

An Introduction to
Mildly Context-Sensitive Grammar Formalisms

— *Tree Adjoining Grammars* —

Gerhard Jäger & Jens Michaelis
Universität Potsdam

{jaeger,michael}@ling.uni-potsdam.de

Mild context-sensitivity (Joshi 1985)

- a **concept** motivated by the intention of characterizing a narrow class of formal grammars which are “**only slightly more powerful than CFGs**,” and which **nevertheless** allow for descriptions of natural languages in a **linguistically significant** way.

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According to Joshi (1985, p. 225) a *mildly context-sensitive language*, L , has to fulfil **three criteria**, to be understood as a “**rough characterization**.” Somewhat paraphrased, these are:

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According to Joshi (1985, p. 225) a *mildly context-sensitive language*, L , has to fulfil **three criteria**, to be understood as a “**rough characterization**.” Somewhat paraphrased, these are:

- (1) the **parsing** problem for L is solvable in **polynomial time**,
- (2) L has the **constant growth property**, and
- (3) there is a finite **upper bound** for L **limiting** the number of **different instantiations of factorized cross-serial dependencies** occurring in a sentence of L .

- A collection of **mildly context-sensitive grammar (MCSG)** formalisms is presented in Joshi et al. 1991:
- ◆ **tree adjoining grammars (TAGs)** (Joshi et al. 1975; Joshi 1985)
- ◆ (restricted) **combinatory categorial grammars (CCGs)** (as formalized e.g. in Weir & Joshi 1988 in accordance with the CCG-version developed in Steedman 1987, 1990)
- ◆ **linear indexed grammars (LIGs)** as they arise from Gazdar 1988
- ◆ **head grammars (HGs)** (Pollard 1984)
- ◆ **multicomponent TAGs (MCTAGs)** (Joshi 1987; Vijay-Shanker et al. 1987) as a generalization of TAGs
- ◆ **linear context-free rewriting systems (LCFRSs)** (Vijay-Shanker et al. 1987) as a generalization of HGs, or, likewise, as a restriction of generalized CFGs (GCFGs) (Pollard 1984)

Mild context-sensitivity

- TAGs, CCGs, LIGs and HGs are weakly equivalent (see e.g. Vijay–Shanker & Weir 1994)
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^a The weak equivalence to LIGs, CCGs and HGs holds for TAGs with local constraints (on tree adjoining) as formally introduced e.g. in Vijay–Shanker & Joshi 1985, following a suggestion in Joshi et al. 1975, and capturing the intended use of local constraints (on adjoining) of the kind proposed in Joshi 1985. The class of TAGs with local constraints properly extends the strong as well as the weak generative capacity of the class of TAGs without such constraints.

^b Note also that HGs as defined e.g. in Vijay–Shanker & Weir 1994 provide a modified version of HGs as originally defined in Pollard 1984. In terms of weak equivalence, HGs of this modified type subsume HGs of the original type, and vice versa. Corresponding proofs can be found in Vijay–Shanker et al. 1986 and Seki et al. 1991, respectively.

^c More precisely, MCTAGs in their *set-local* variant, i.e. MCTAGs which, during the course of a derivation, allow the members of a derived sequence of auxiliary trees to be (simultaneously) adjoined at distinct nodes to the members of a single elementary tree sequence (cf. Definition 2.7.1 in Weir 1988).

Finite labeled trees

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- label_t the **labeling (function)**, a function from N_t into a set of **labels**.

Objects specified by a tree adjoining grammar

V_N a set of **nonterminals**

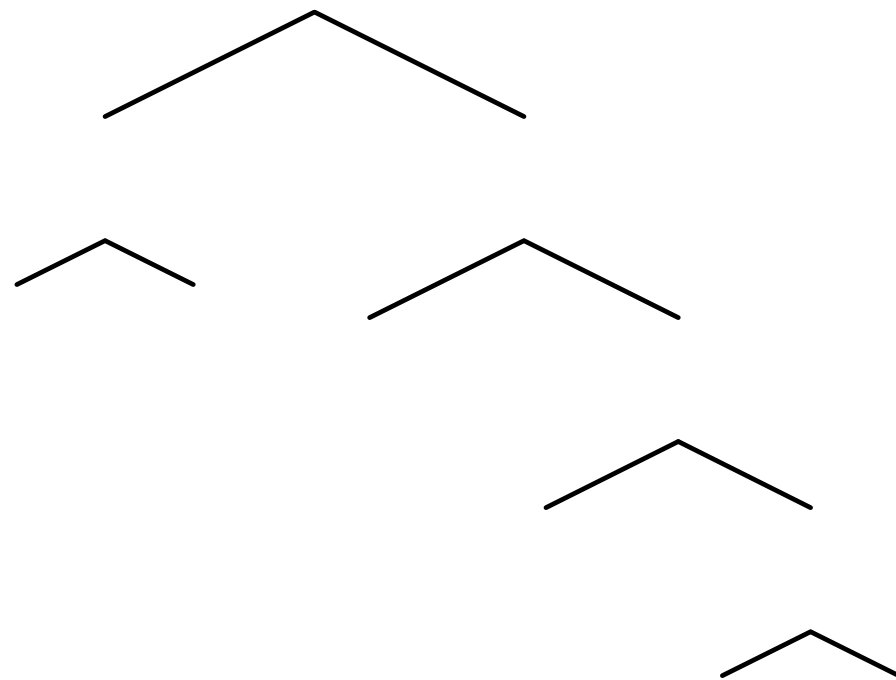
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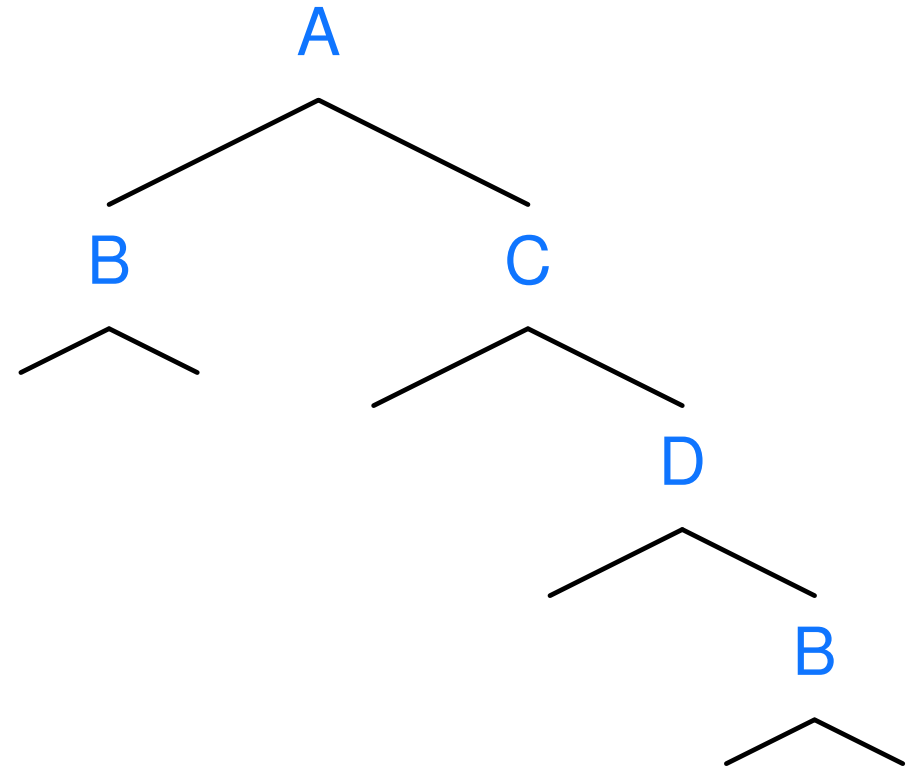
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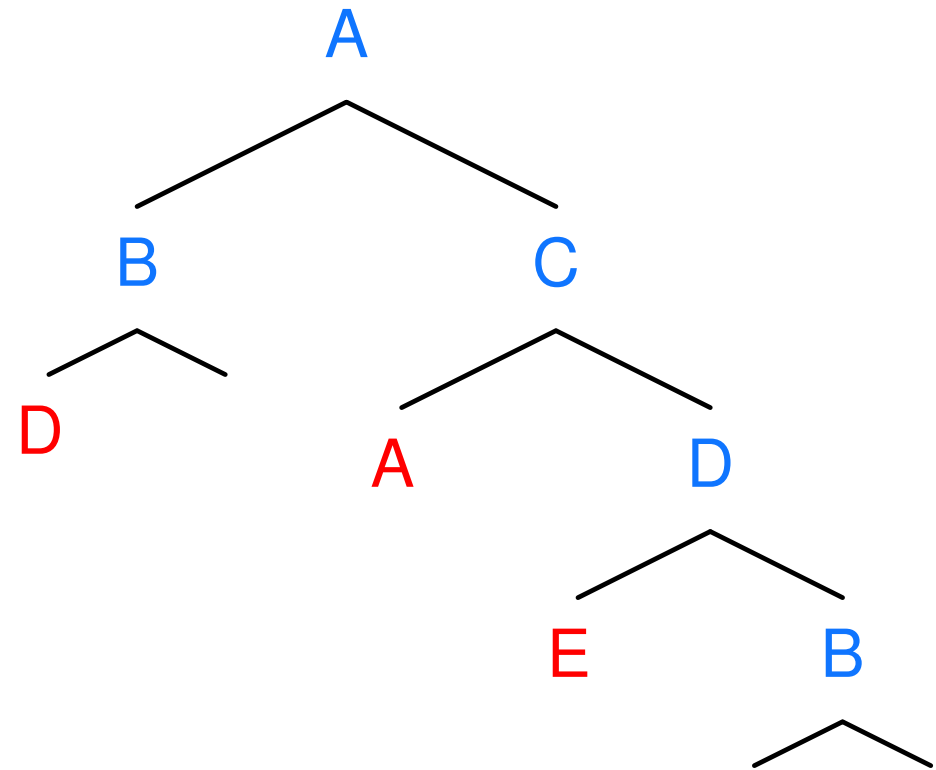
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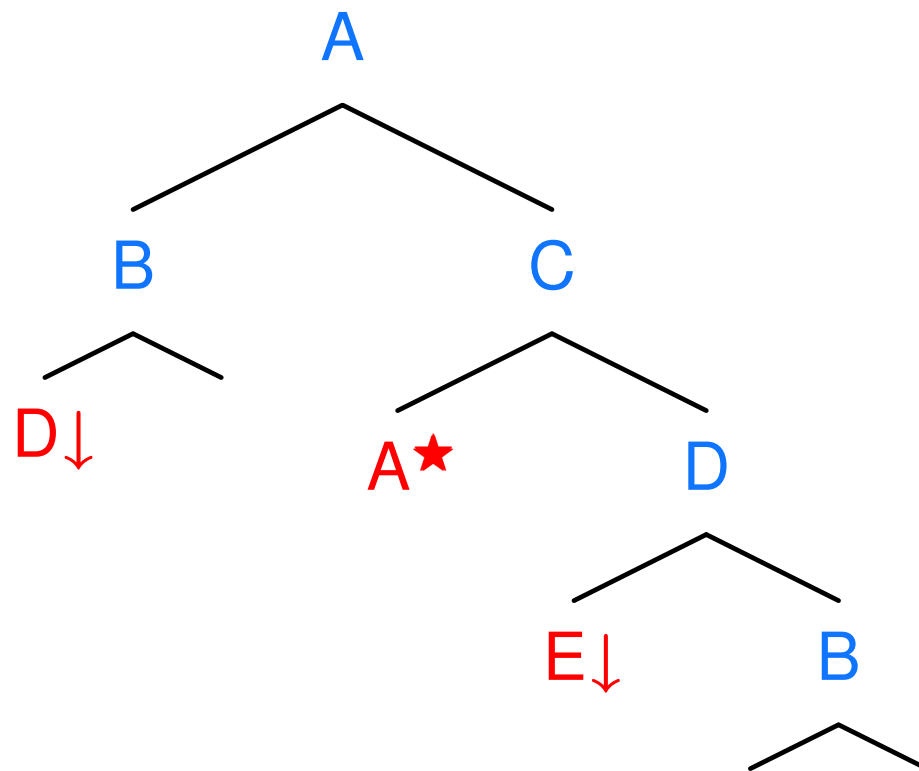
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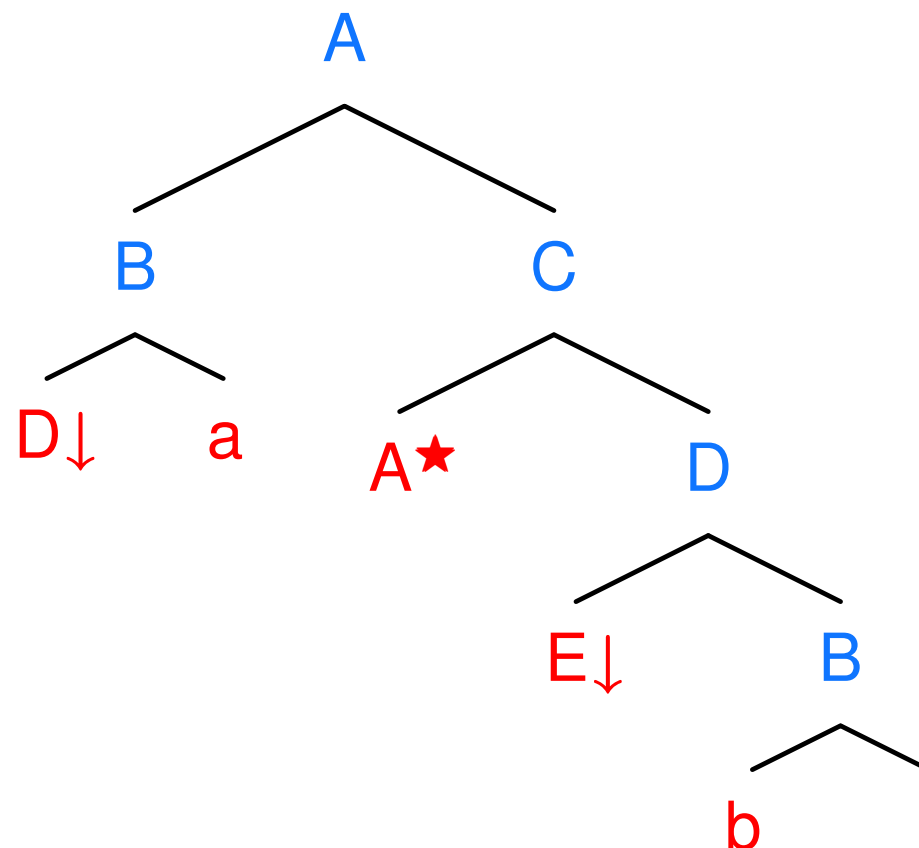
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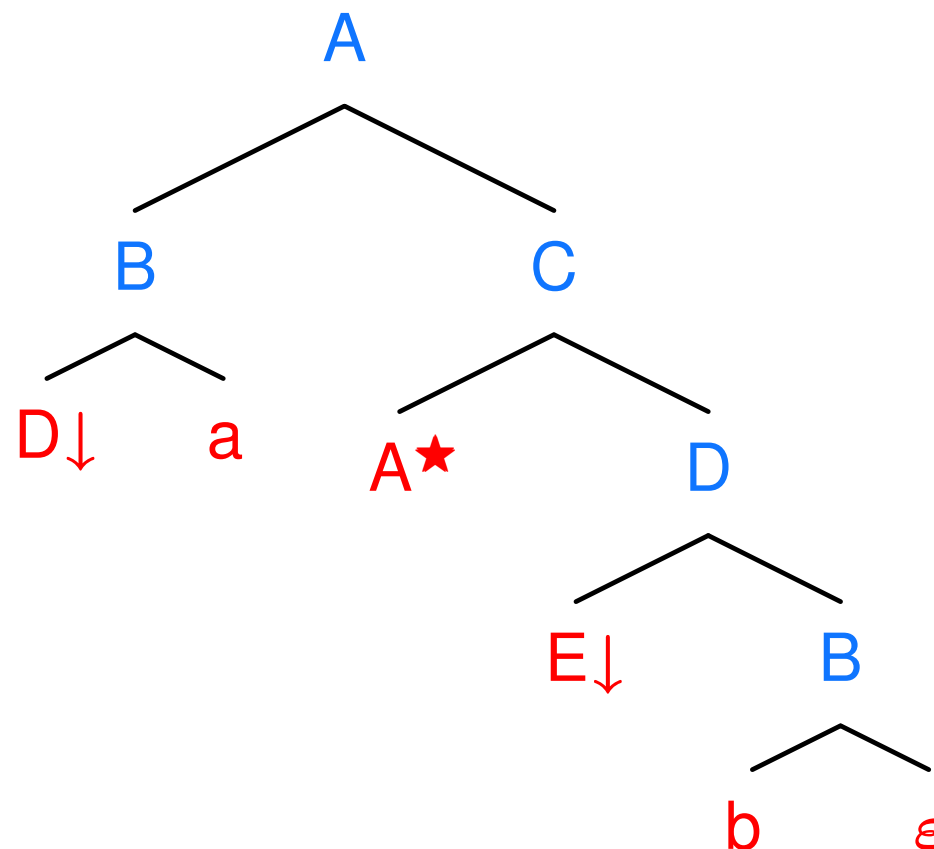
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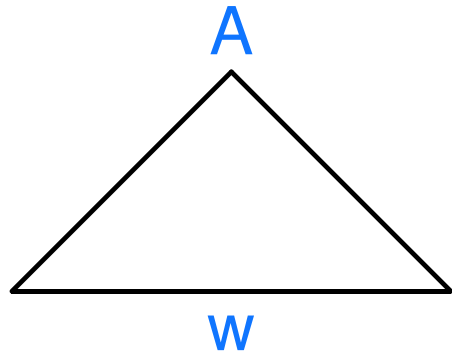
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Deriving (labeled) trees by a tree adjoining grammar

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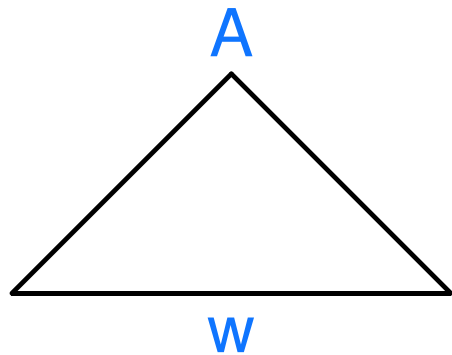


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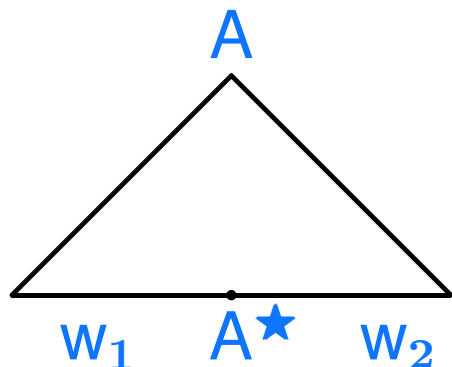
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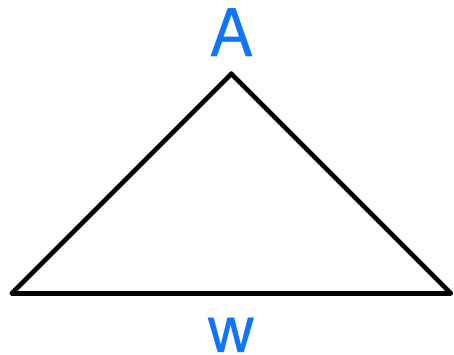


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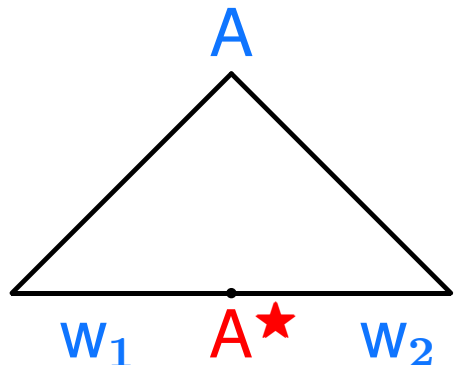
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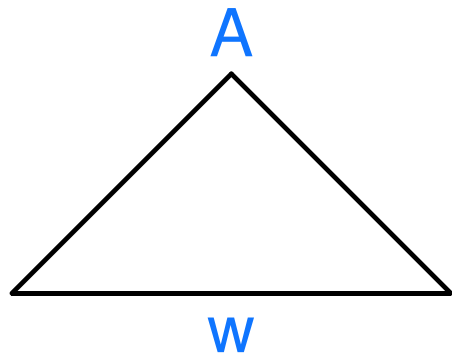
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foot node

