions.

- The applications of these functions are triggered by particular instances of syntactic features appearing in the leaf-labels of the trees to which the functions are applied.

- After having been applied the triggering instances are deleted and count as checked.

- Different structure building operations are triggered by different types of syntactic features.

Syntactic features (the set Syn)

- Syn is partitioned into . . .

Base	$= \{\mathtt{x}, \mathtt{y}, \mathtt{z}, \ldots\}$	*(basic) categories*
Select	$= \{\mathtt{=x}, \mathtt{=y}, \mathtt{=z}, \ldots\}$	*(merge-)selectors*
Licensees	$= \{\mathtt{-x}, \mathtt{-y}, \mathtt{-z}, \ldots\}$	*(move)-licensees*
Licensors	$= \{\mathtt{+x}, \mathtt{+y}, \mathtt{+z}, \ldots\}$	*(move)-licensors*

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- NonSyn* $= \{$ *book* , *which* , *Mary_ read* , . . . , $\emptyset$, . . . $\}$

Structure building functions

$$\text{merge} : \text{Exp(Feat)} \times \text{Exp(Feat)} \xrightarrow{\text{part}} \text{Exp(Feat)}$$

$\langle \phi , \psi \rangle \in$ Domain(merge) $:\Longleftrightarrow$

- ψ displays feature $\mathtt{x} \in$ Base
- ϕ displays feature $\mathtt{=x} \in$ Select

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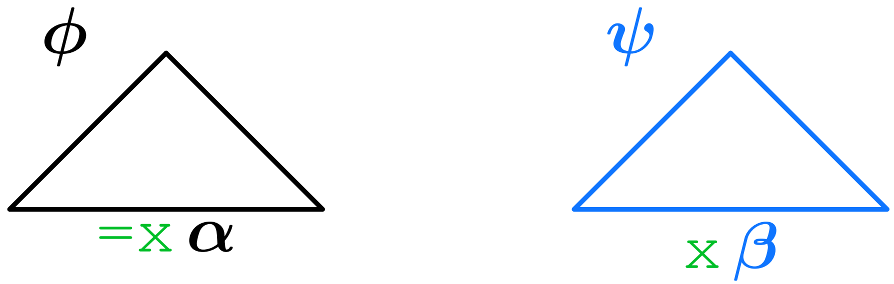

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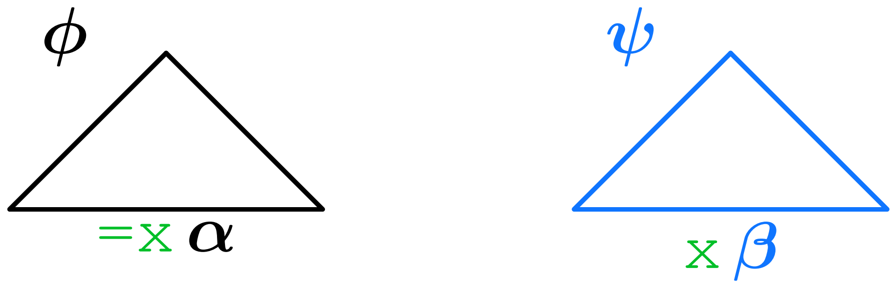

$\rightsquigarrow$

ϕ simple

ϕ complex

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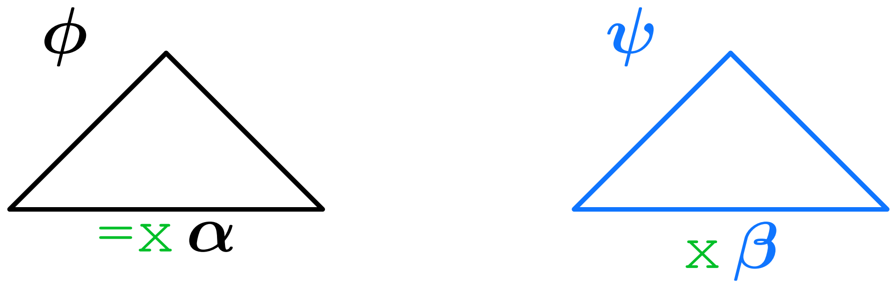

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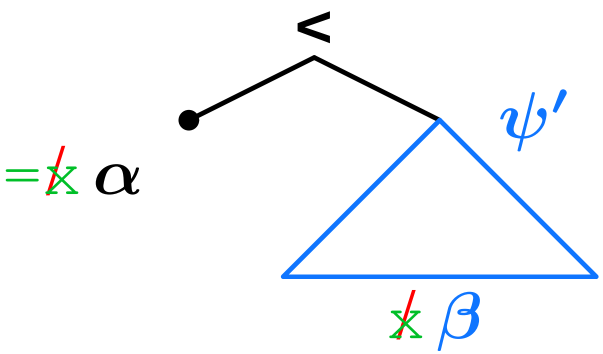

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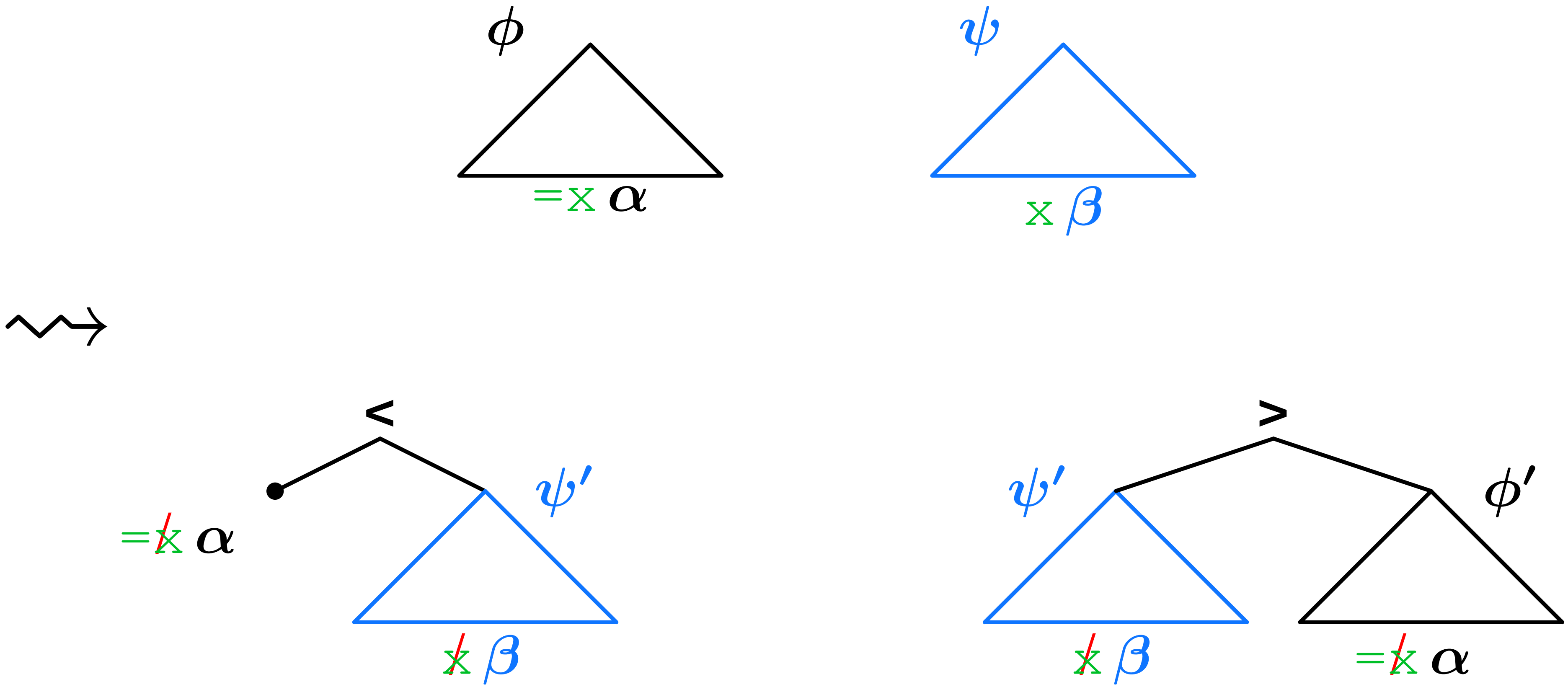
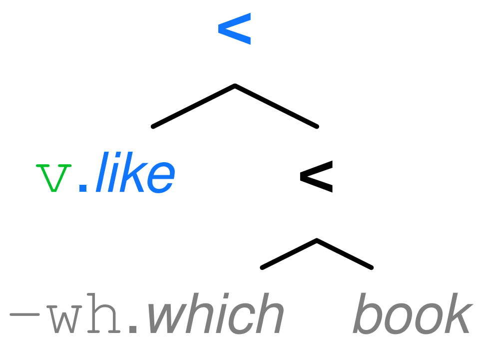


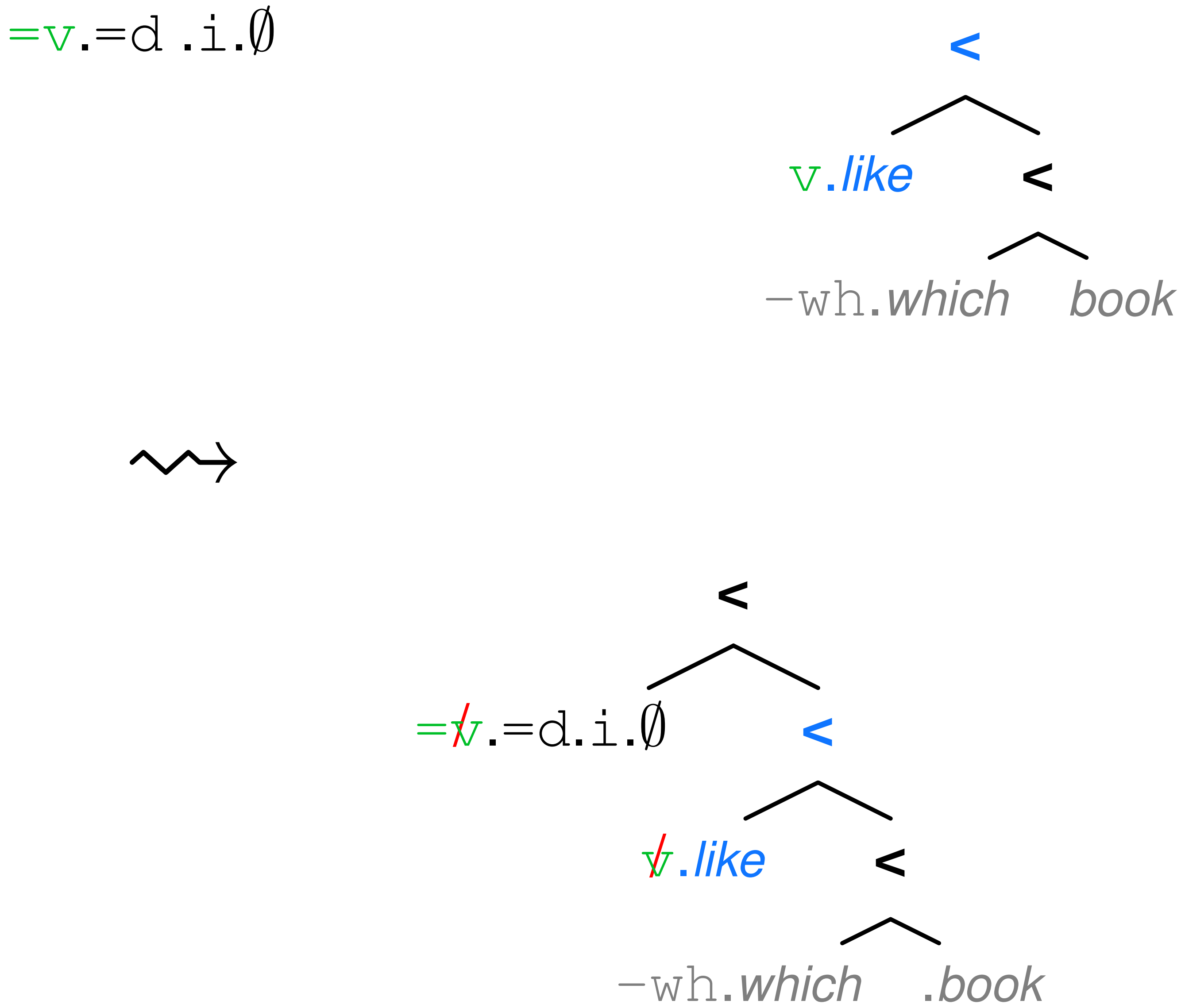
ϕ simple

ϕ complex

merge (selecting tree is simple)