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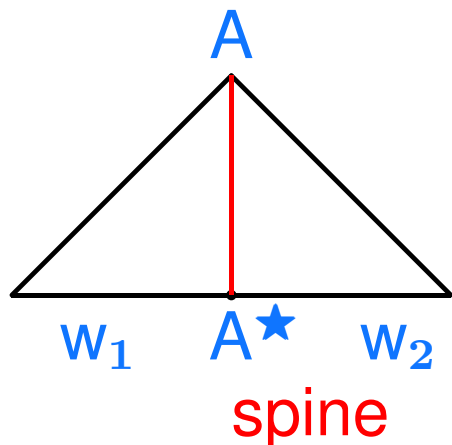
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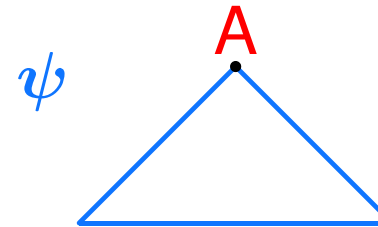
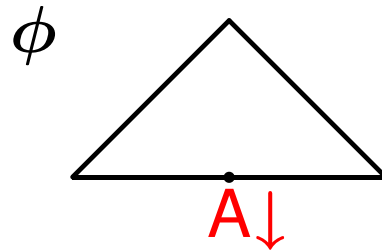
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substitution : $\text{Trees}(V) \times \text{Trees}(V) \xrightarrow{\text{part}} 2^{\text{Trees}(V)}$

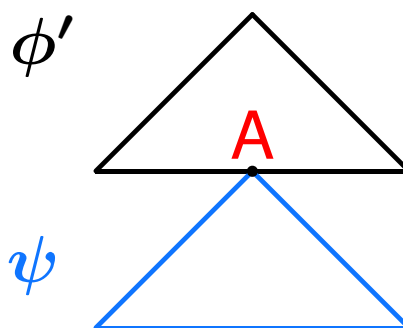
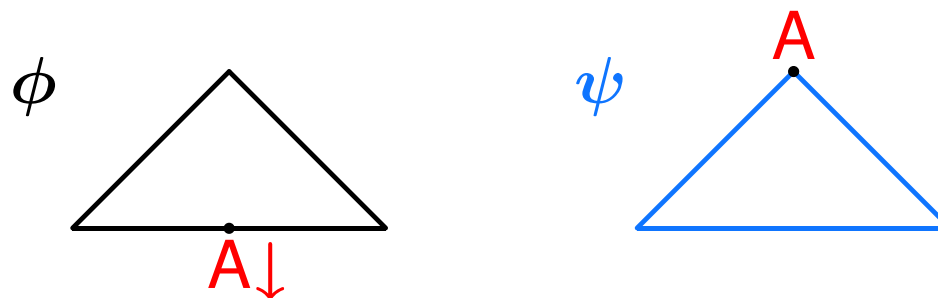
$\langle \phi, \psi \rangle \in \text{Domain}(\text{substitution}) : \iff$

- ϕ has a **leaf** labeled $A \downarrow$ for some $A \in V_N$
- ψ 's **root** is labeled A

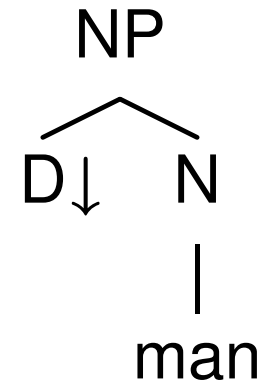
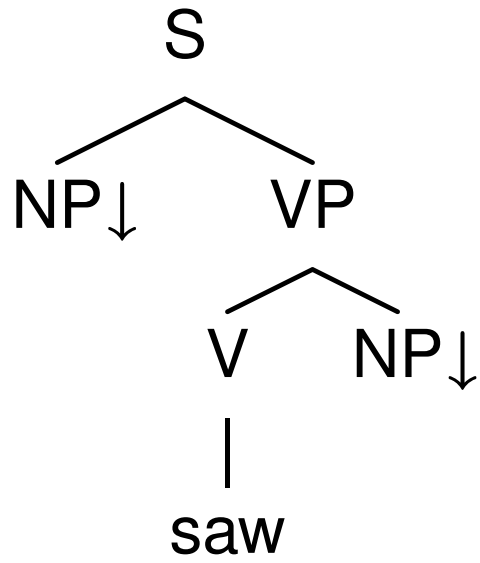
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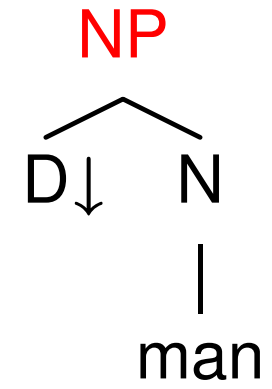
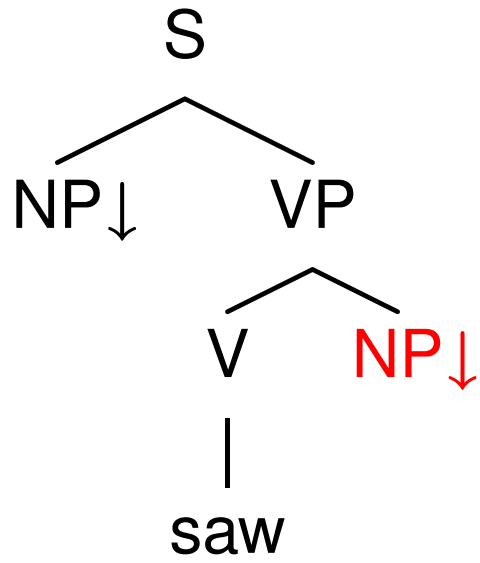
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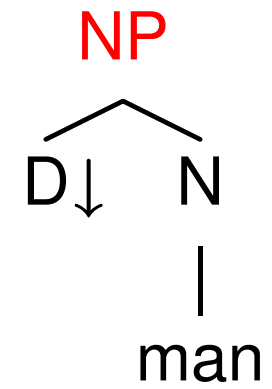
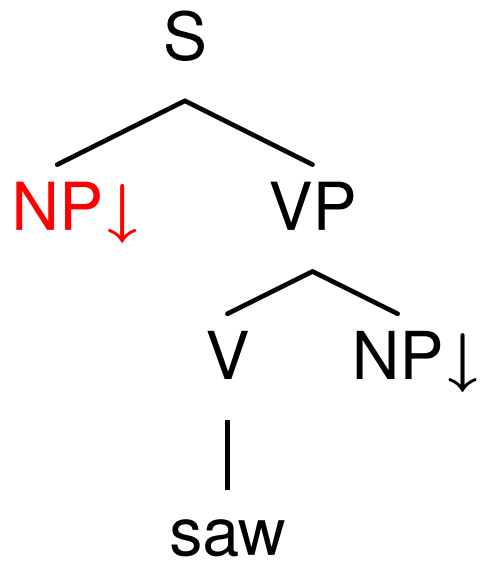
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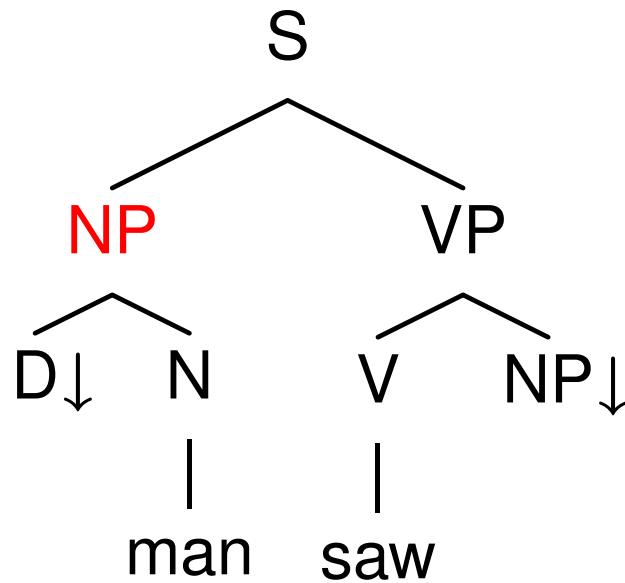
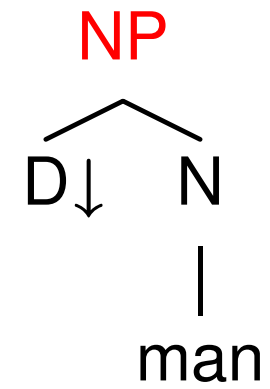
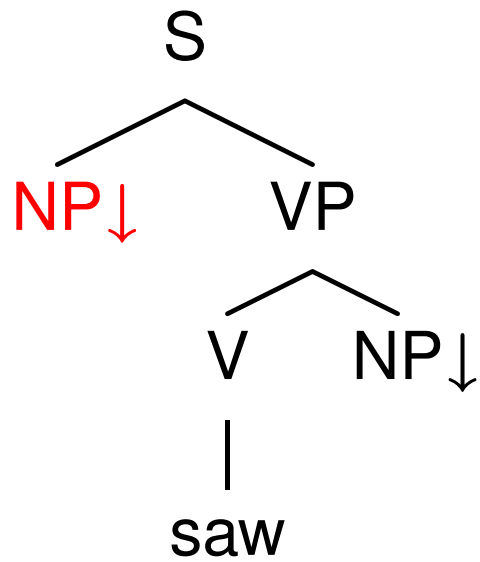
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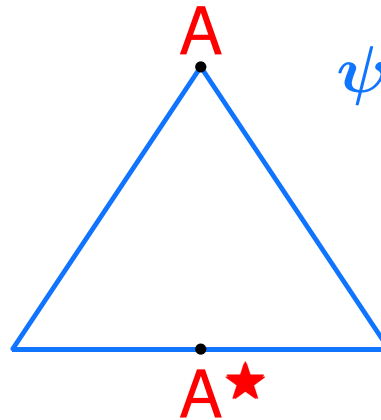
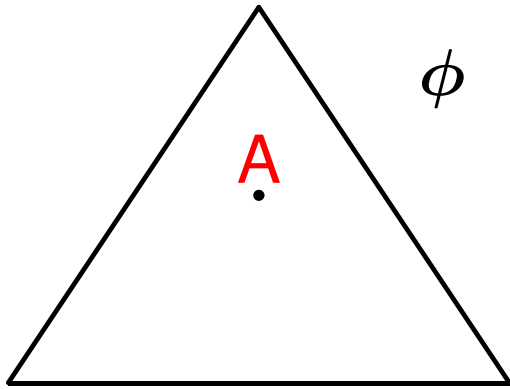


adjoining : $\text{Trees}(V) \times \text{Trees}(V) \xrightarrow{\text{part}} 2^{\text{Trees}(V)}$

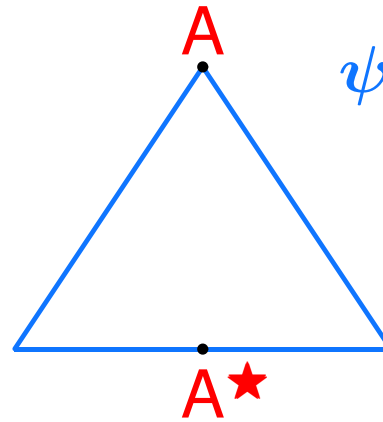
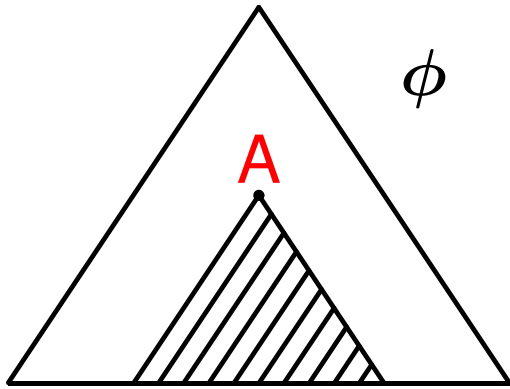
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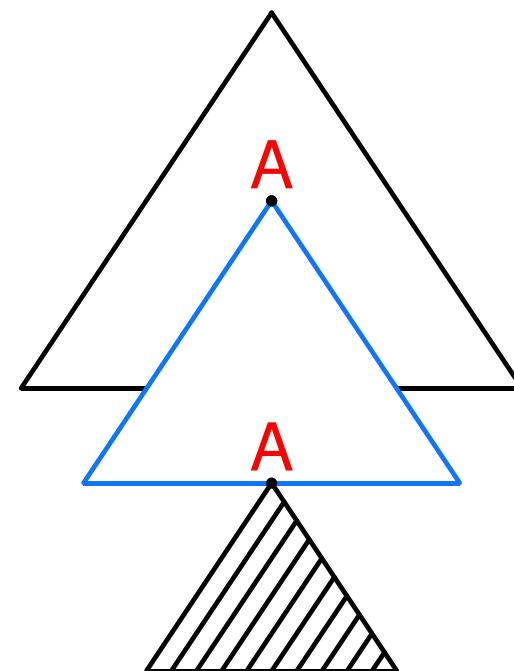
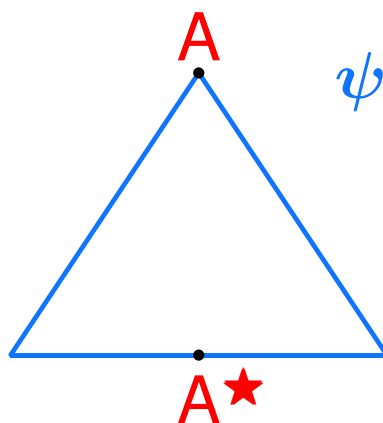
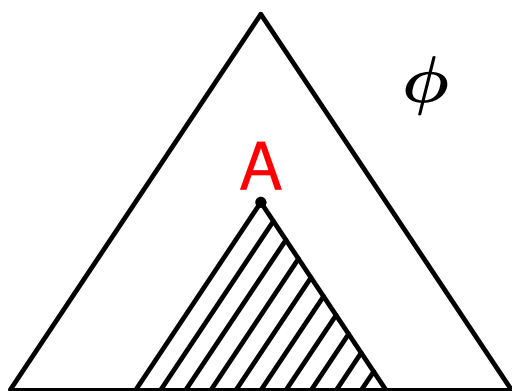
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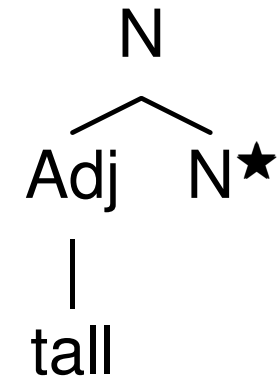
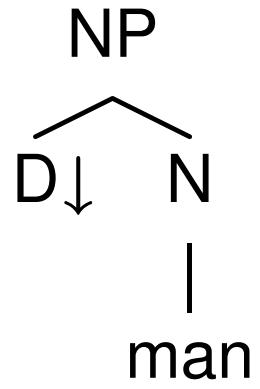
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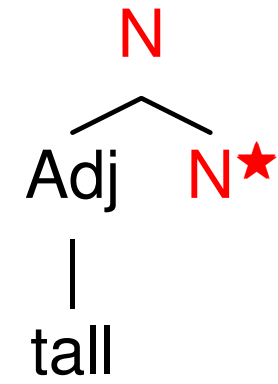
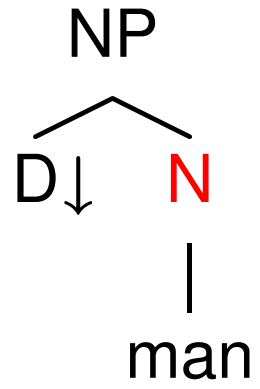
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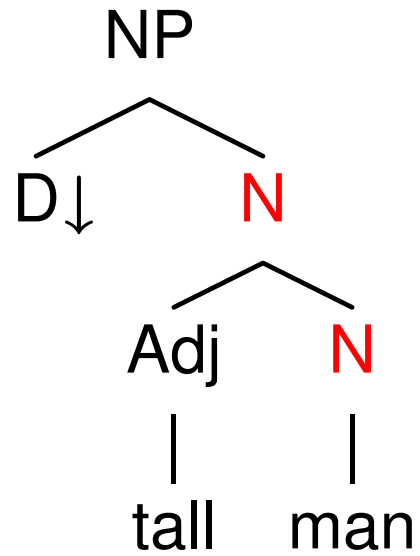
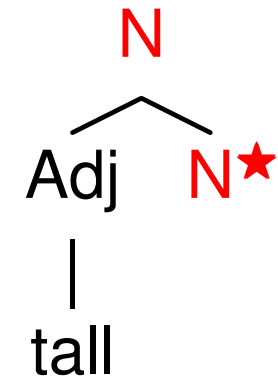
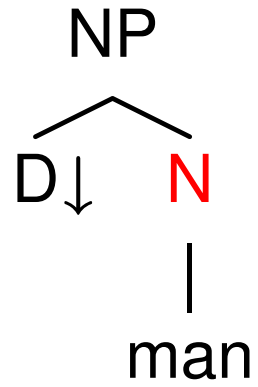
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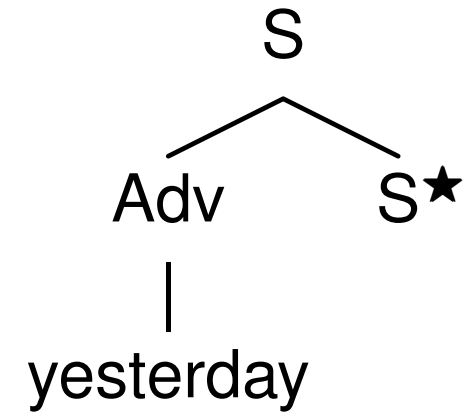
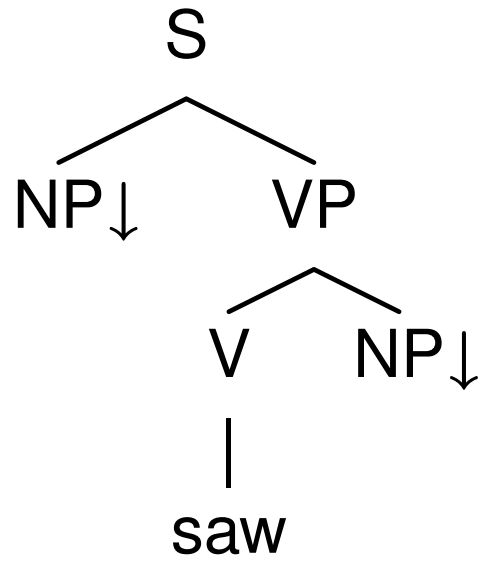
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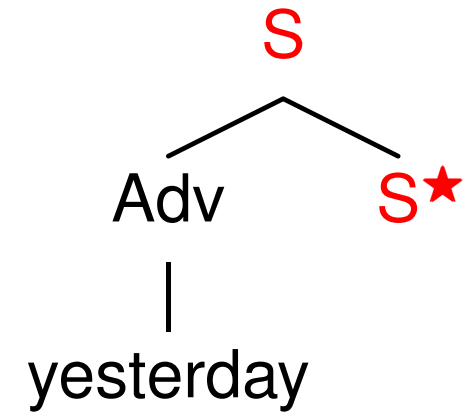
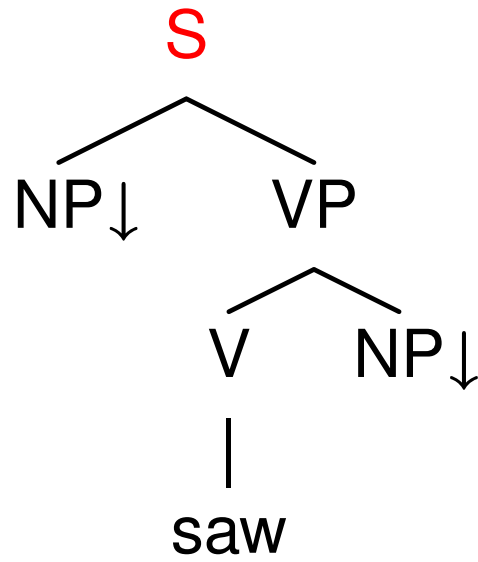
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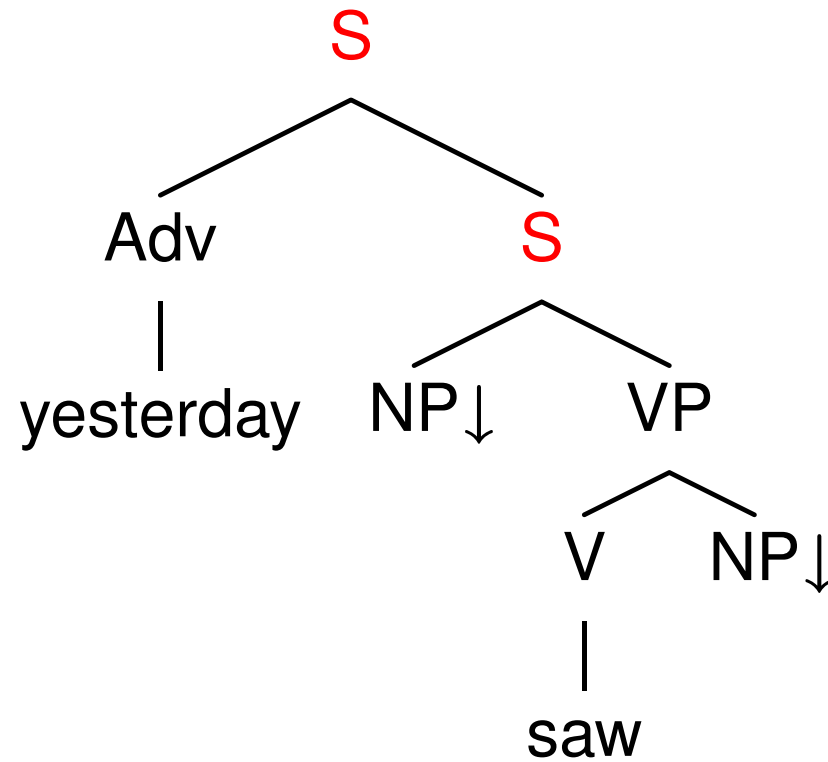
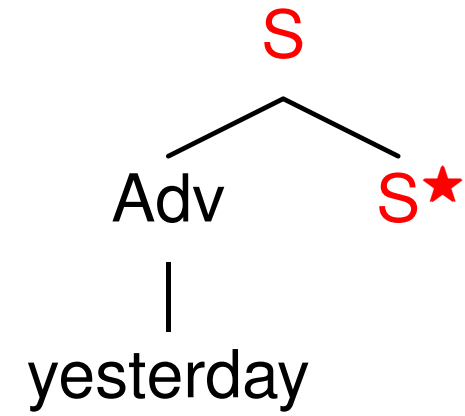
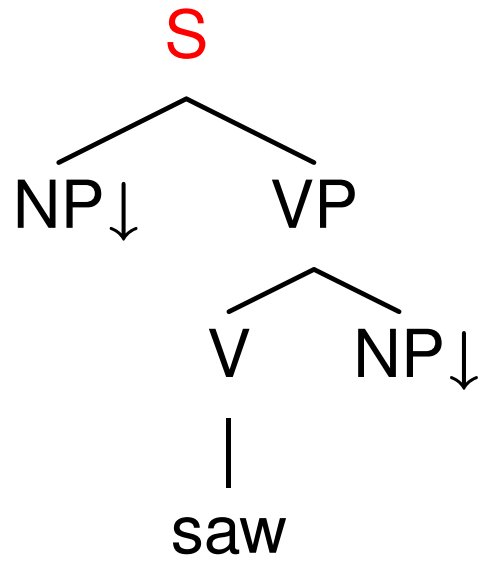
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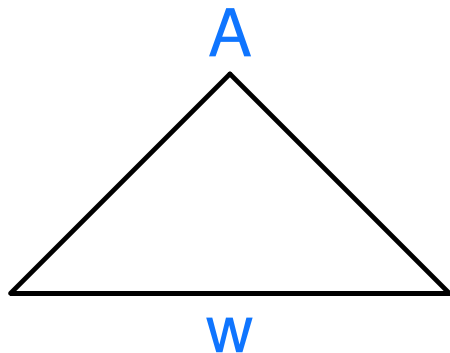
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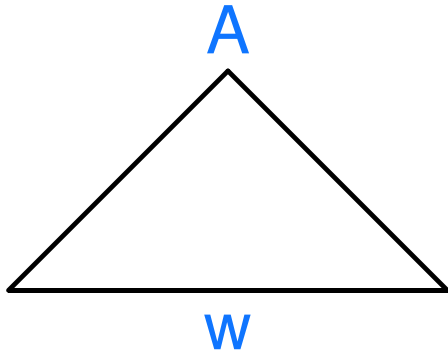
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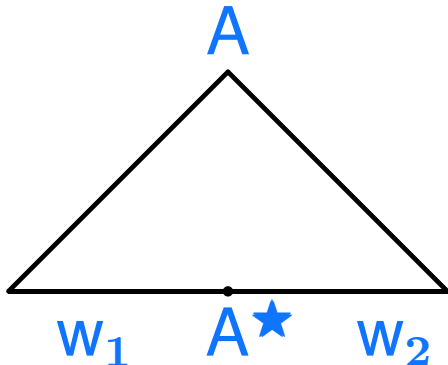