=v.=d.i.∅

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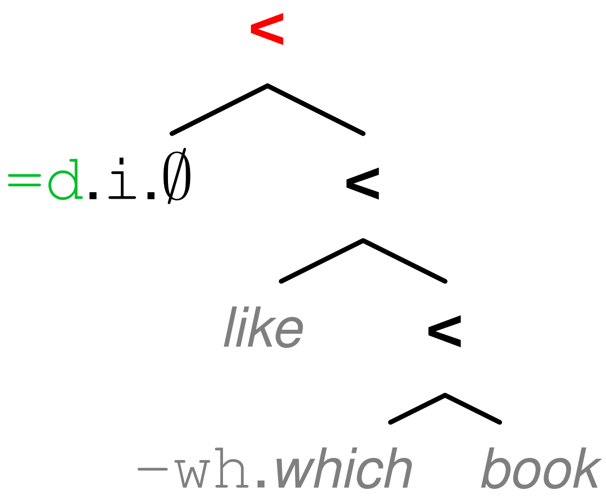
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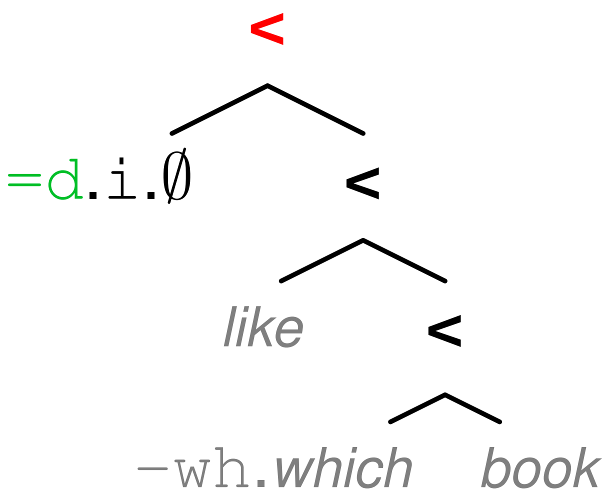
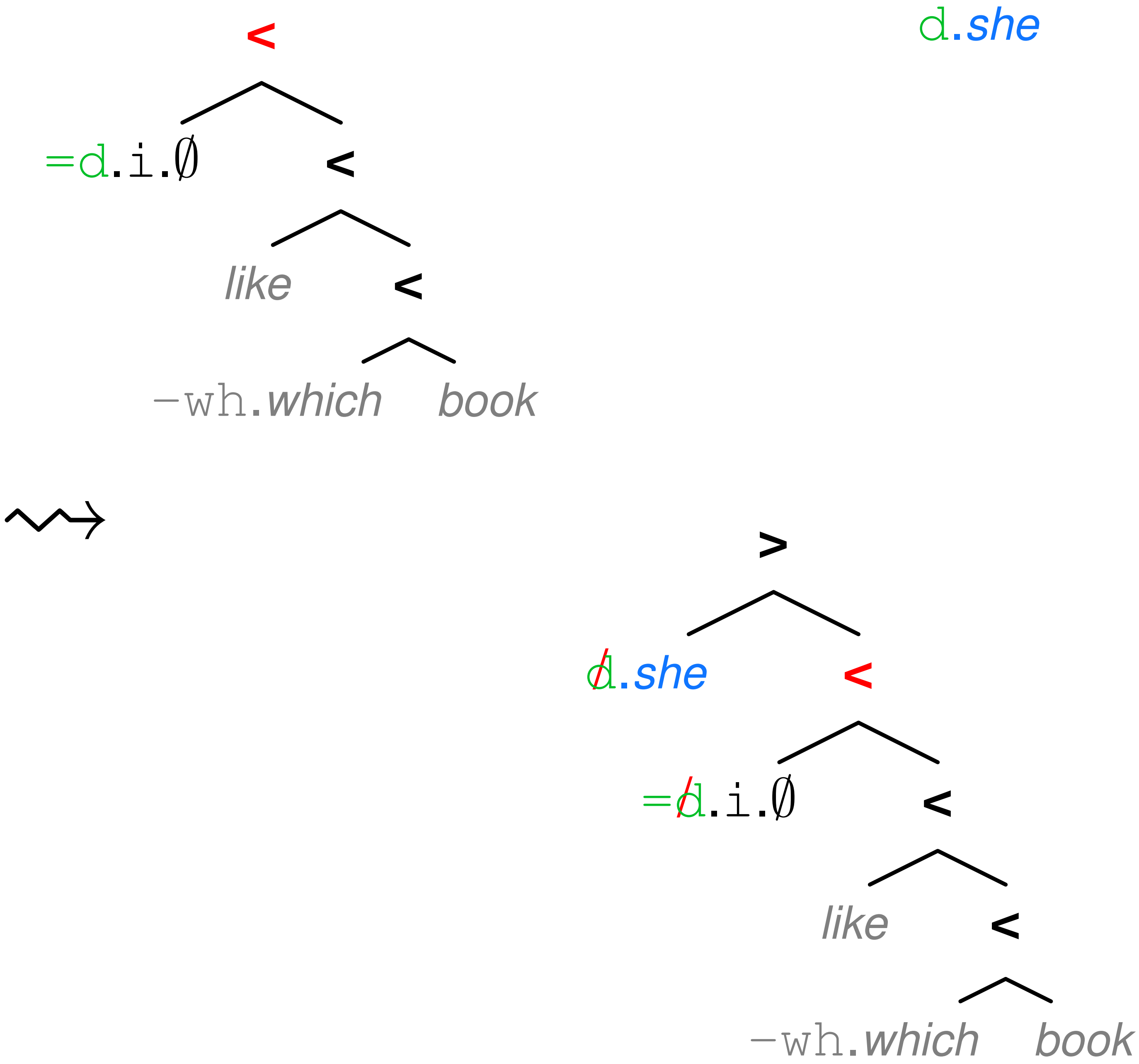
merge (selecting tree is complex)



d.*she*

merge (selecting tree is complex)



Structure building functions

$\mathsf{move} : \mathsf{Exp}(\mathsf{Feat}) \xrightarrow{\text{part}} \mathsf{Exp}(\mathsf{Feat})$

$\phi \in \mathsf{Domain}(\mathsf{move}) \; :\Longleftrightarrow$

- ϕ displays feature $+\mathtt{x} \in$ Licensors
- there is exactly one maximal projection ψ within ϕ that displays feature $-\mathtt{x} \in$ Licensees (SMC)

Structure building functions

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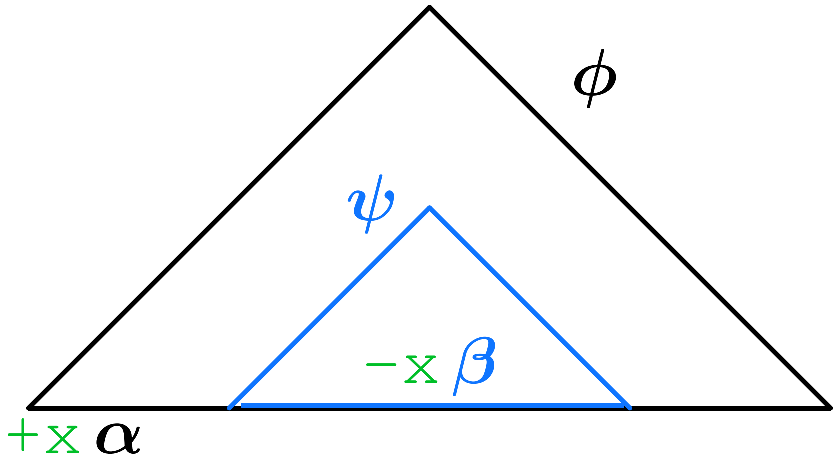

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