$A \in V_N$

$w \in \text{Strings}(V_N\{\downarrow\} \cup V_T)$

$t \in T_{\text{Aux}}$

t is a finite labeled tree $\langle N_t, \triangleleft_t^*, \prec_t, \text{label}_t \rangle$ such that

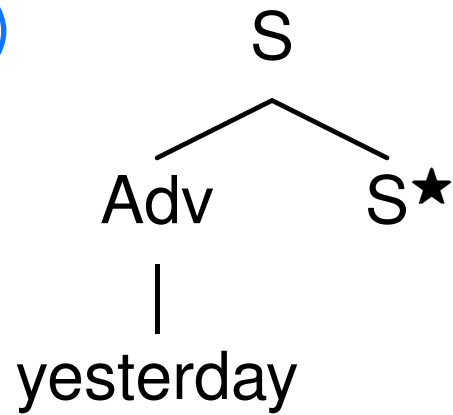


$A \in V_N$

$w_1, w_2 \in \text{Strings}(V_N\{\downarrow\} \cup V_T)$

Elementary trees: examples

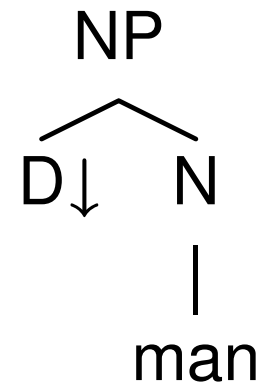
(β_{yest})



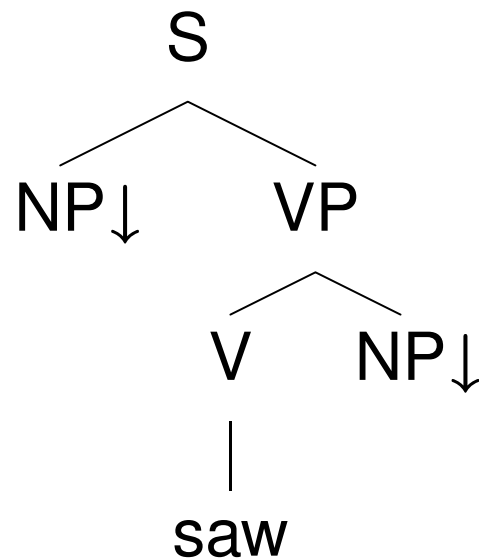
(α_a)



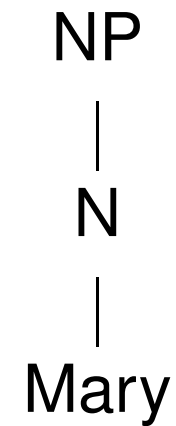
(α_{man})



(α_{saw})



(α_{Mary})



Tree adjoining languages

Closure(G), the **closure** of a TAG $G = \langle V_N, V_T, T_{Ini}, T_{Aux}, S \rangle$,

is the **closure** of $T_{Ini} \cup T_{Aux}$ **under** finitely many applications of **substitution** and **adjoining**.

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The **tree** and **string language** generated by G

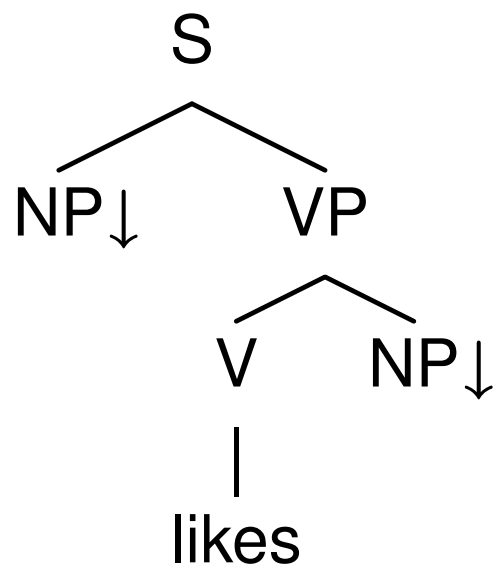
$$T(G) = \{ t \mid t \in \text{Closure}(G) \text{ and complete} \}$$

$$L(G) = \{ \text{yield}(t) \mid t \in T(G) \}$$

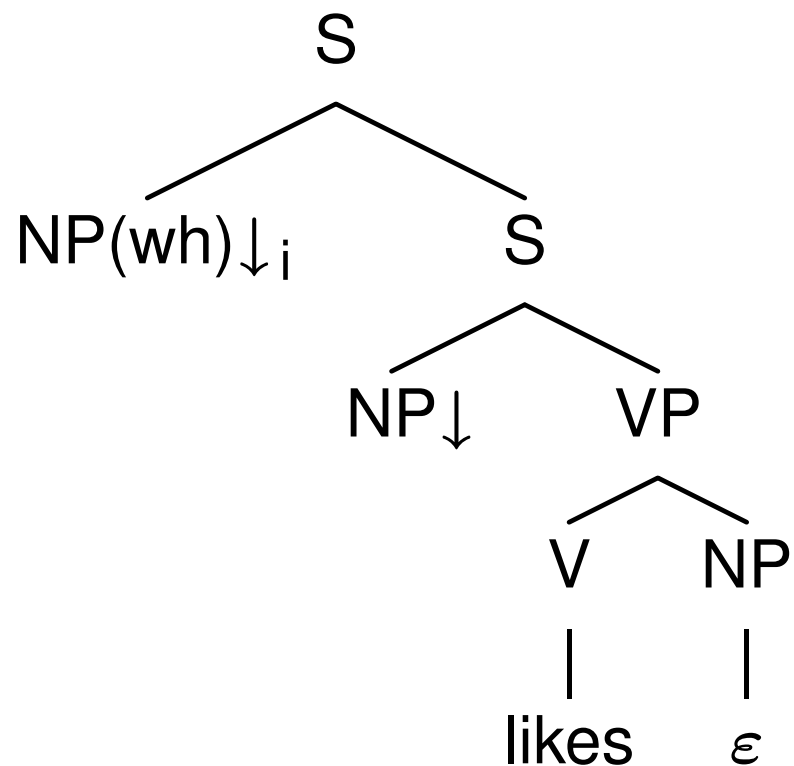
- Wh-movement
- Verbclusters

Linguistic applications: elementary trees for 'likes'

(α_1)

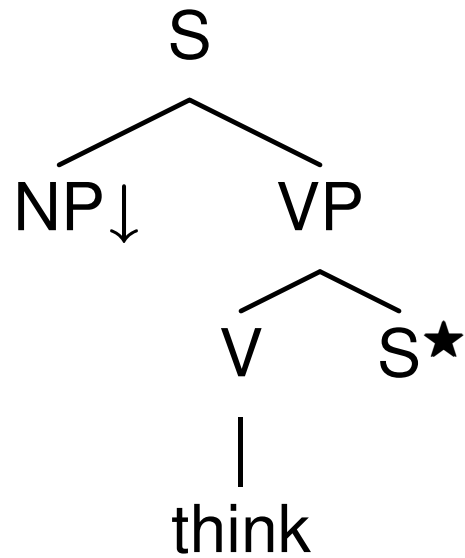


(α_2)

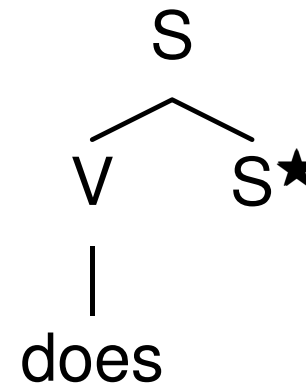


Linguistic applications: sample elementary trees

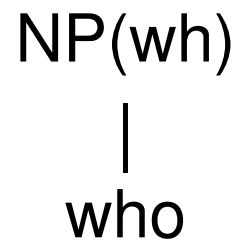
(β_1)



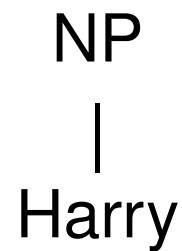
(β_2)



(α_3)



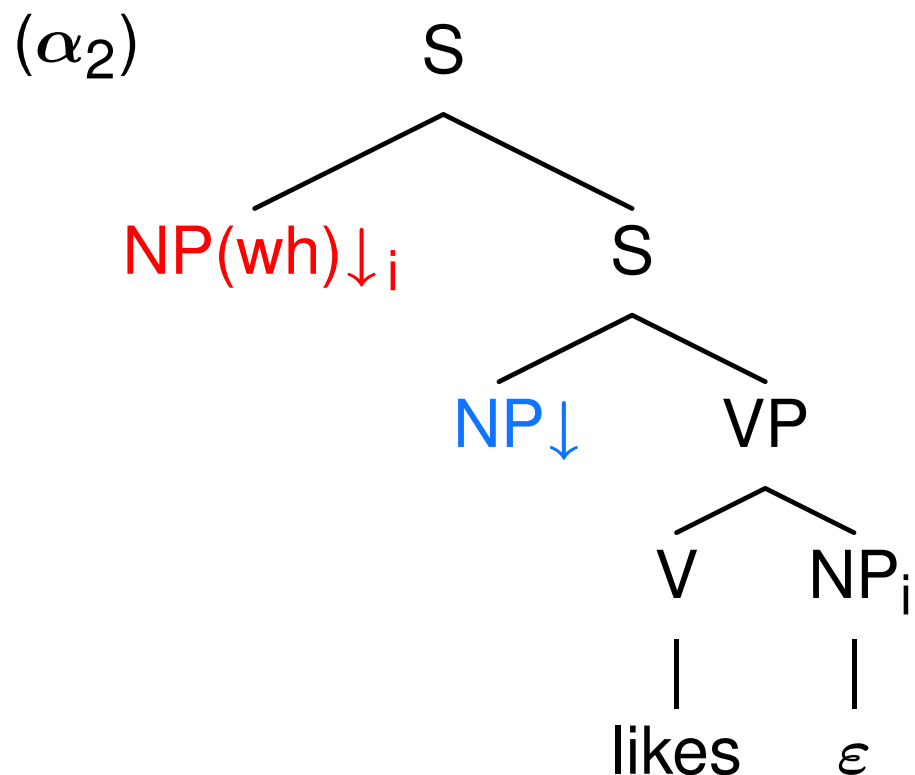
(α_4)



(α_5)



Linguistic applications: sample derivation

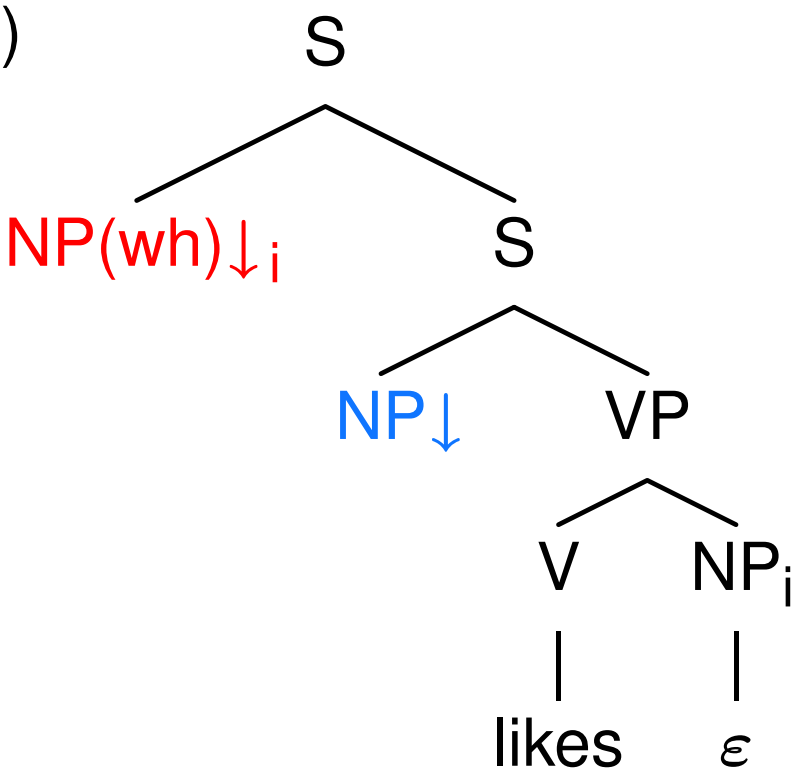


(α_3) NP(wh)
|
who

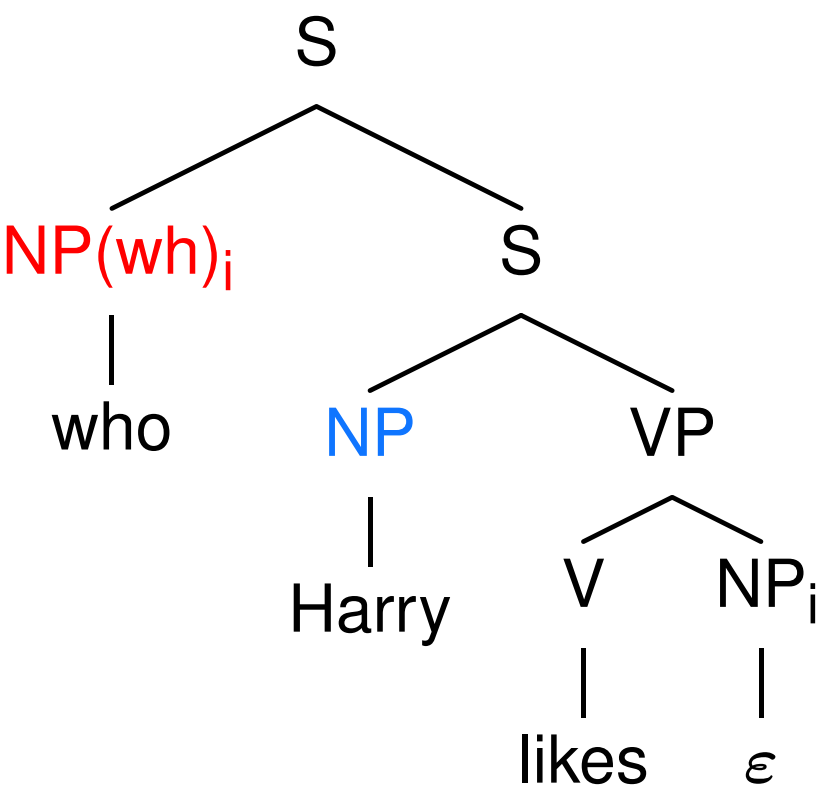
(α_4) NP
|
Harry

Linguistic applications: sample derivation

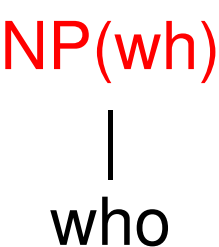
(α_2)



(γ_1)



(α_3)

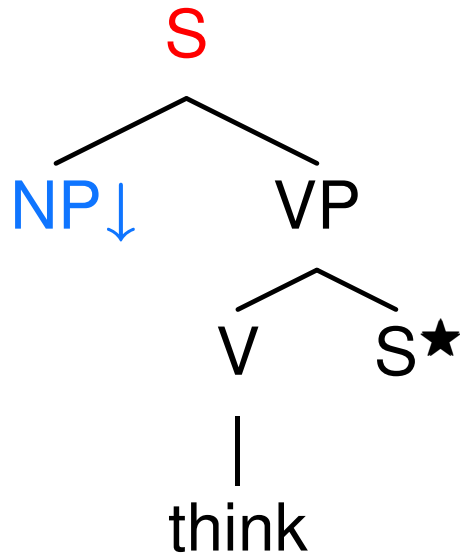


(α_4)