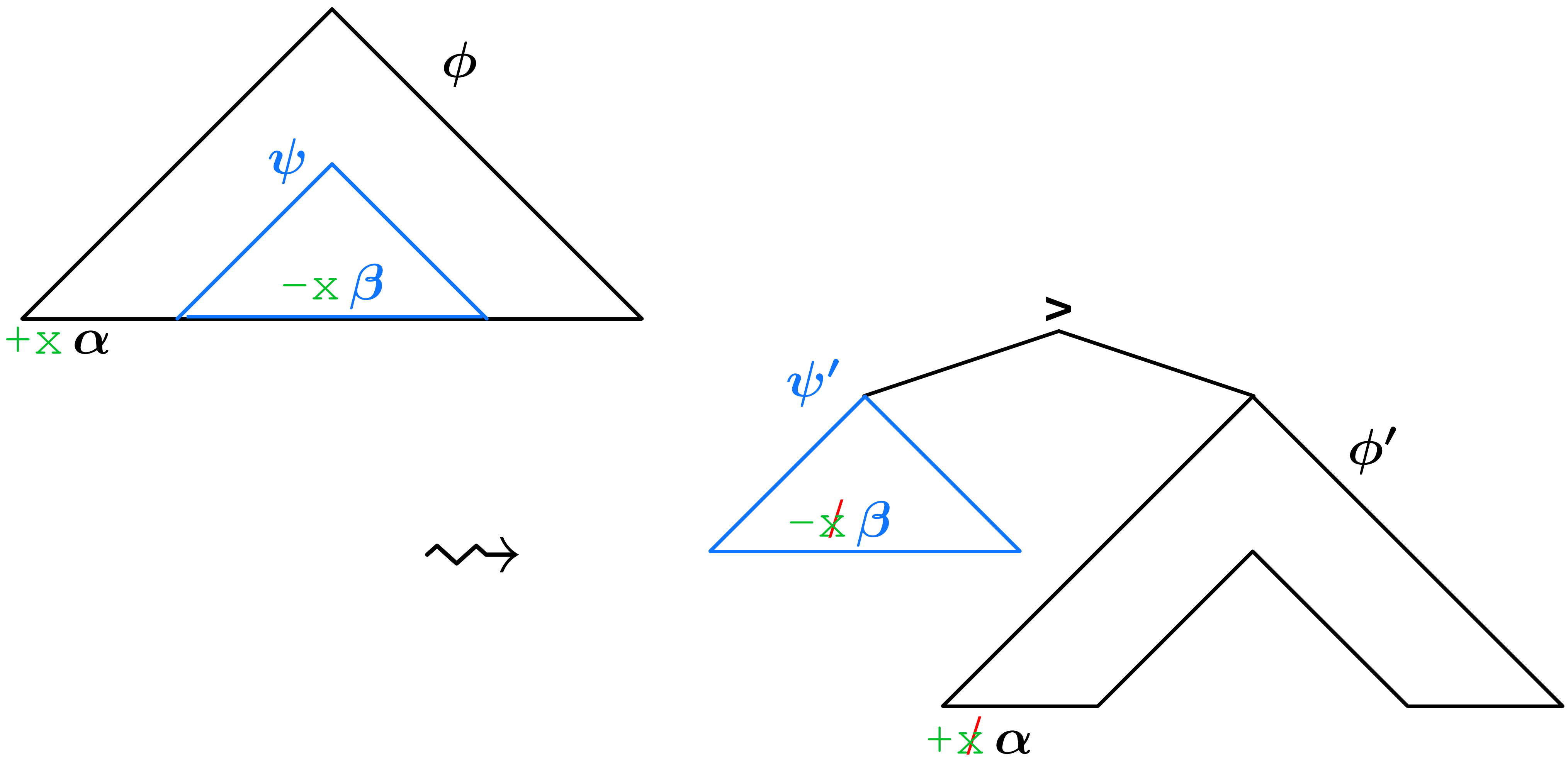
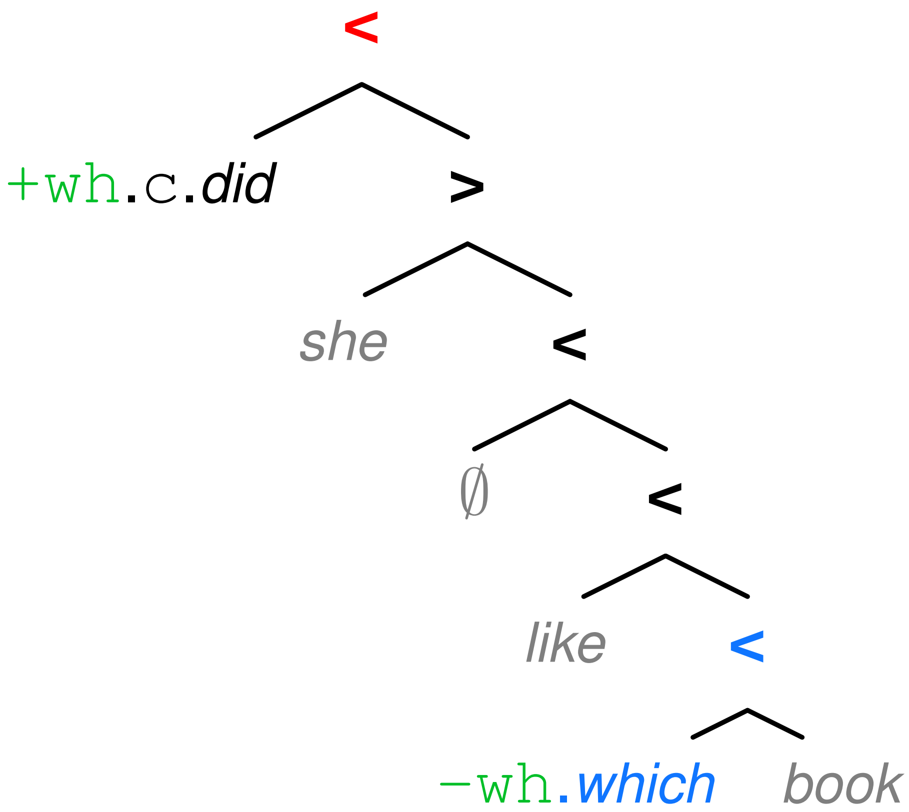
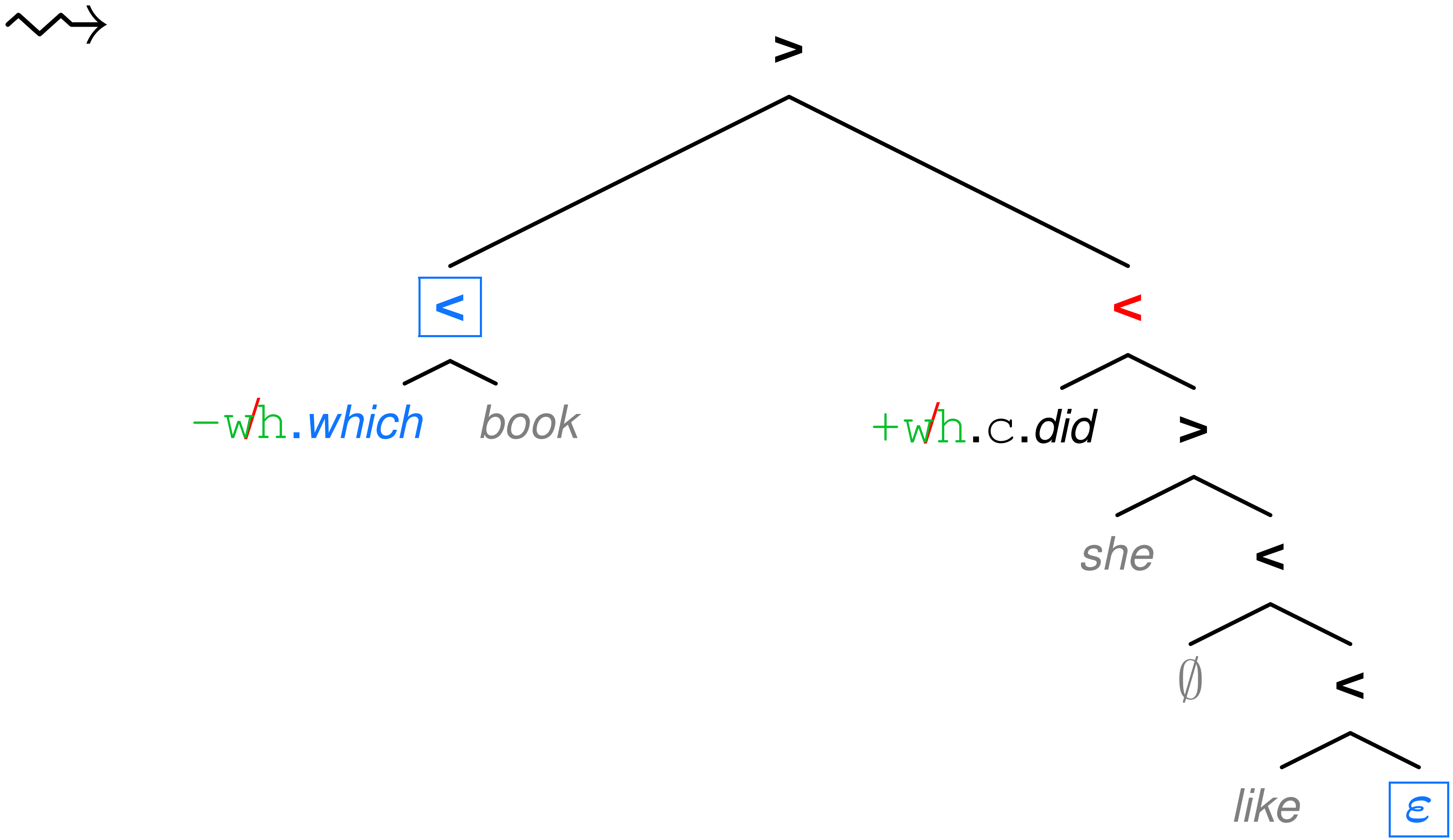


move

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Base	Select	Licensees	Licensors
x	=x	−x	+x

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- Lex is a finite set of simple expressions with labels from $\text{Syn}^*\text{NonSyn}^*$ [lexicon]

An MG-lexicon

n.*book*	d.*she*	=d.v.*like*
=n.d.−wh.*which*	=v.=d.i.∅	=i.c.*that*
=i.+wh.c.*did*	i.*Mary_ read*	

$$\text{NonSyn}^* = \{\, \textit{book}\,,\ \textit{which}\,,\ \textit{Mary\_ read}\,,\ \ldots\,,\ \emptyset\,,\ \ldots \}$$

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  Base: x — Select: $=x$ — Licensees: $-x$ — Licensors: $+x$

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- $c \in \text{Base}$

Closure(G), the closure of an MG $G = \langle \mathrm{Feat}, \mathrm{Lex}, \mathbf{\Omega}, \mathrm{c} \rangle$,

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no leaf-label has an unchecked instance of a syntactic feature, except for the head-label, which is from $\{\mathrm{c}\}\,\mathrm{NonSyn}^*$.

Minimalist languages

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The tree and string language generated by G

$$T(G) = \{ \tau \mid \tau \in \text{Closure}(G) \text{ and complete} \}$$

$$L(G) = \{ \text{yield}_{\text{NonSyn}}(\tau) \mid \tau \in T(G) \}$$

Generative capacity

- MGs are weakly equivalent to LCFRSs

  (Michaelis 2001a, 2001b; Harkema 2001)

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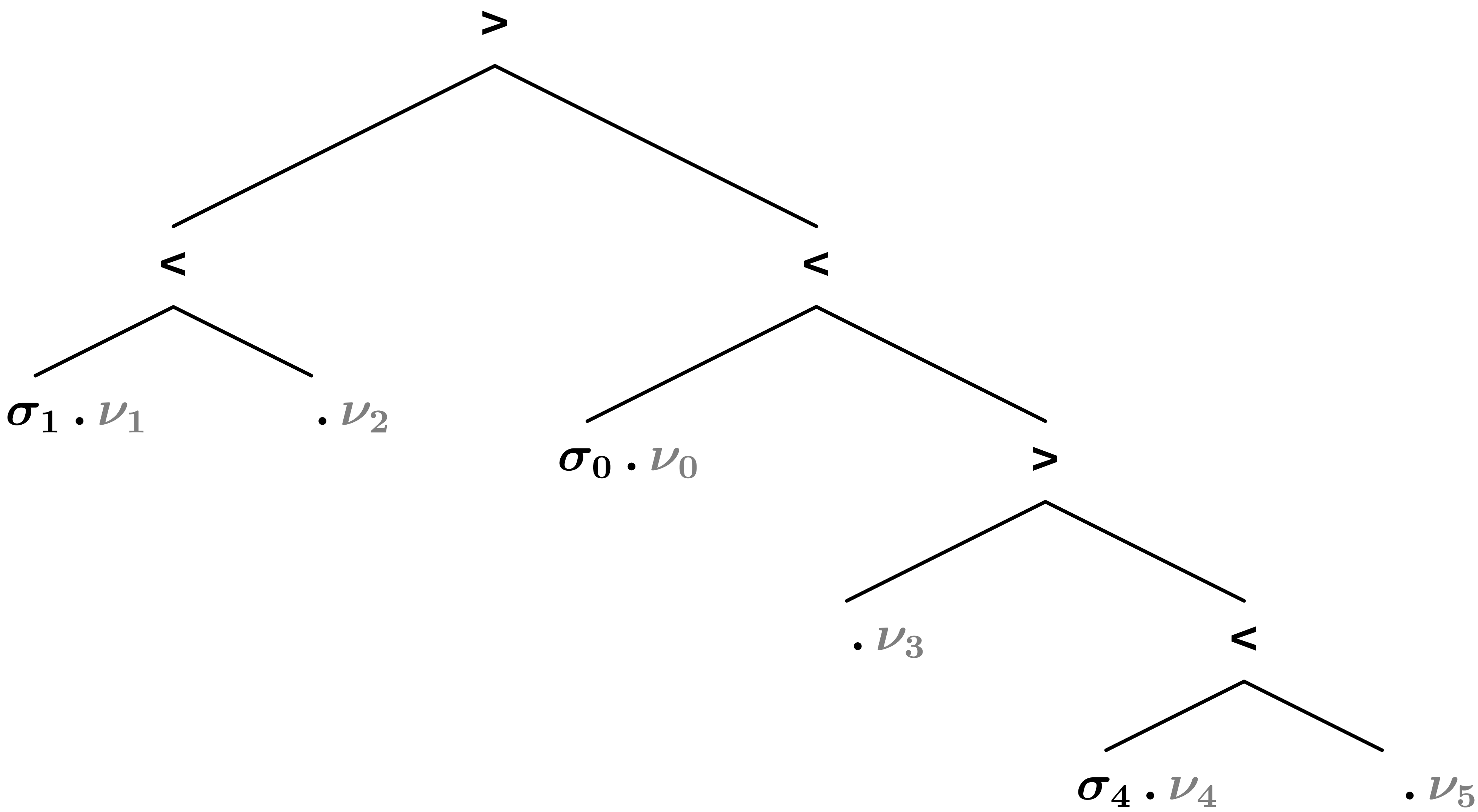
  Thus, in particular, for each MG there is an LCFRS deriving the same string language. This can be shown applying the methods which were developed in Michaelis 2001a for exactly this purpose, and which led to the succinct, chain-based MG-reformulation presented in Stabler & Keenan 2000 — reducing “classical” MGs to their “bare essentials.”

Reducing an MG



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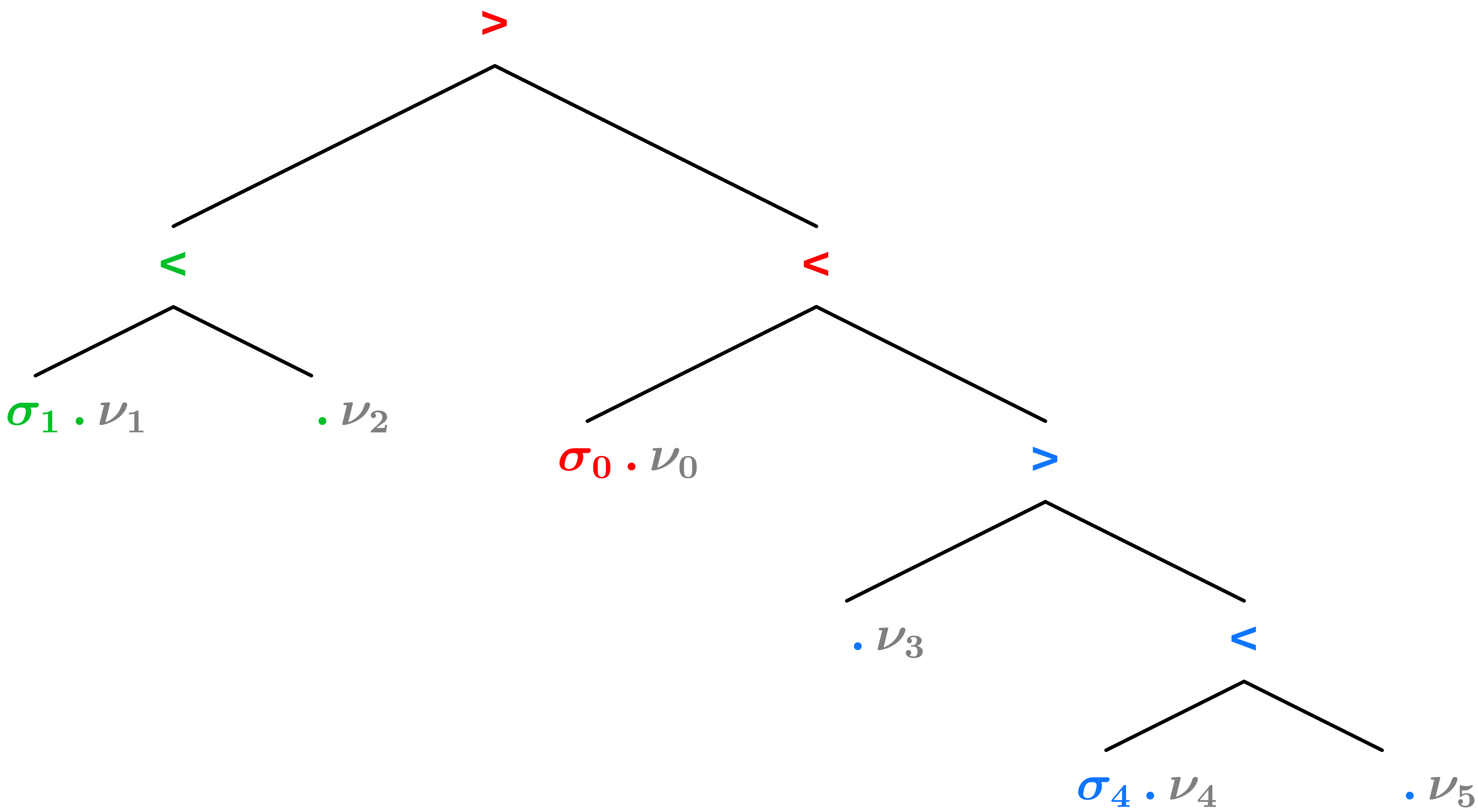