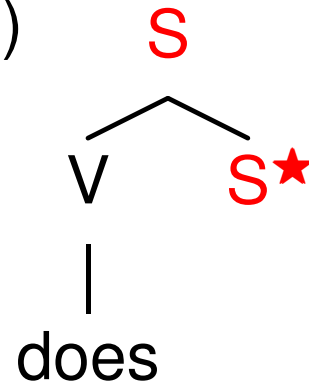


Linguistic applications: sample derivation

(β_1)



(β_2)

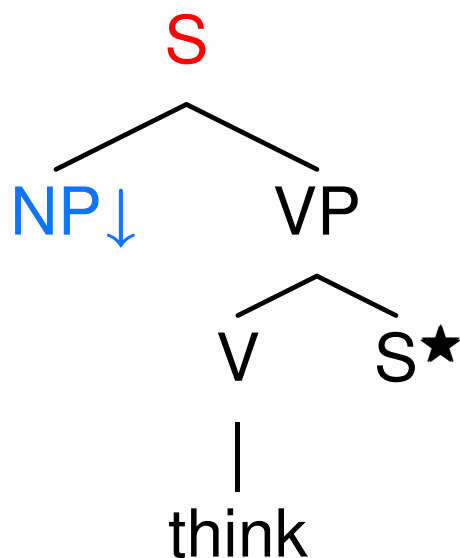


(α_5)

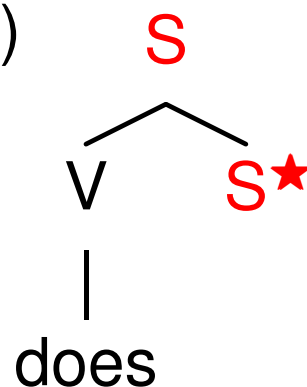


Linguistic applications: sample derivation

(β_1)



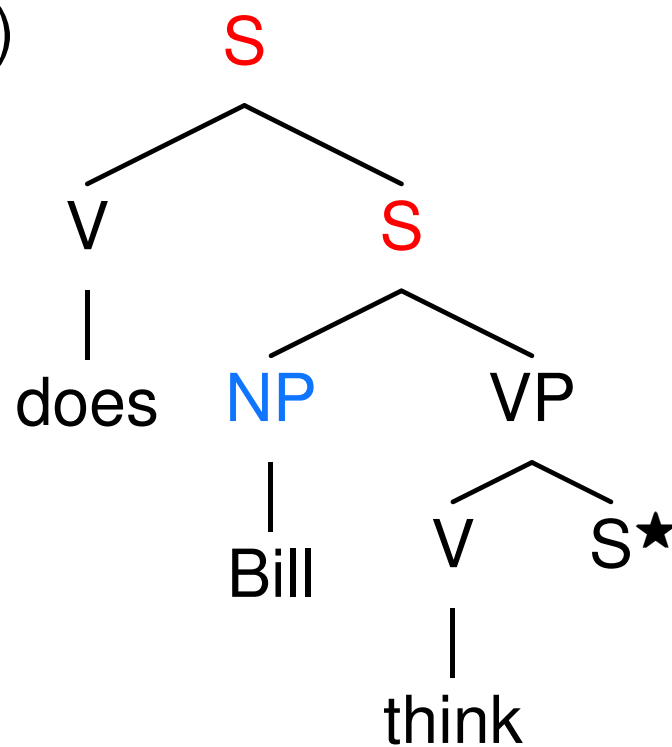
(β_2)



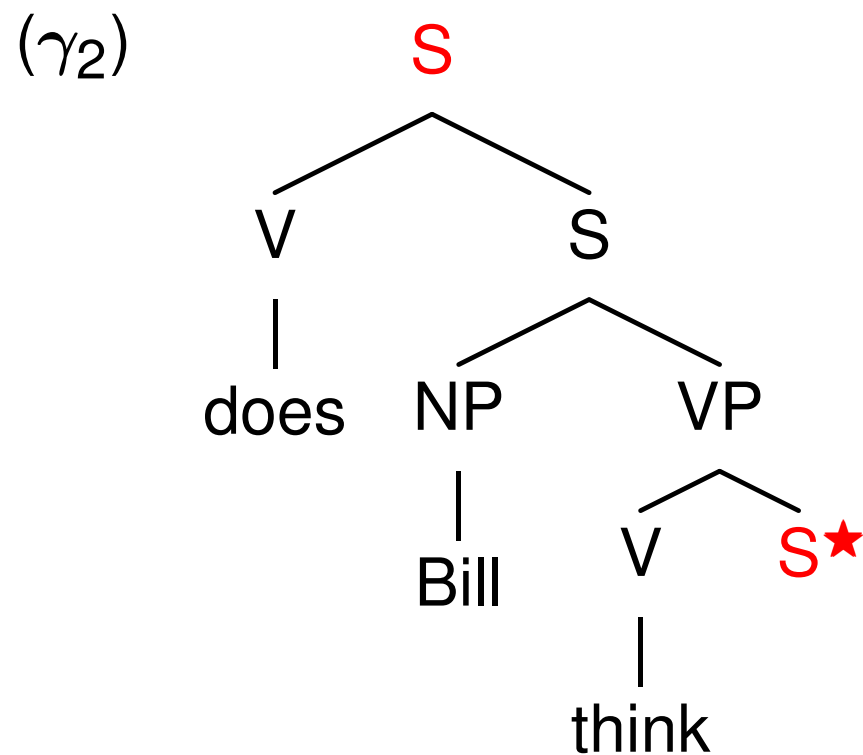
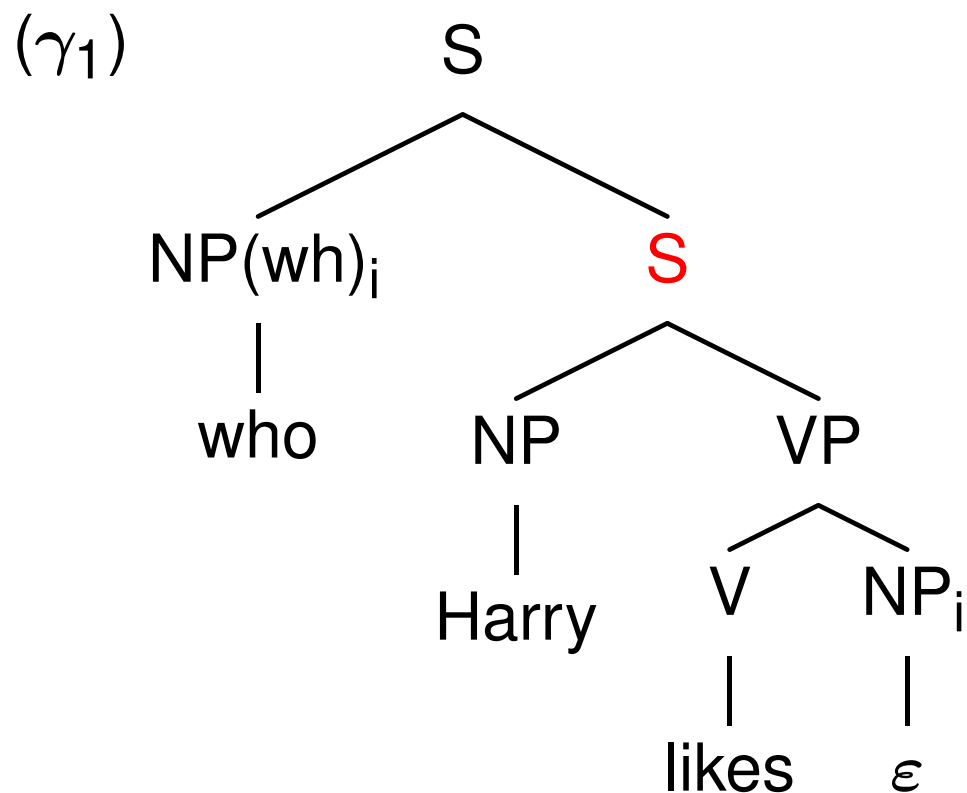
(α_5)



(γ_2)

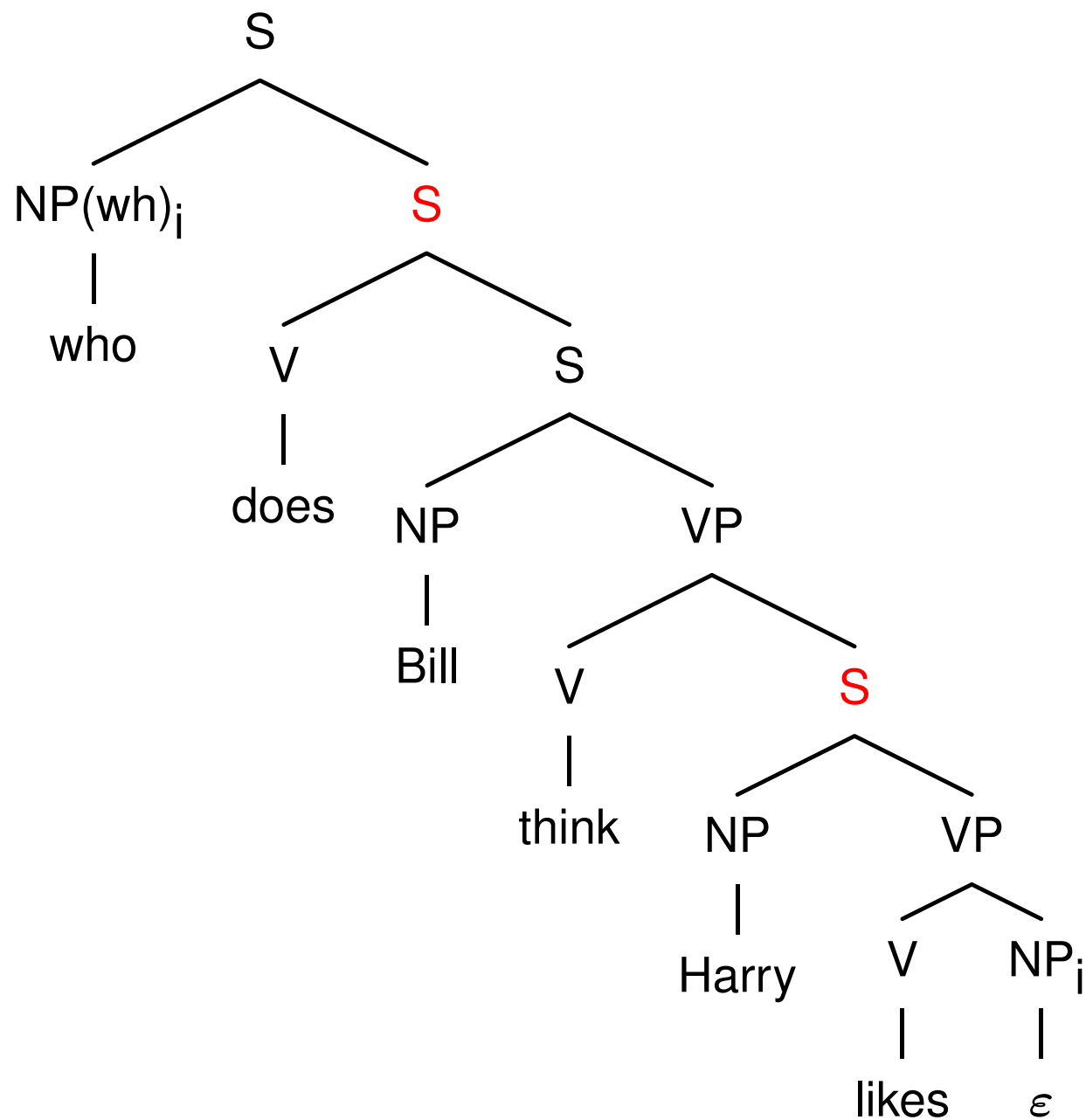


Linguistic applications: sample derivation



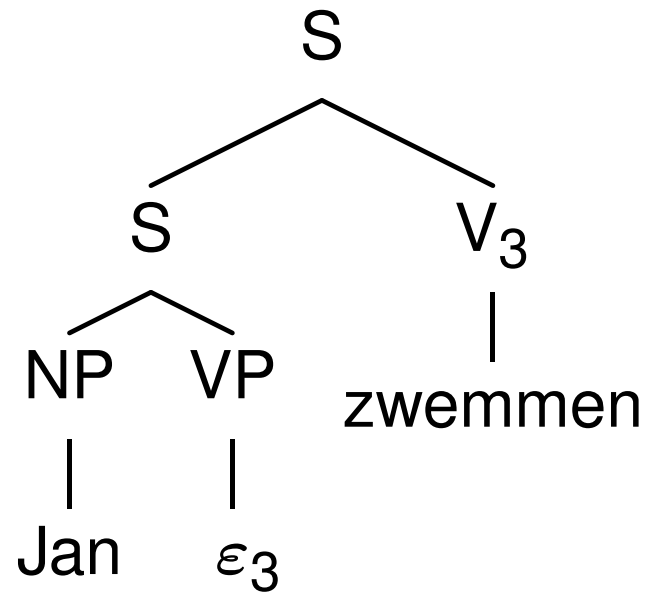
Linguistic applications: sample derivation

(73)



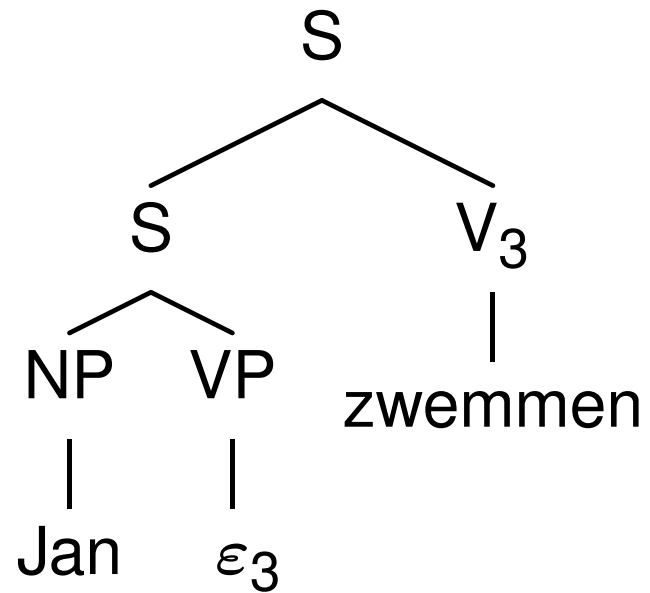
Linguistic applications: “verb clusters”

(α_7)

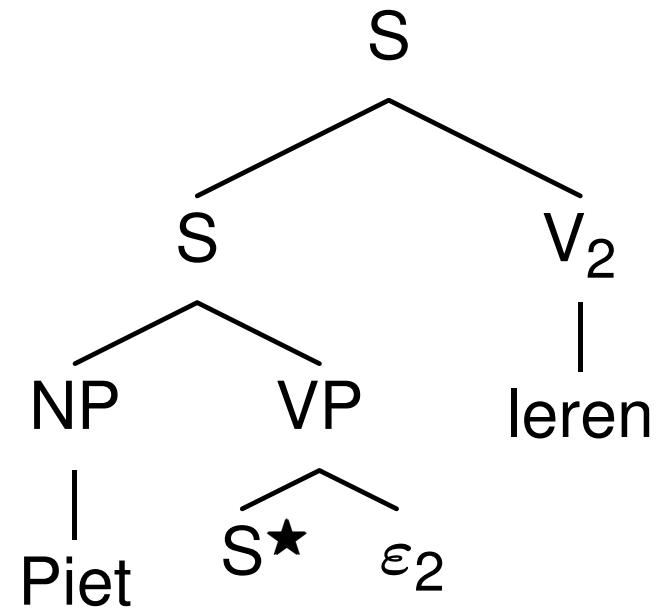


Linguistic applications: “verb clusters”

(α_7)

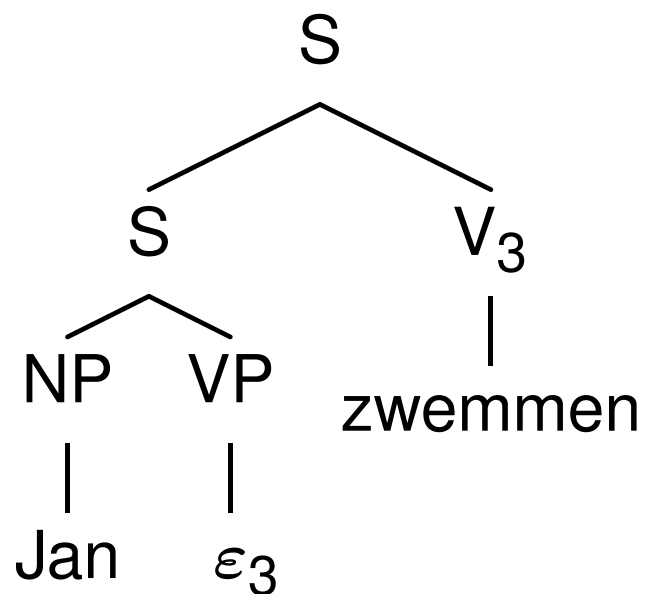


(β_4)

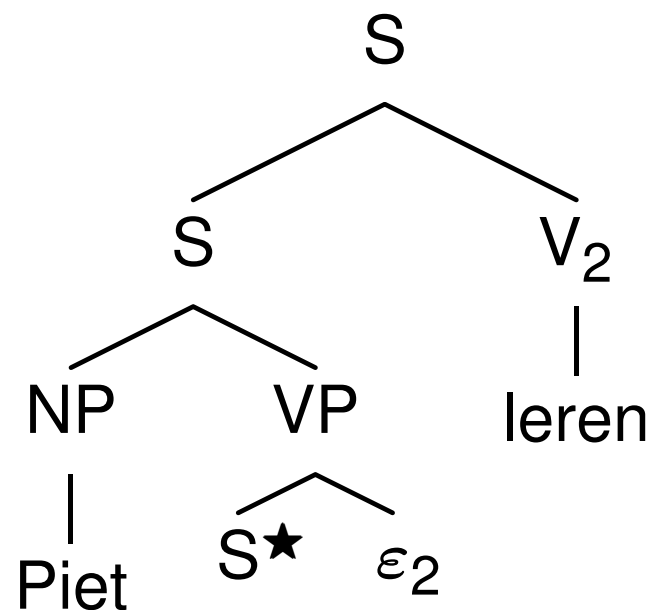


Linguistic applications: “verb clusters”

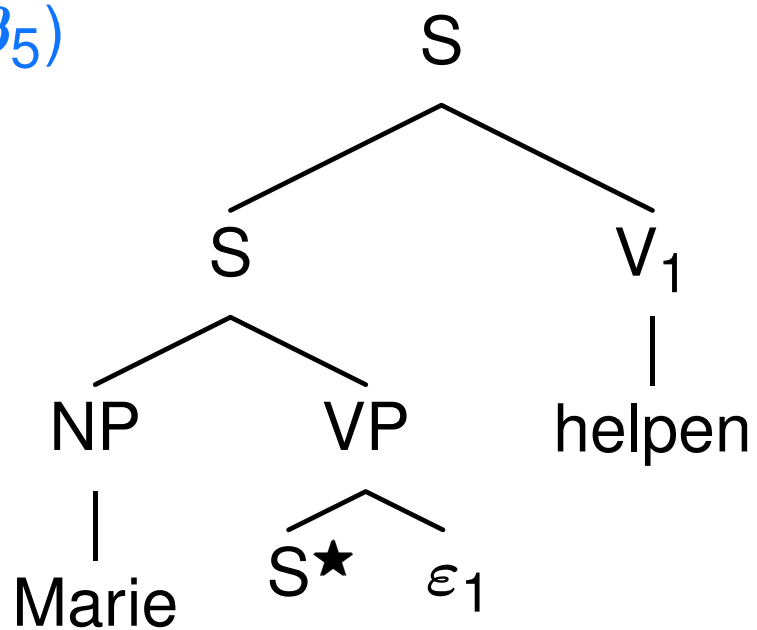
(α_7)



(β_4)

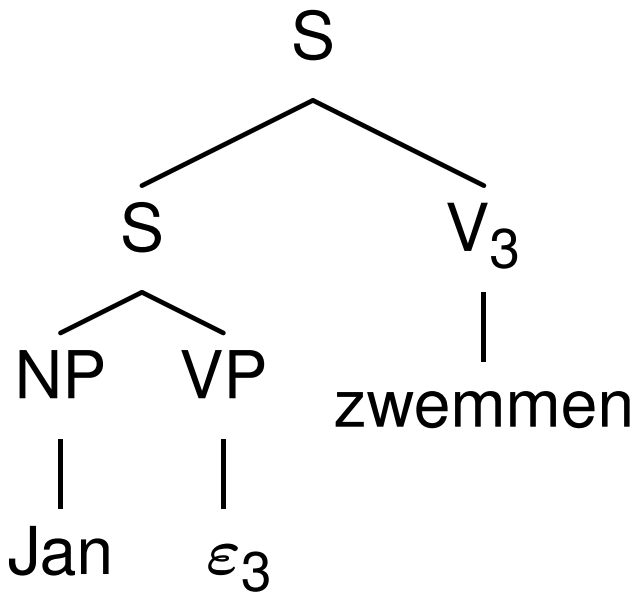


(β_5)