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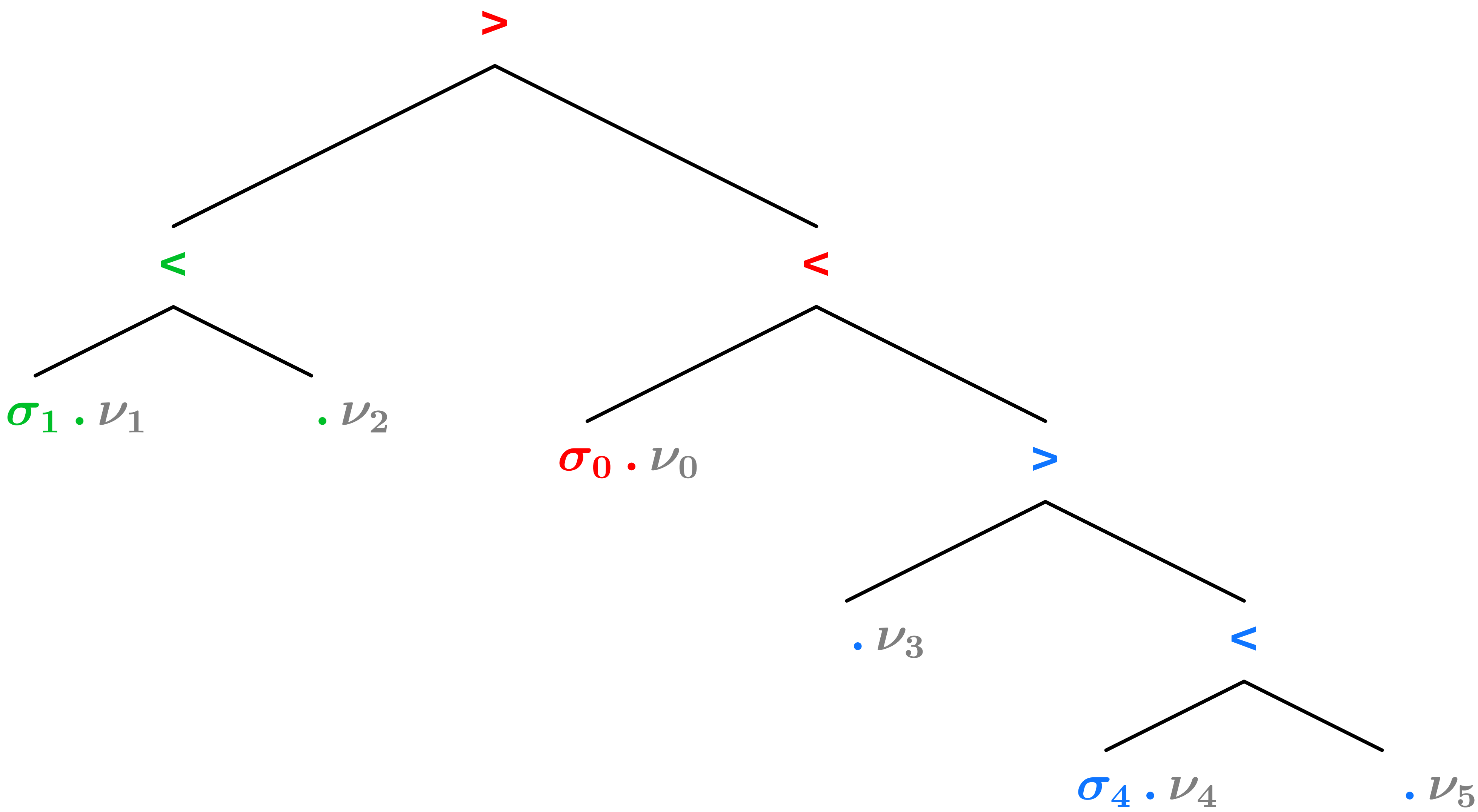

$$\left\langle \sigma_0 . \nu_0 \;,\; \sigma_1 . \nu_1 \nu_2 \;,\; \sigma_4 . \nu_3 \nu_4 \nu_5 \right\rangle$$

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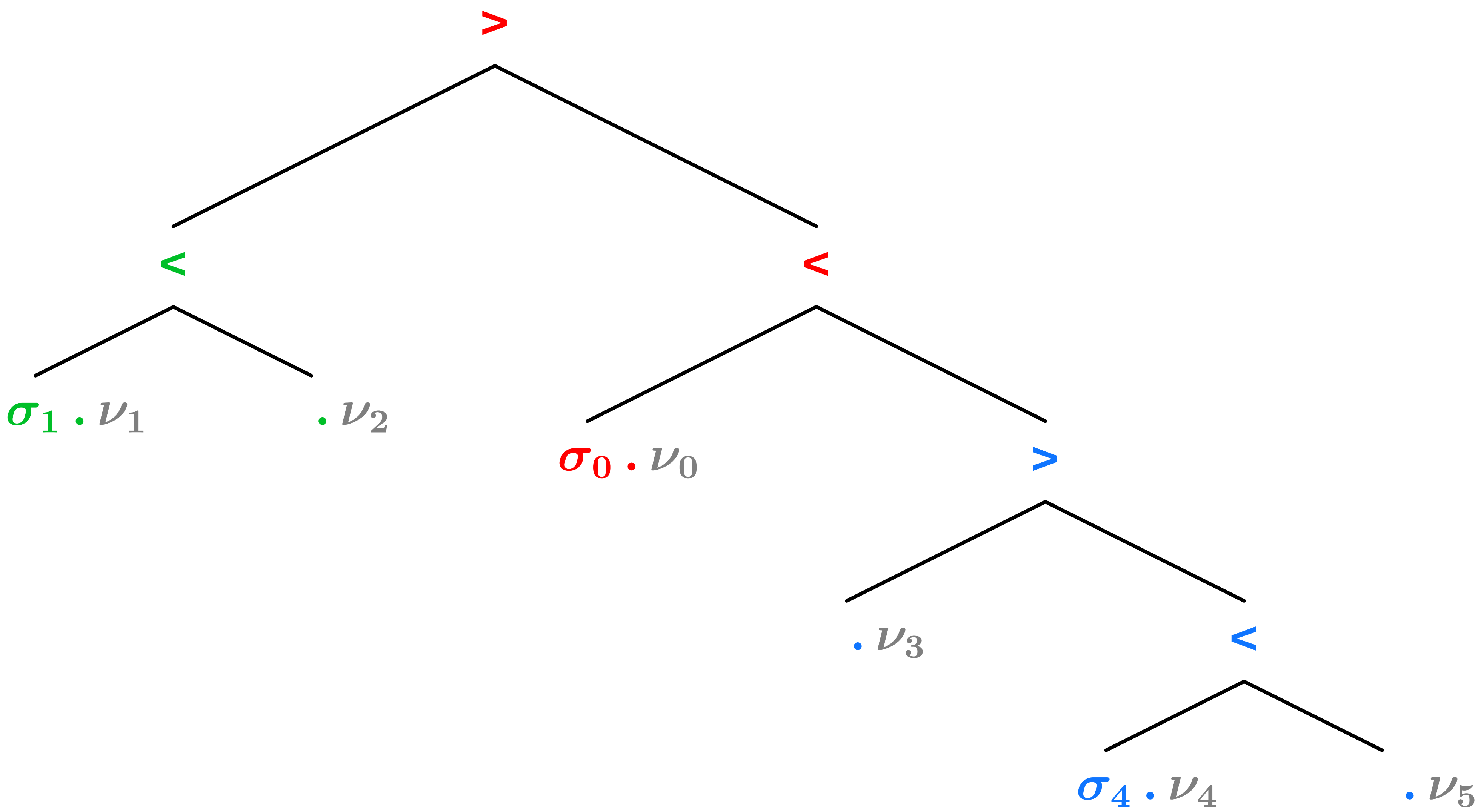

$$\left\langle\ \sigma_0 \cdot \quad ,\ \sigma_1 \cdot \quad ,\ \sigma_4 \cdot \quad \right\rangle$$

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In fact, this kind of structure is characteristic of each $\tau \in \mathrm{Closure(G)}$ involved in creating a complete expression in G. Recall that 'move' is defined only in case that there is exactly one maximal projection in τ that has a particular licensee feature allowing the projection's "movement into a specifier position."

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$\text{RClosure}(G)$ is the set of all relevant $\tau \in \text{Closure}(G)$

A finite partition of RClosure(G)

Basic idea : consider $\tau \in \text{RClosure}(G)$

- Reduce τ to a tuple such that for each maximal projection displaying an unchecked syntactic feature, there is exactly one component of the tuple consisting of the projection's head-label, but with the suffix of non-syntactic features truncated.

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Relevance :
The resulting tuple has at most m+1 components , $m = |\text{Licensees}|$.

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Each tuple component is the suffix of the syntactic prefix of the label of a lexical item.

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$\rightsquigarrow$ regarding the partition, applications of 'merge' and 'move' do not depend on the chosen representatives

A weakly equivalent LCFRS

- The nonterminating rules :

  (1) $\mathsf{T} \longrightarrow \mathsf{merge}_{\mathsf{U},\mathsf{V}}(\mathsf{U},\mathsf{V})$

  T , U and V the “representatives” of some $\boldsymbol{\tau}$, $\boldsymbol{\upsilon}$ and $\boldsymbol{\phi} \in$ RClosure , respectively, such that $\boldsymbol{\tau} =$ merge$(\boldsymbol{\upsilon}, \boldsymbol{\phi})$.

  (2) $\mathsf{T} \longrightarrow \mathsf{move}_{\mathsf{U}}(\mathsf{U})$

  T and U the “representatives” of some $\boldsymbol{\tau}$ and $\boldsymbol{\upsilon} \in$ RClosure , respectively, such that $\boldsymbol{\tau} =$ move$(\boldsymbol{\upsilon})$.

- The terminating rules :

  (3) $\mathsf{T} \longrightarrow \boldsymbol{\nu}$

  T the “representative” of $\boldsymbol{\tau} \in$ Lex with label $\boldsymbol{\sigma\nu}$, where $\boldsymbol{\sigma} \in$ Syn , and $\boldsymbol{\nu} \in$ NonSyn .

- "Reconstruction" of the non-syntactic material is possible by means of the regular functions of the LCFRS :

$$\mathrm{T} \overset{*}{\underset{G}{\Longrightarrow}} \langle \nu_0, \ldots, \nu_{\mathrm{m}} \rangle \in \mathrm{Strings(NonSyn)}^{\mathrm{m+1}}$$

(a) T the "representative" of $\tau \in$ RClosure(G) .

(b) For $\{ \mathrm{l}_1 , \ldots \mathrm{l}_{\mathrm{m}} \}$, an enumeration of Licensees,

ν_0 is the "non-extractable" part of the non-syntactic yield of τ , i.e. , that part of the non-sytactic yield of τ which by no means would be pied-piped if some proper subtree of τ became subject to movement.

For $1 \leq \mathrm{i} \leq \mathrm{m}$,

if there is no subtree of τ displaying licensee l_i , then $\nu_\mathrm{i} = \varepsilon$.

Otherwise , ν_i is the "non-extractable" part of the non-syntactic yield of τ_i , the subtree of τ displaying licensee l_i .

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