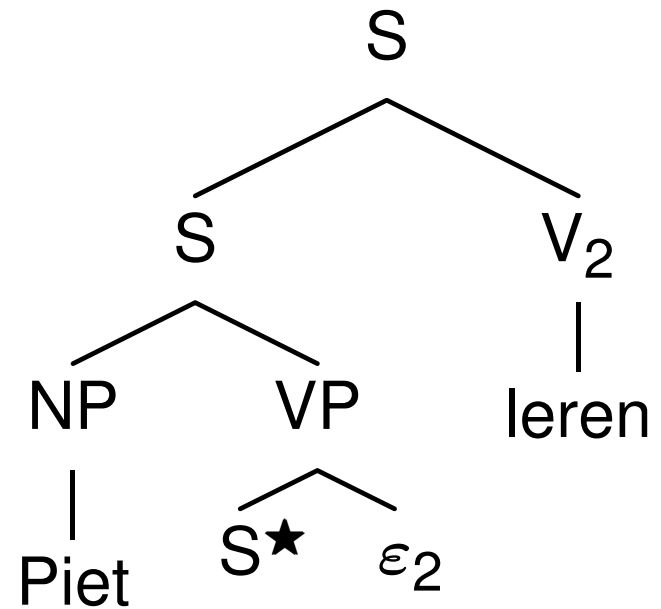


Linguistic applications: “verb clusters”

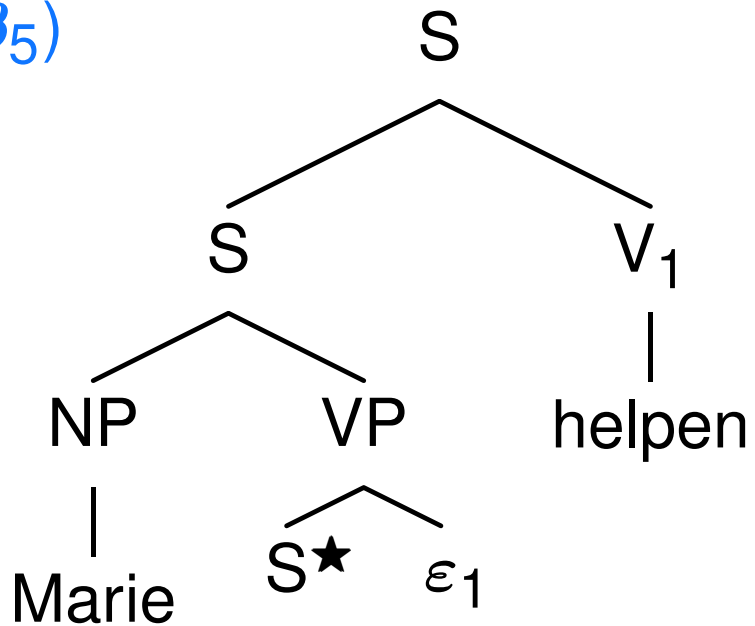
(α_7)



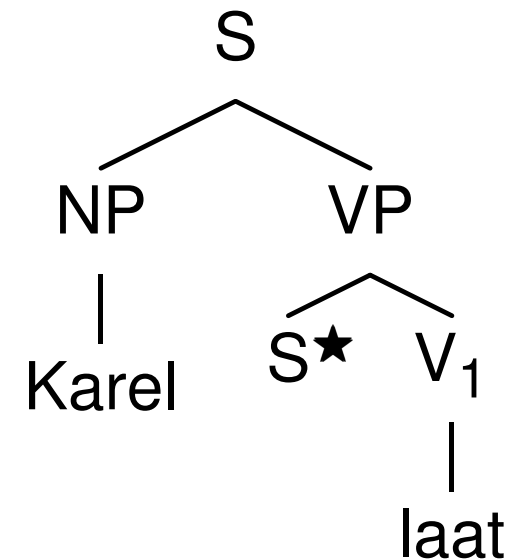
(β_4)



(β_5)



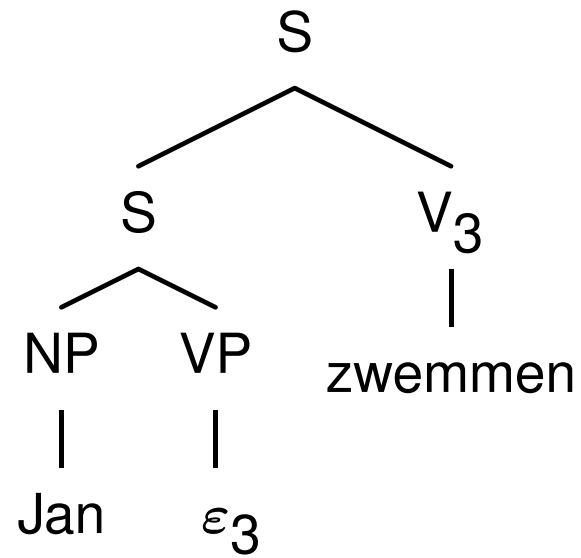
(β_6)



‘dat Karel Marie Piet Jan laat helpen leren zwemmen’

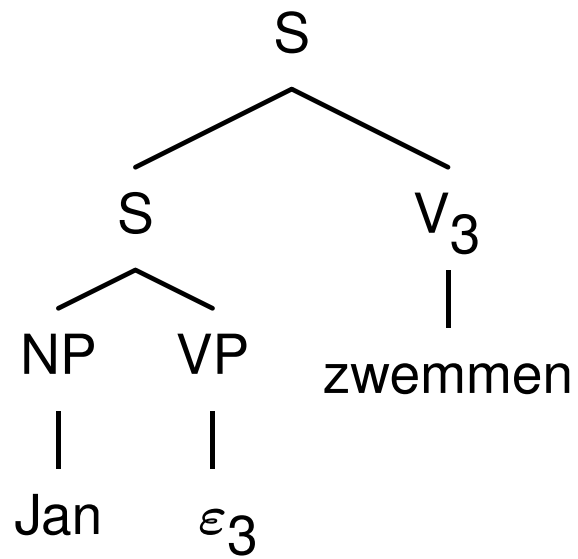
'dat Karel Marie Piet Jan laat helpen leren zwemmen'

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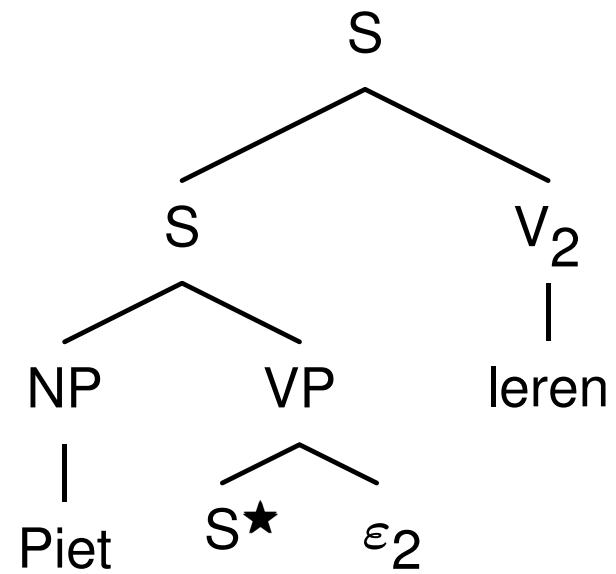


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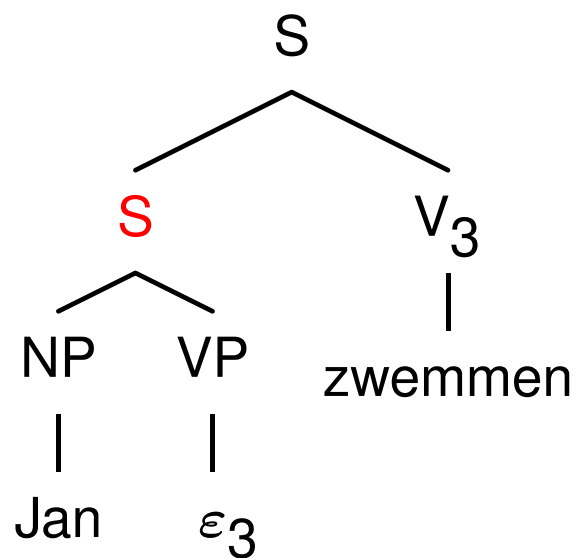


(β_4)

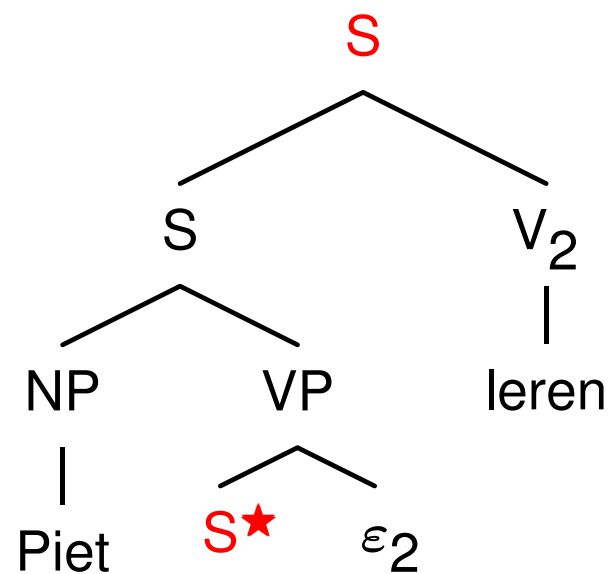


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(α_7)

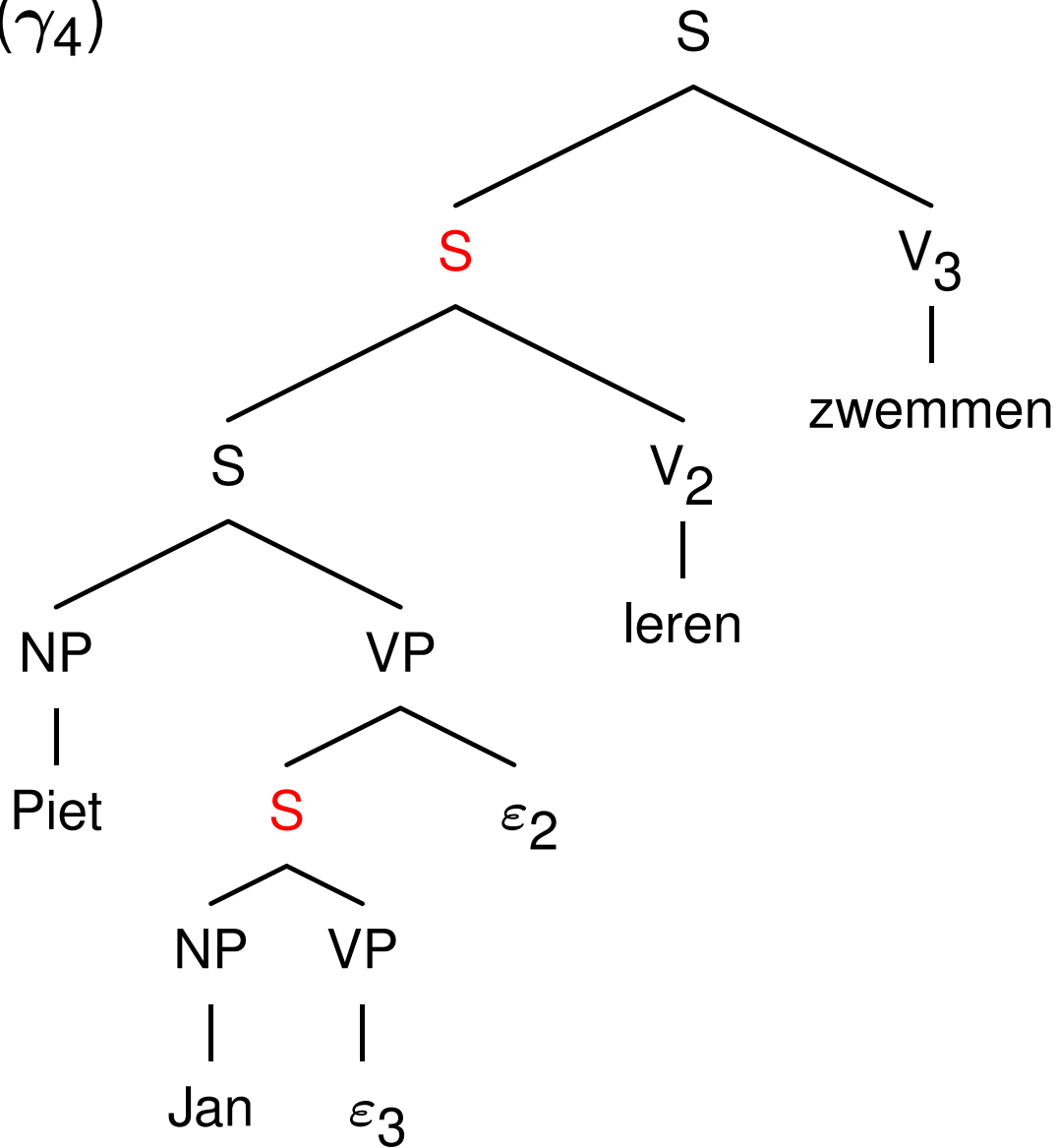


(β_4)



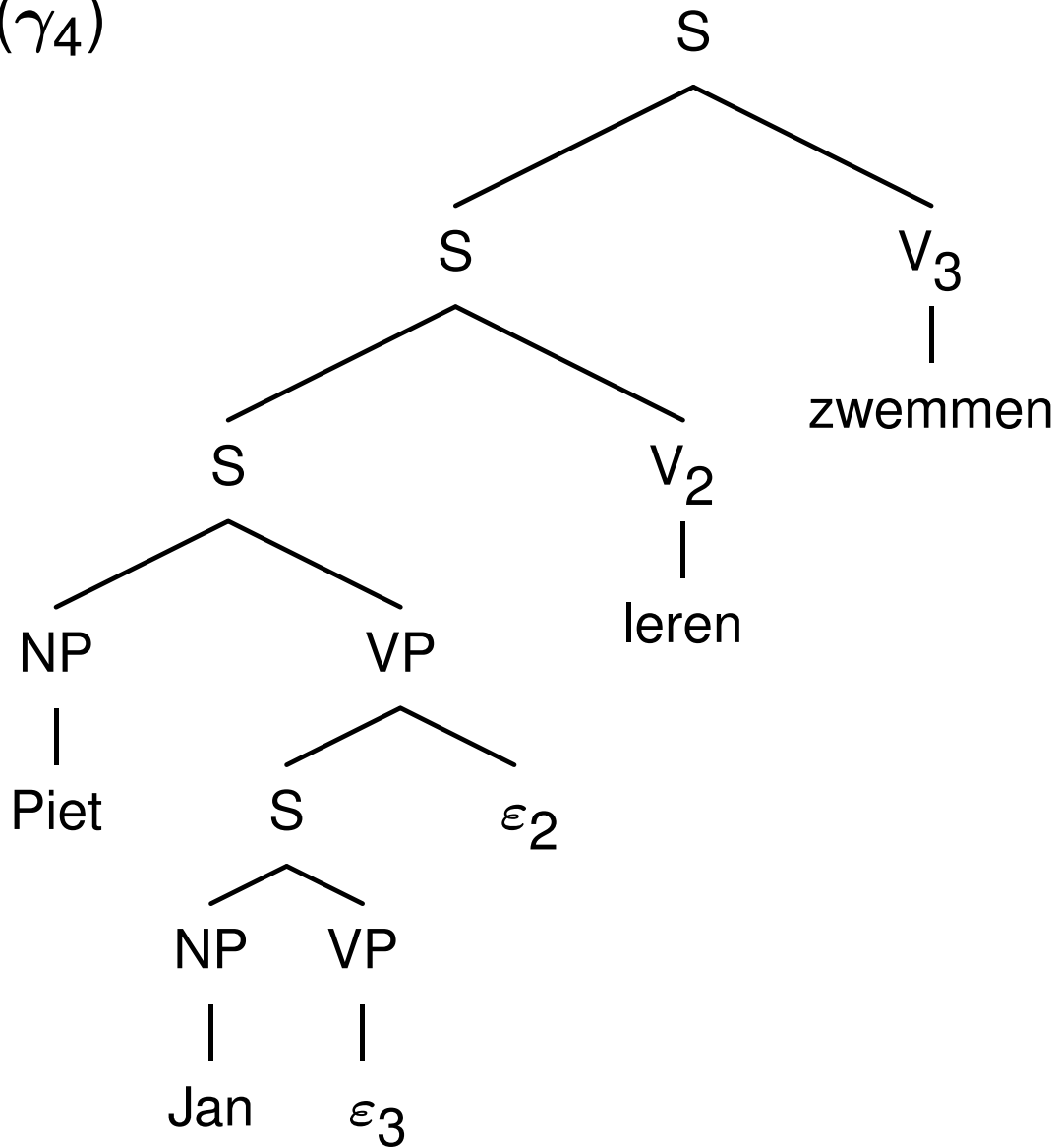
'dat Karel Marie Piet Jan laat helpen leren zwemmen'

(74)



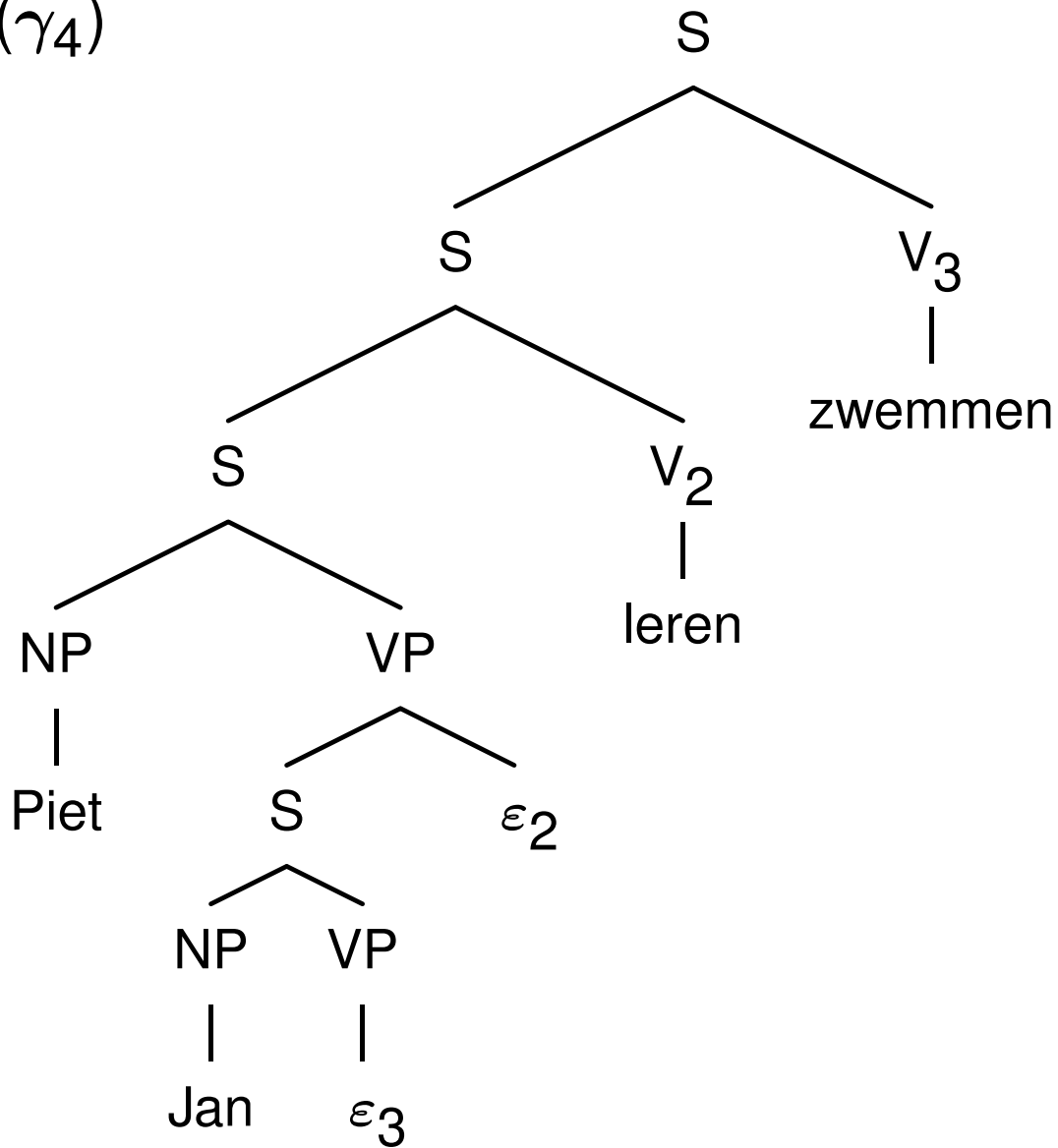
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(74)

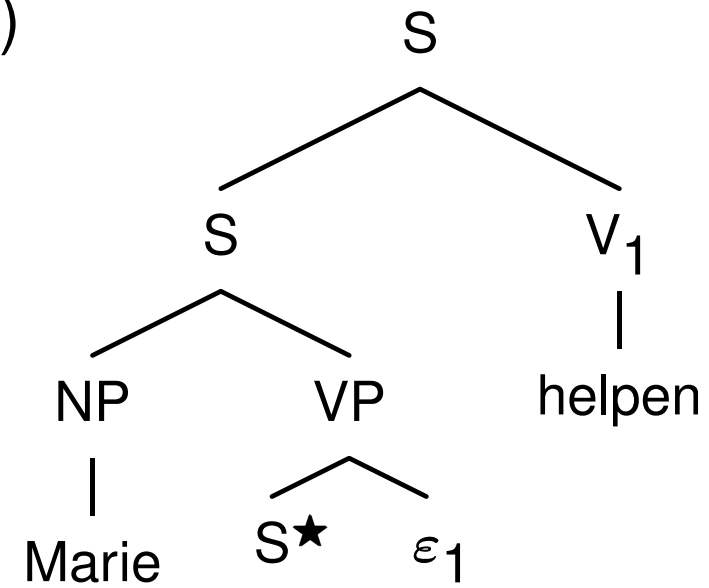


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(γ_4)

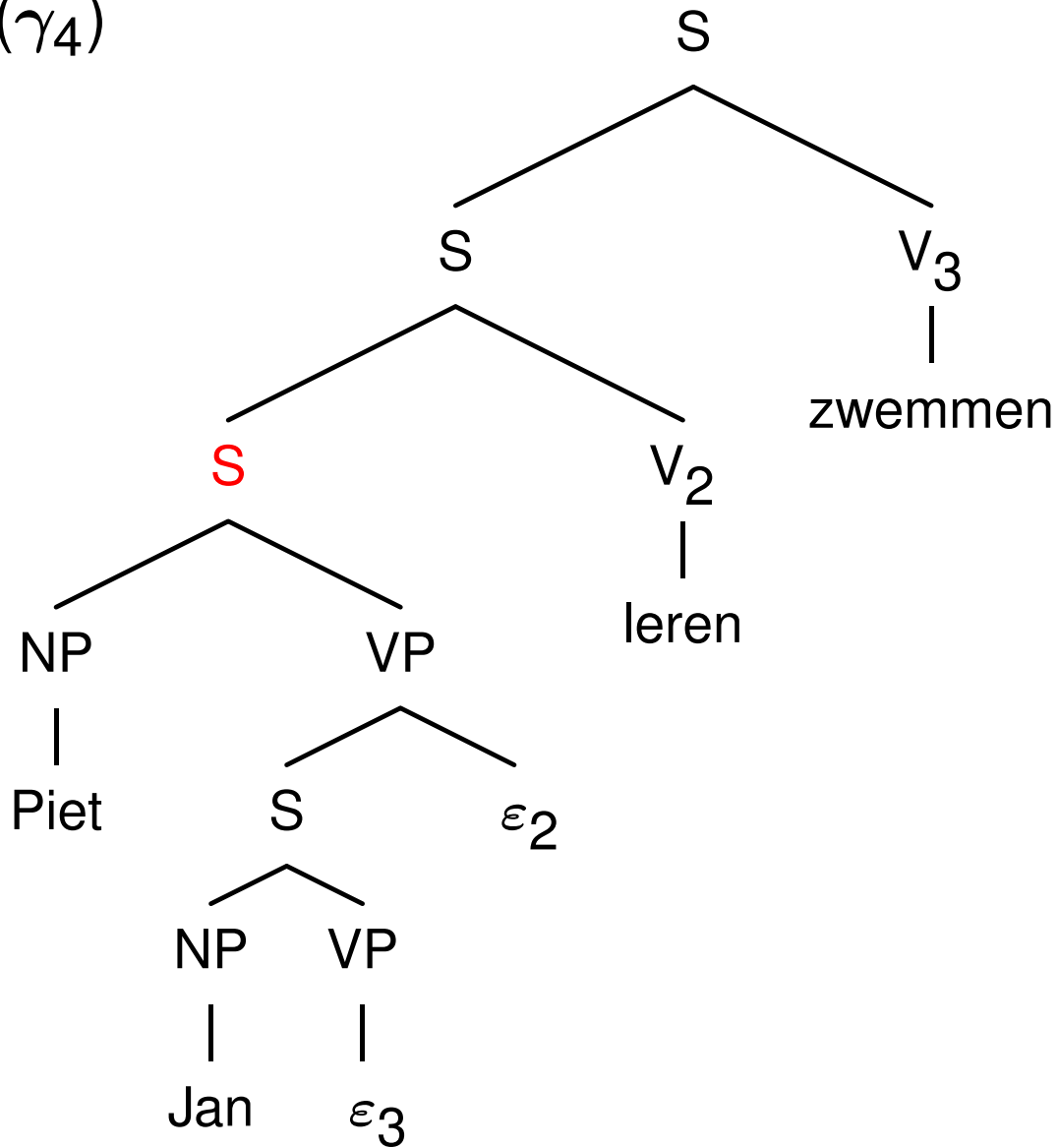


(β_5)

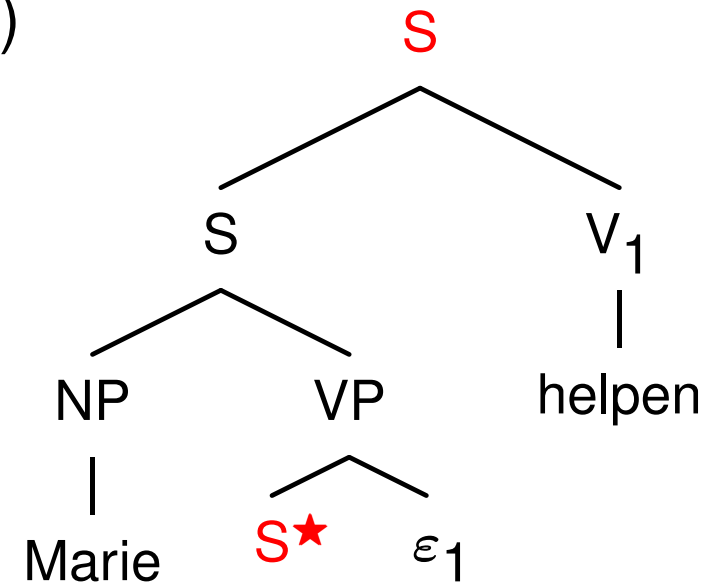


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(γ_4)

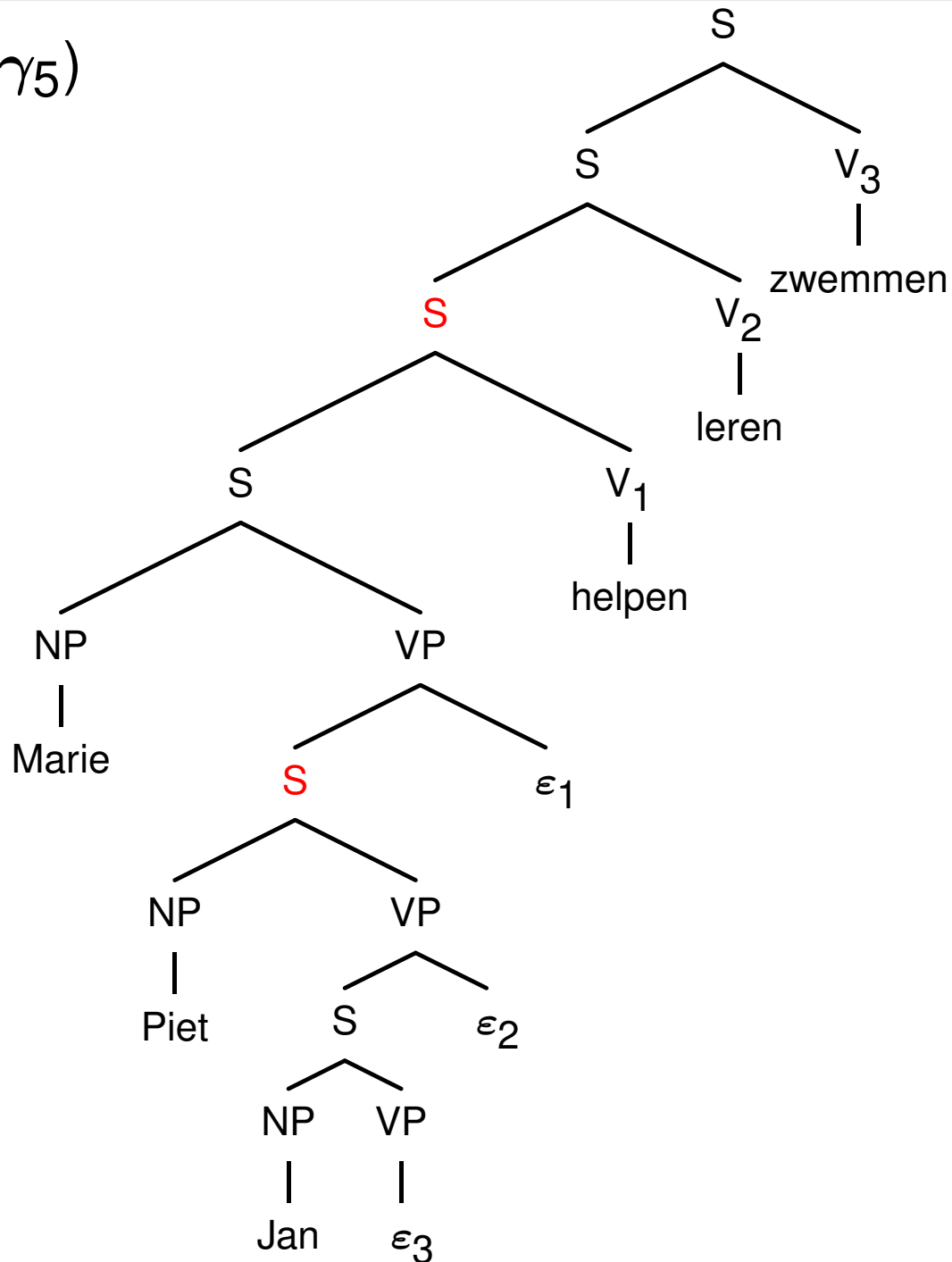


(β_5)



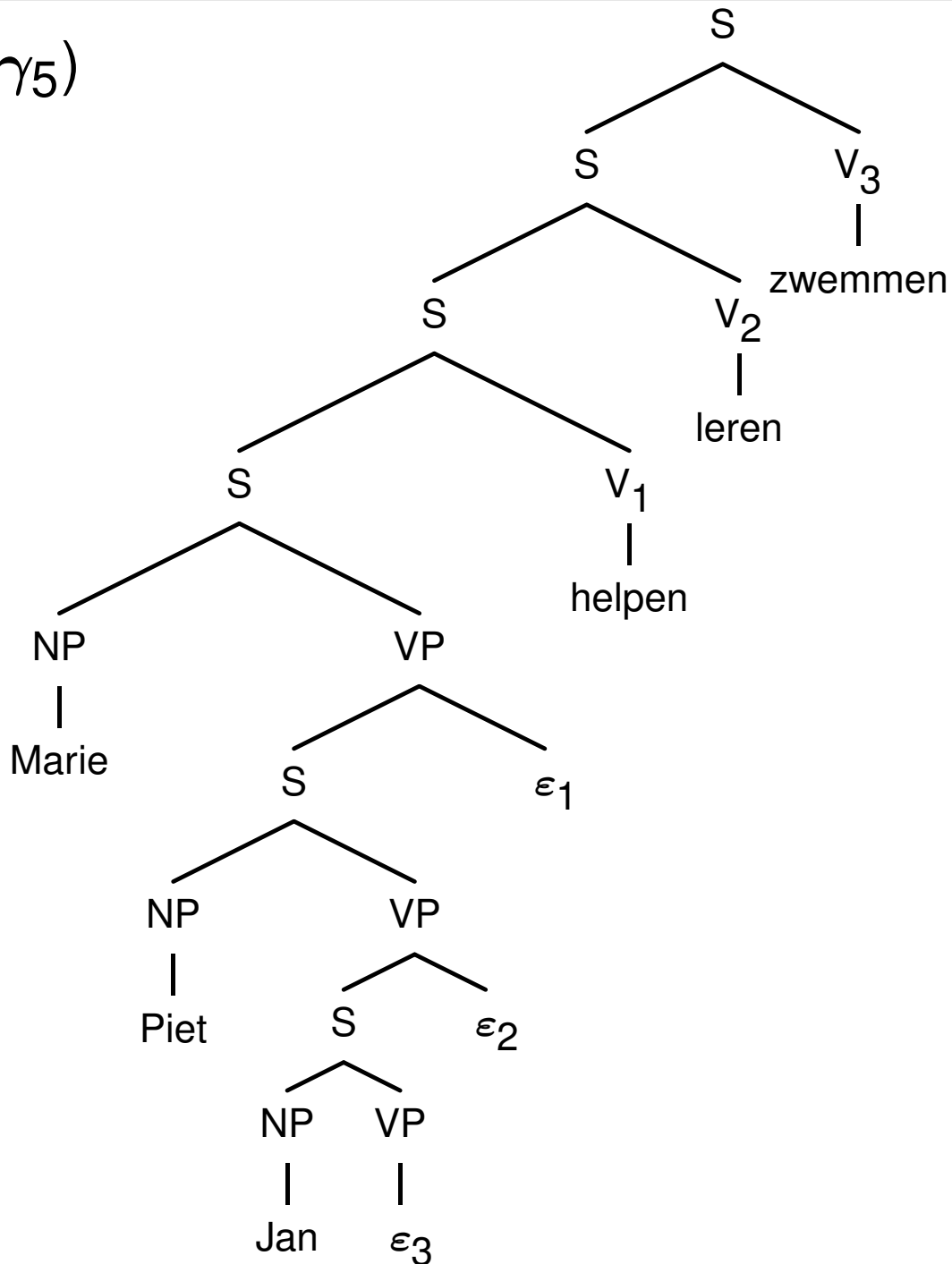
'dat Karel Marie Piet Jan laat helpen leren zwemmen'

(75)



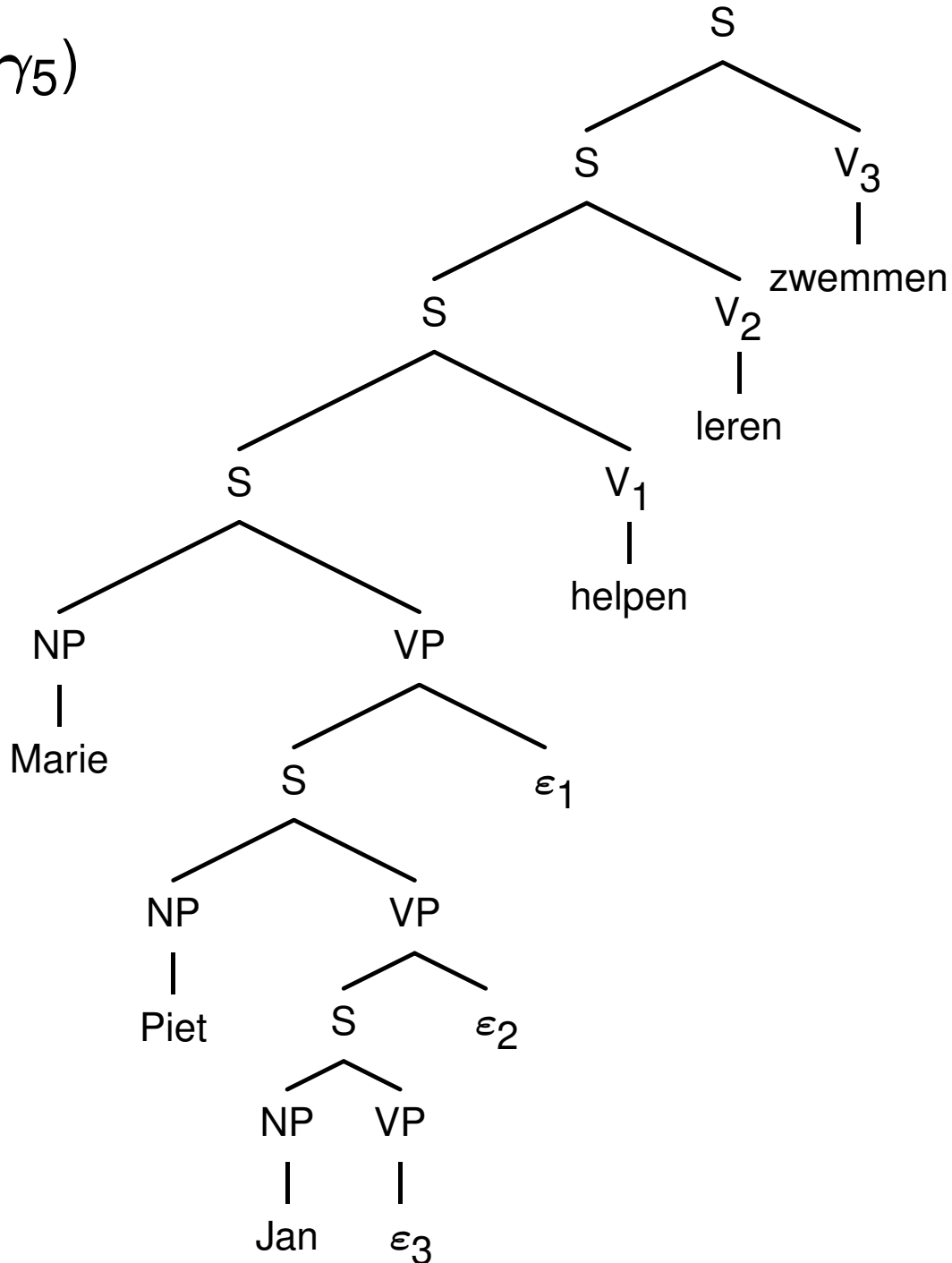
'dat Karel Marie Piet Jan laat helpen leren zwemmen'

(75)

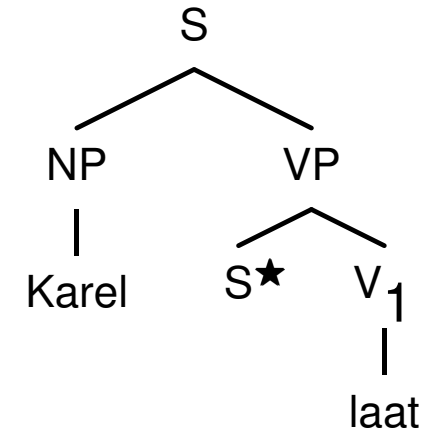


'dat Karel Marie Piet Jan laat helpen leren zwemmen'

(γ_5)

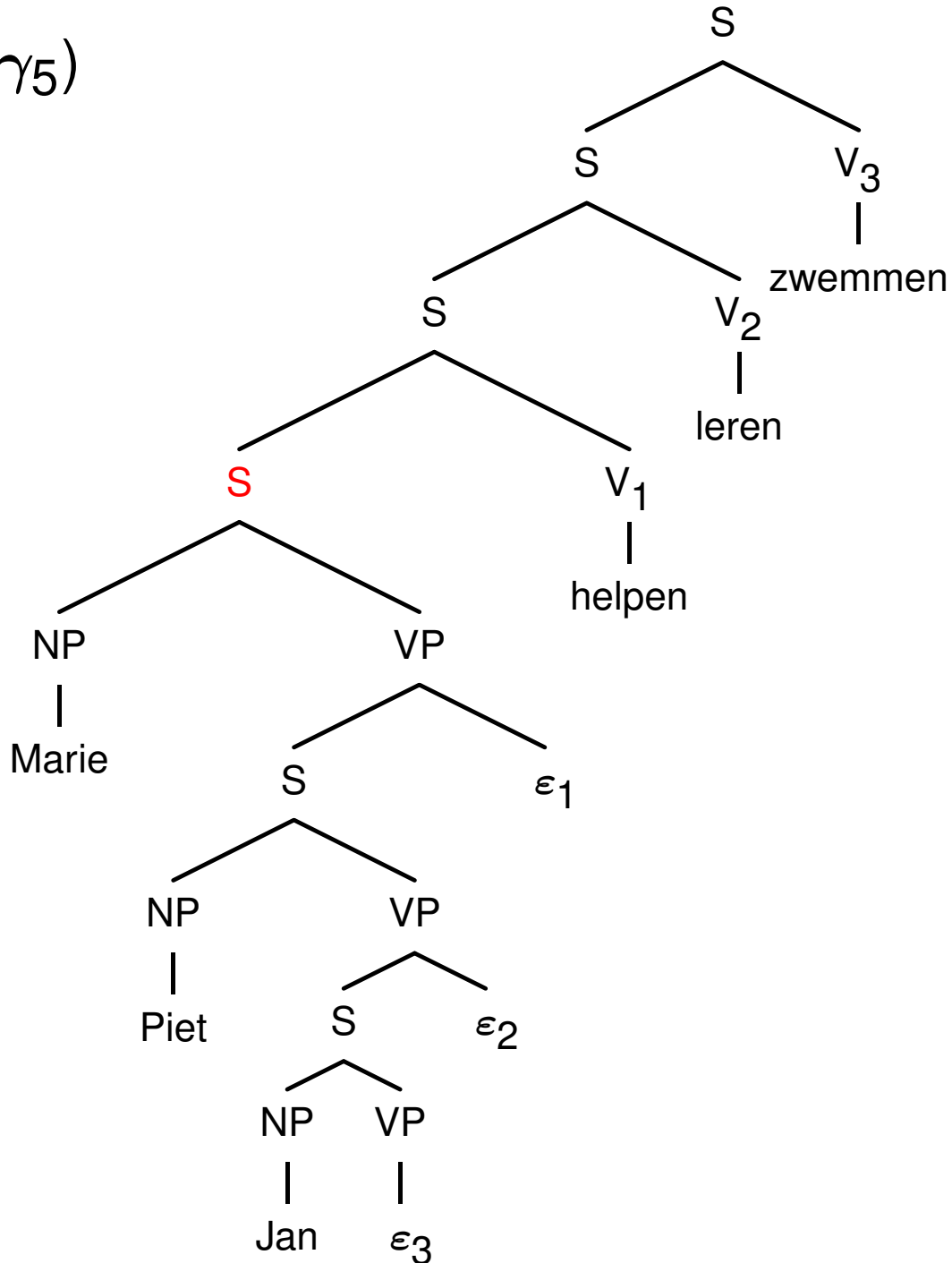


(β_6)

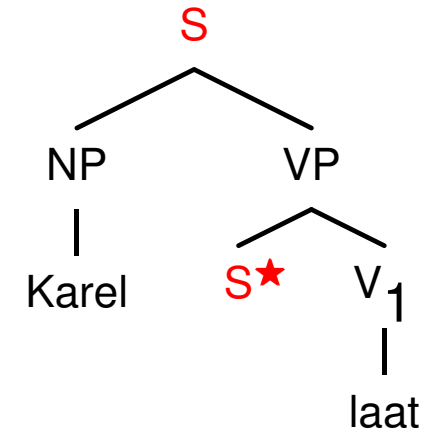


'dat Karel Marie Piet Jan laat helpen leren zwemmen'

(γ_5)



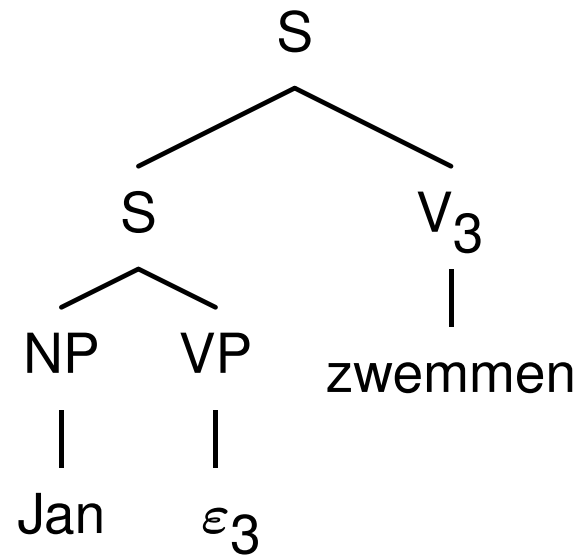
(β_6)



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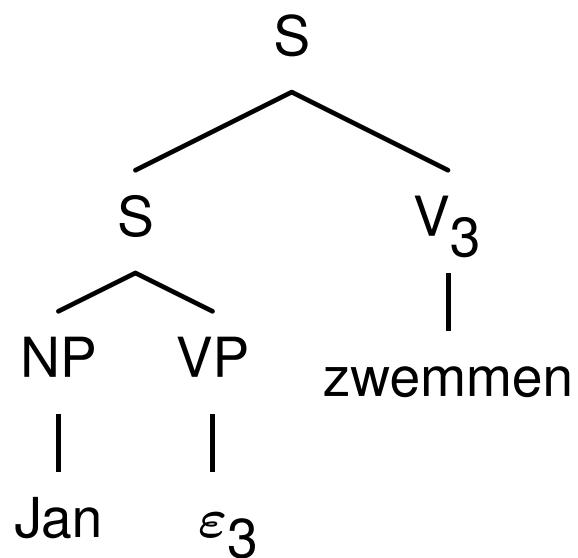
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(α_7)

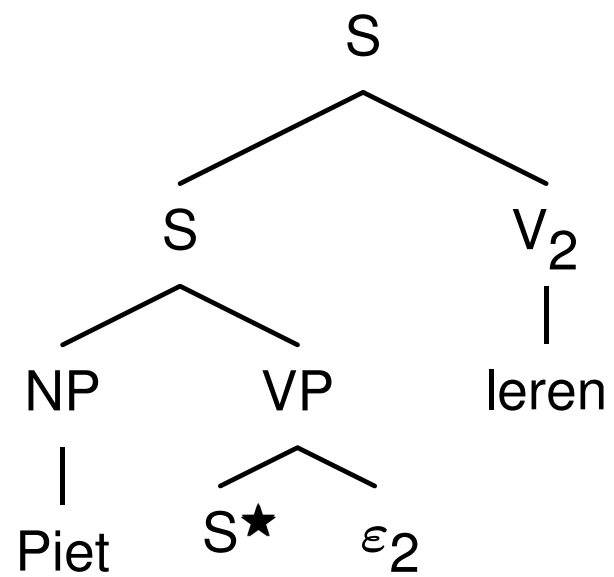


'dat Karel Piet Jan laat leren zwemmen'

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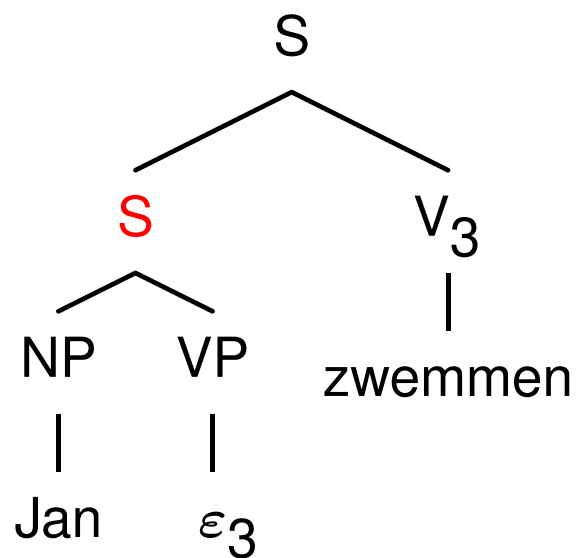


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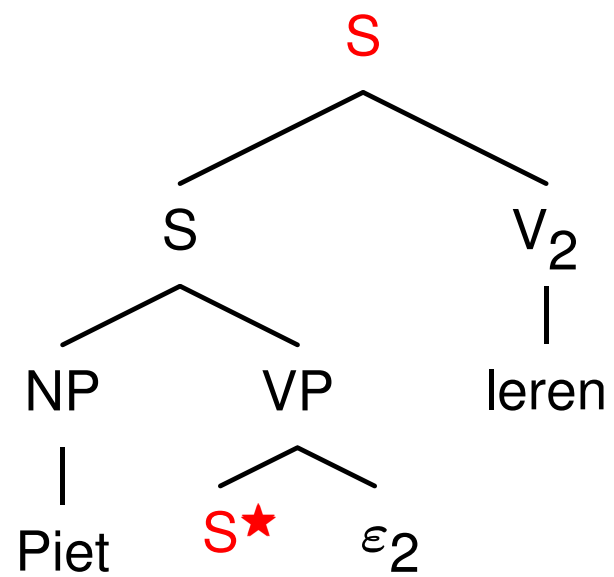


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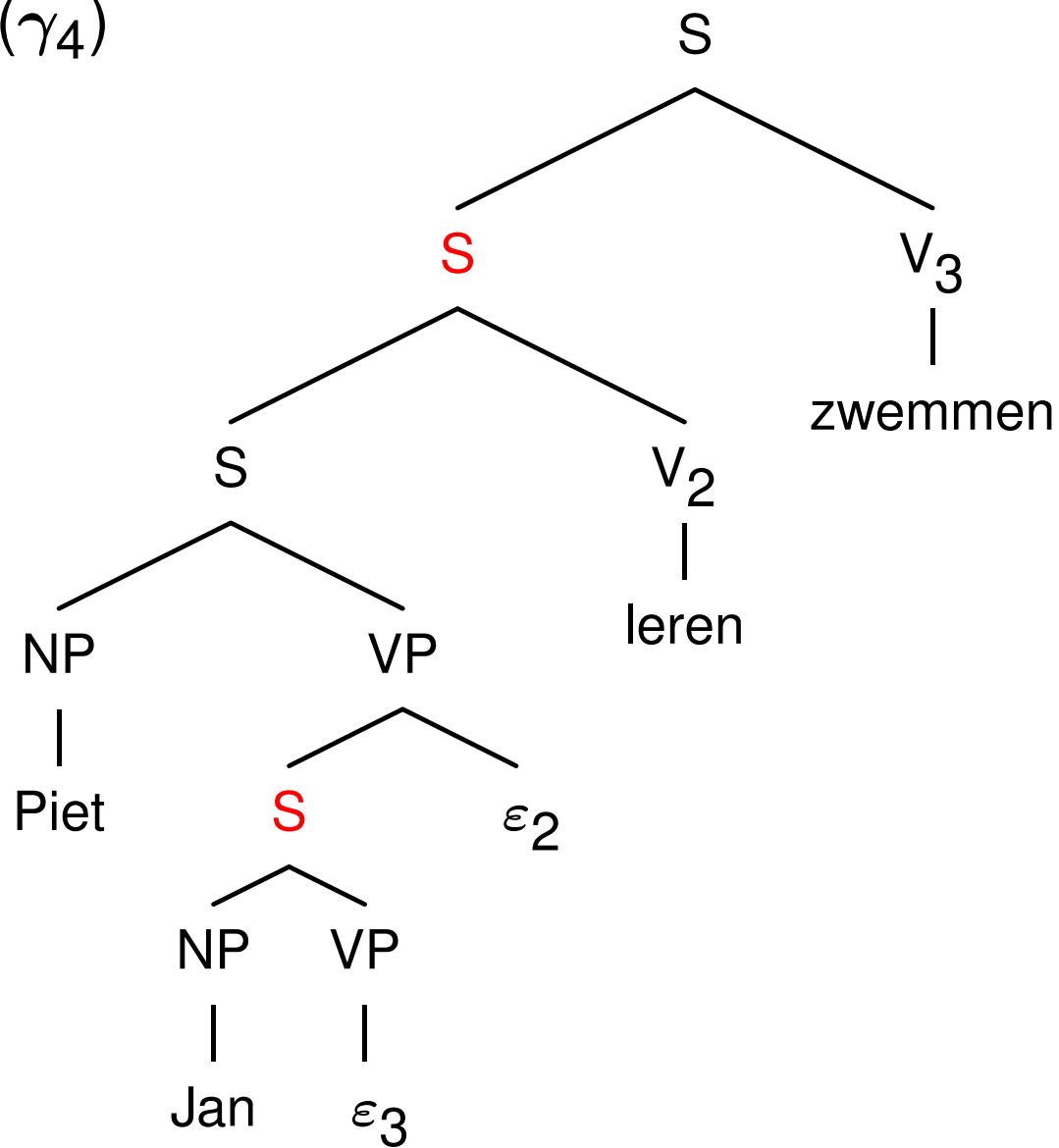


(β_4)



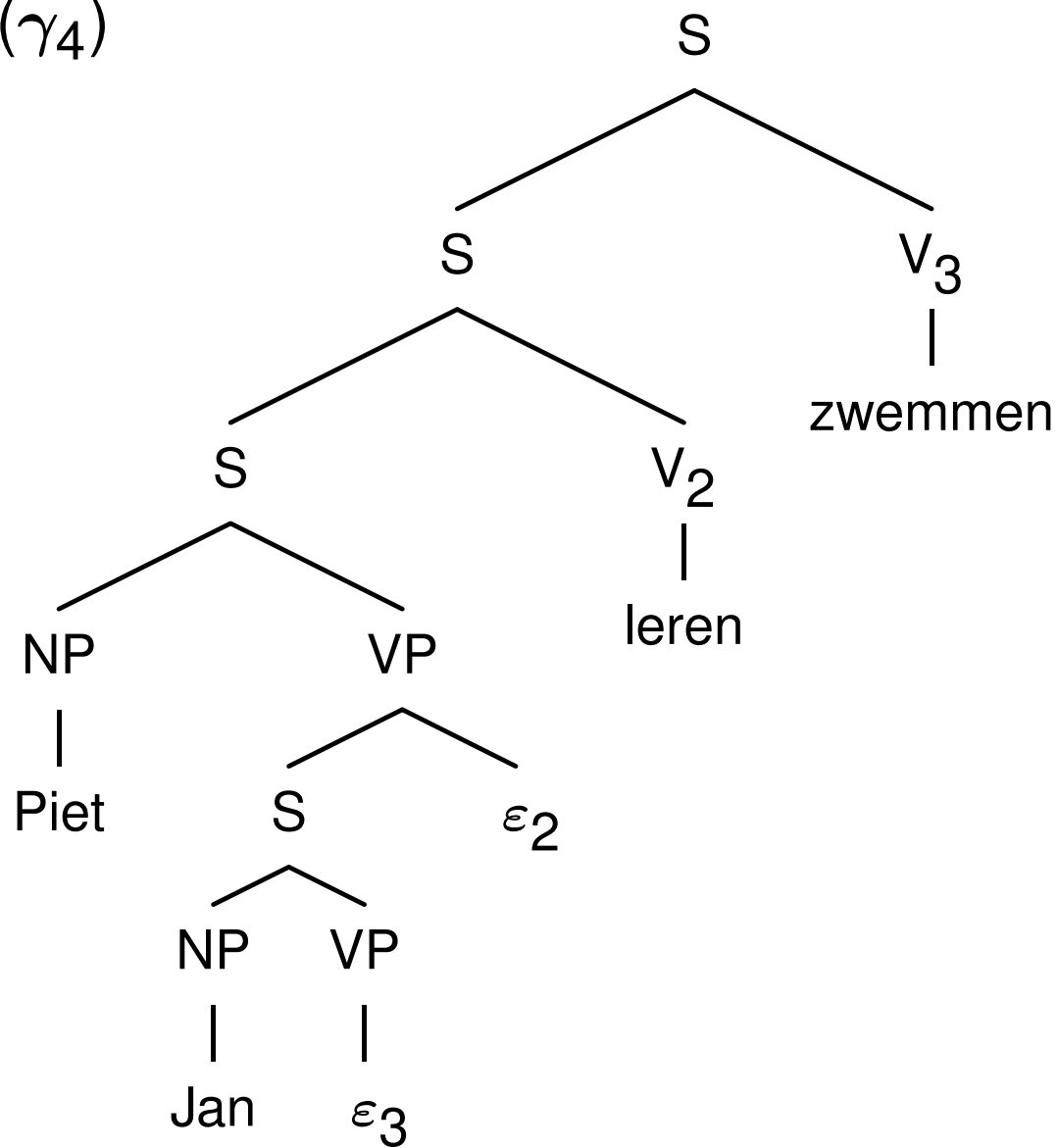
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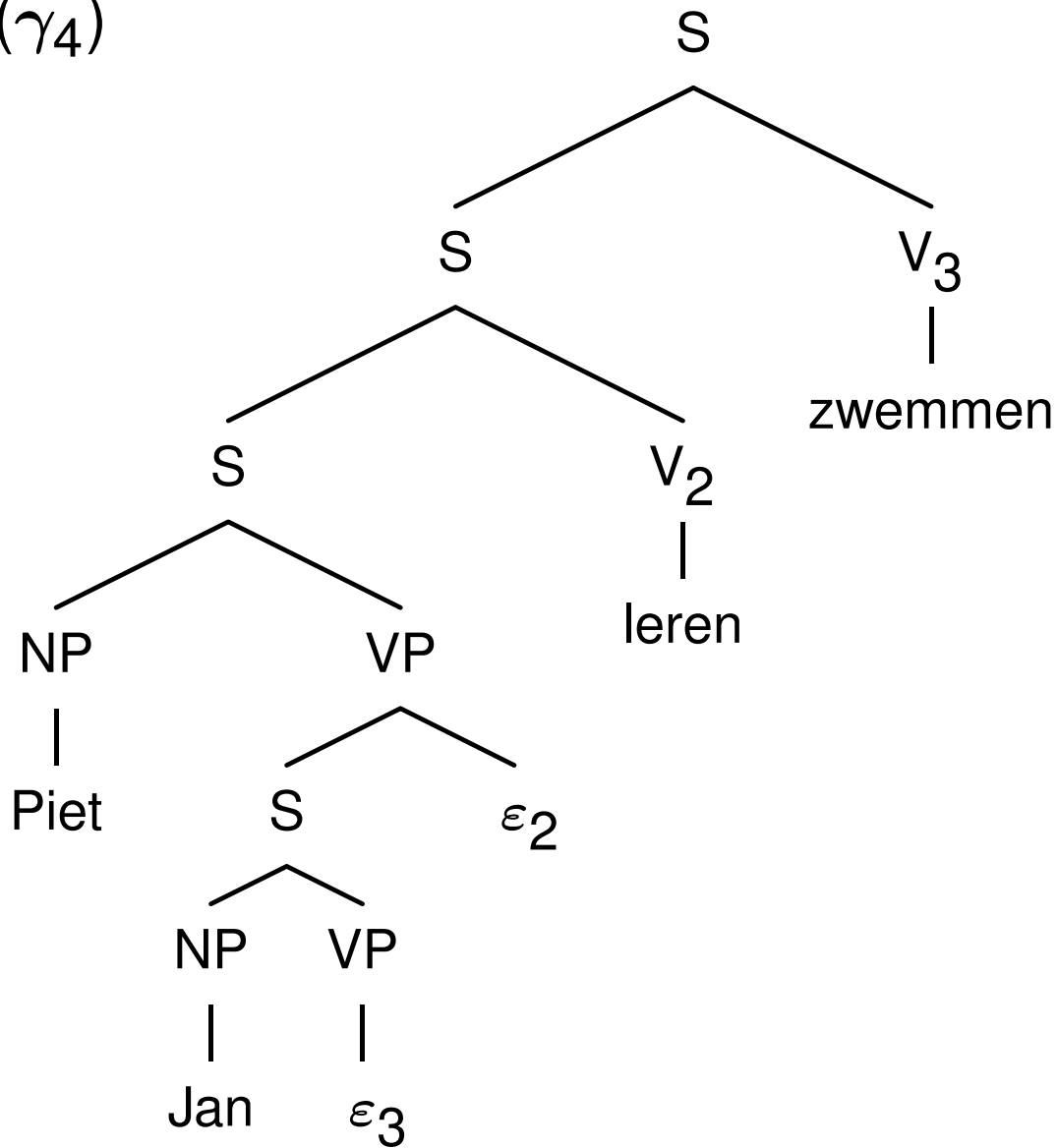
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(74)

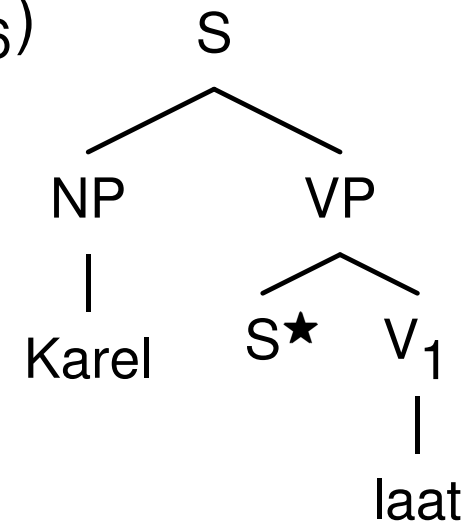


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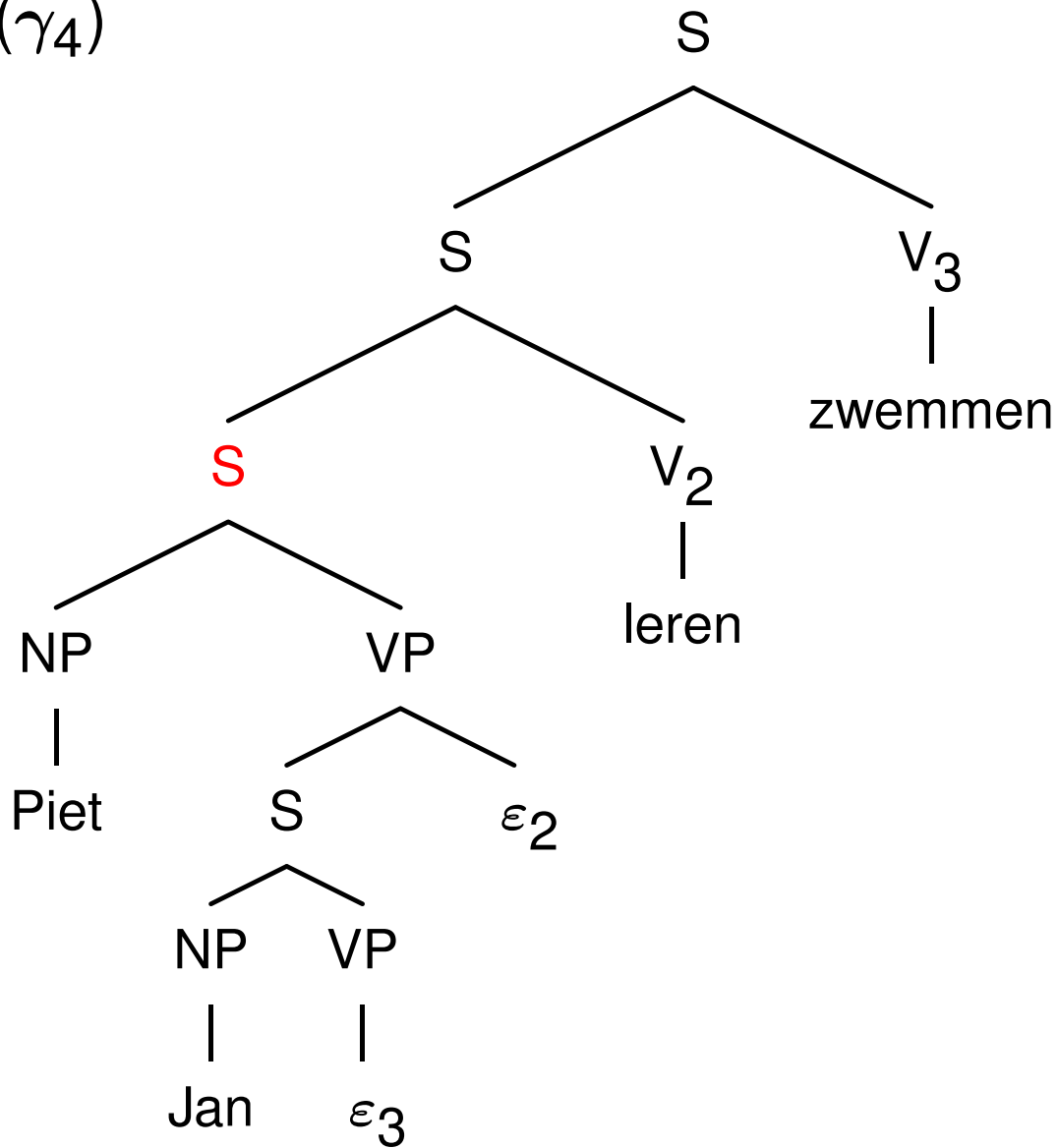


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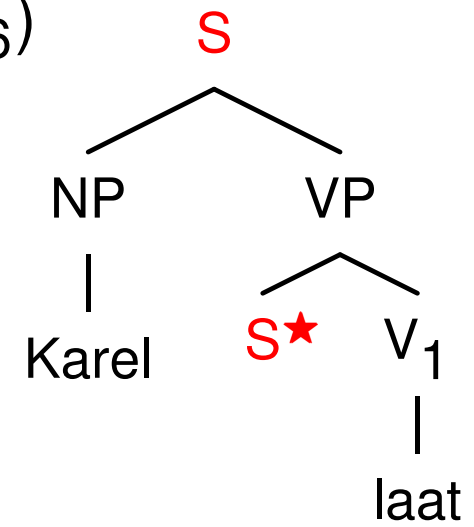


'dat Karel Piet Jan laat helpen leren zwemmen'

(γ_4)

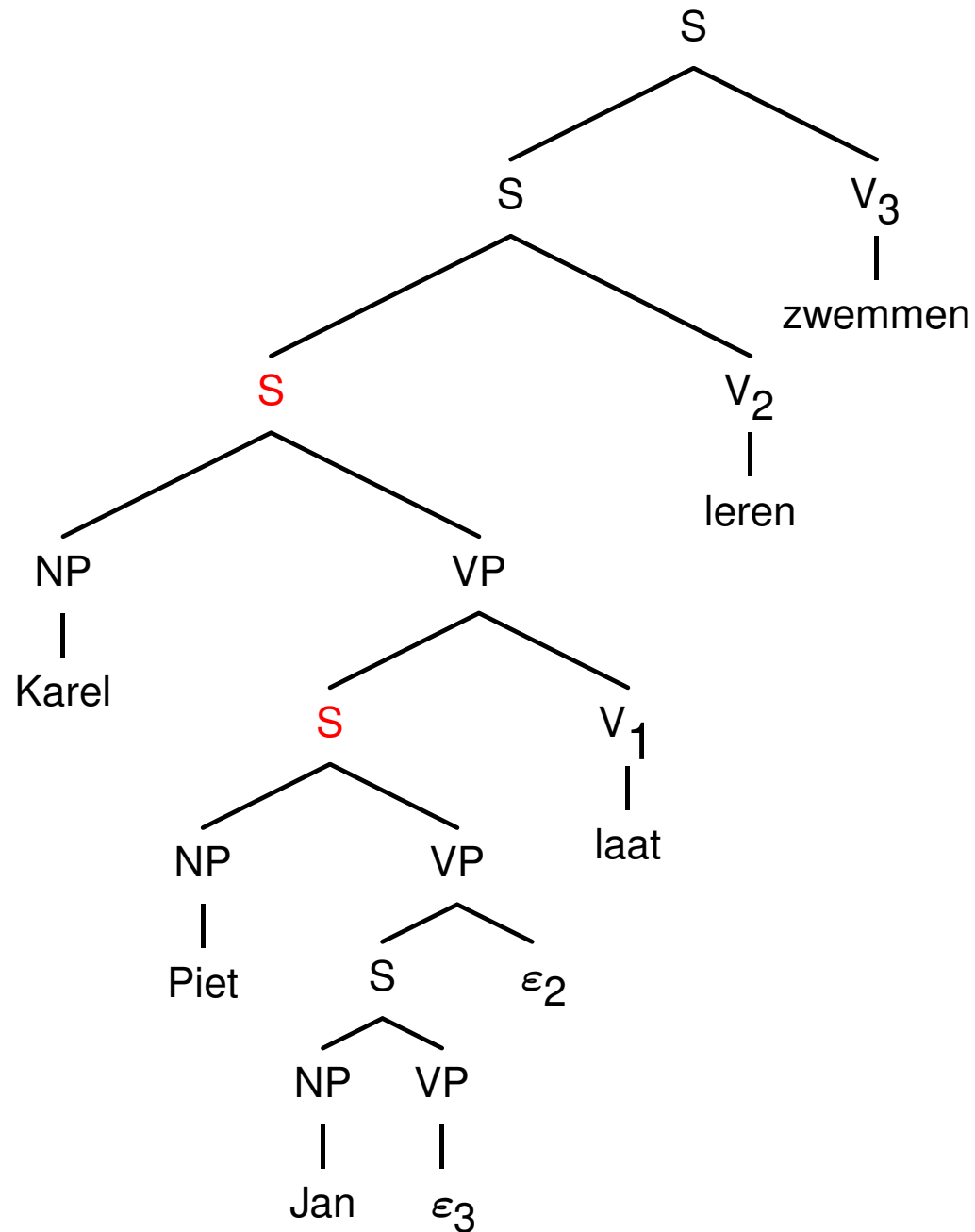


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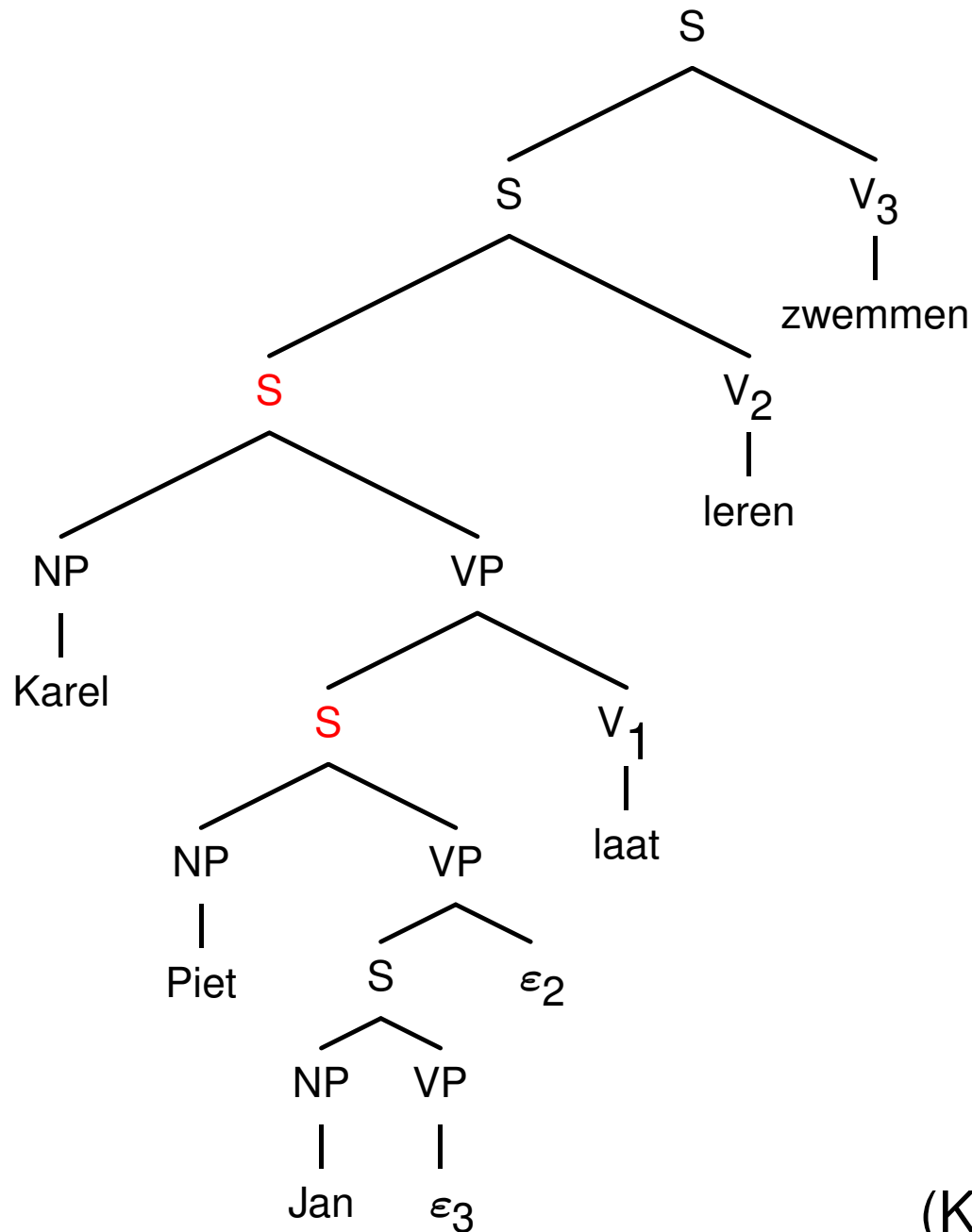
'dat Karel Piet Jan laat leren zwemmen'

(76)



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(76)



Some formal properties

- TAGs (even TIGs) strongly lexicalize CFGs

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- Lexicalized tree adjoining grammars (LTAGs):
each elementary tree has at least one lexical anchor

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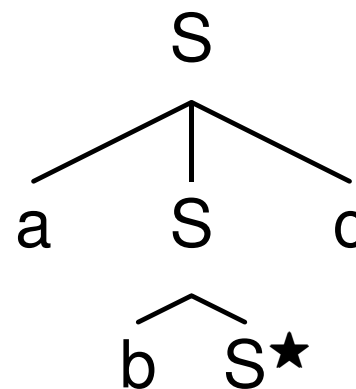
But so far, $TACL$ is not an AFL, because in particular, it is **not closed under intersection with regular languages**.

■ Consider the TAG G_{ex1} whose elementary trees are the following :

(α_{ex1})



(β_{ex1})

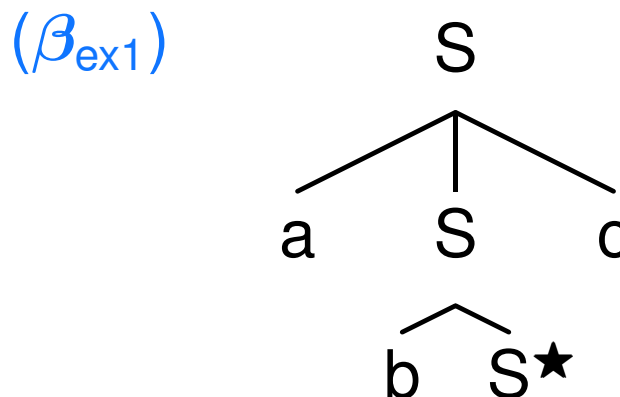
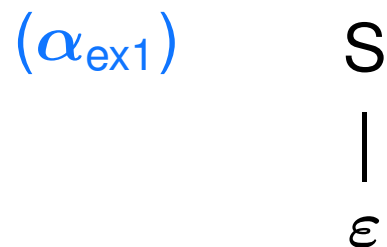


Some formal properties

$$\blacksquare \text{ CFL } \subsetneq \text{ TACL } \subsetneq \text{ CSL}$$

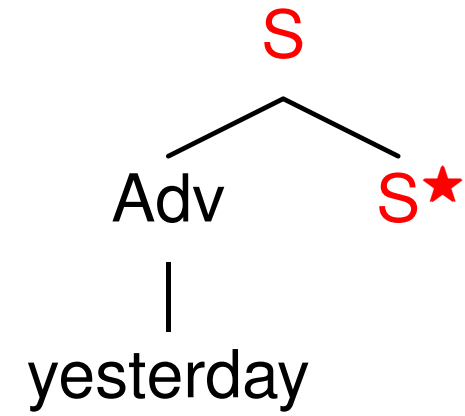
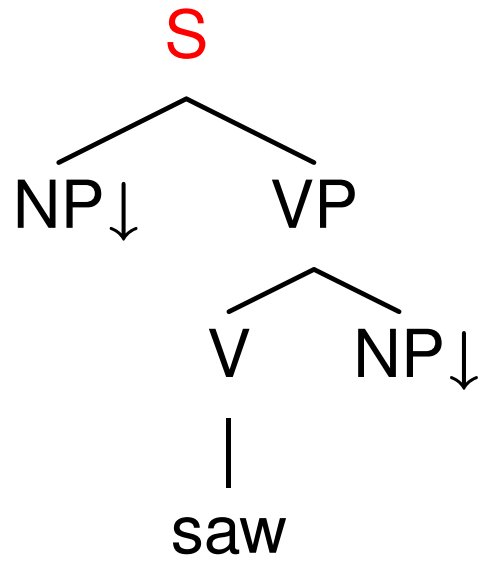
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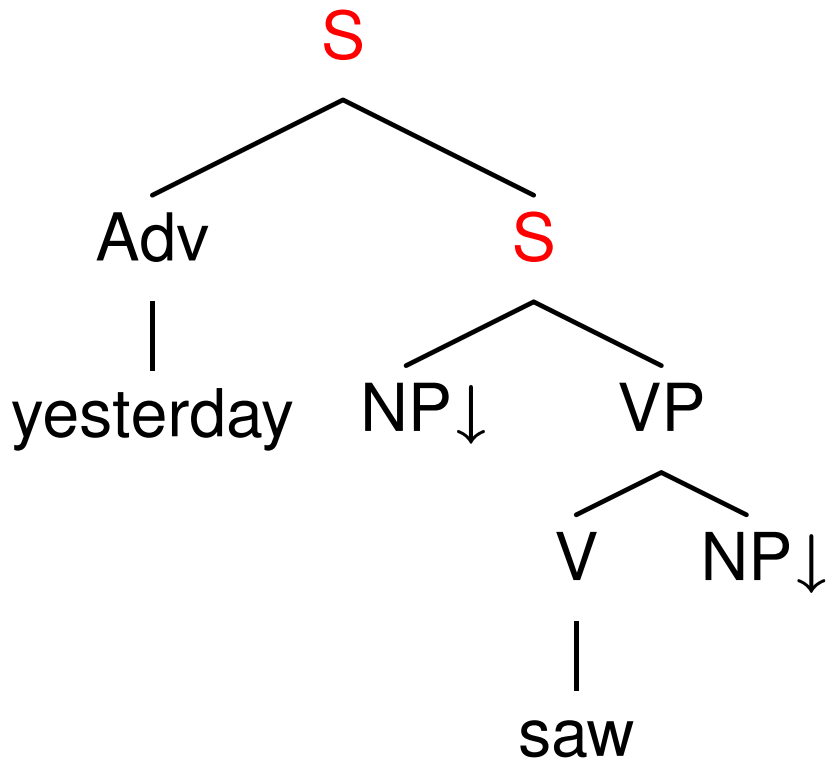
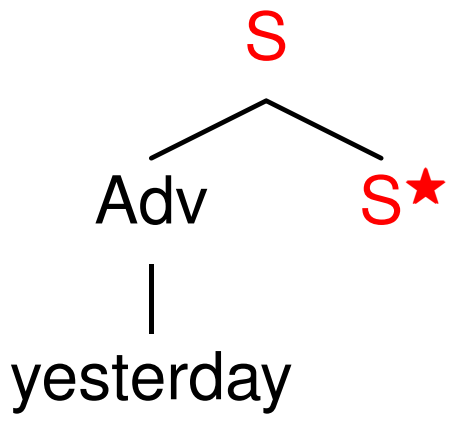
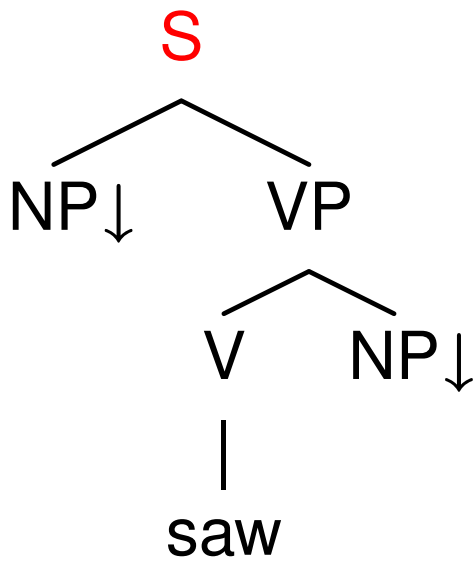


$$L(G_{\text{ex1}}) \cap \{a^k b^l c^m \mid k, l, m \geq 0\} = \{a^n b^n c^n \mid n \geq 0\}$$

Unrestricted adjoining

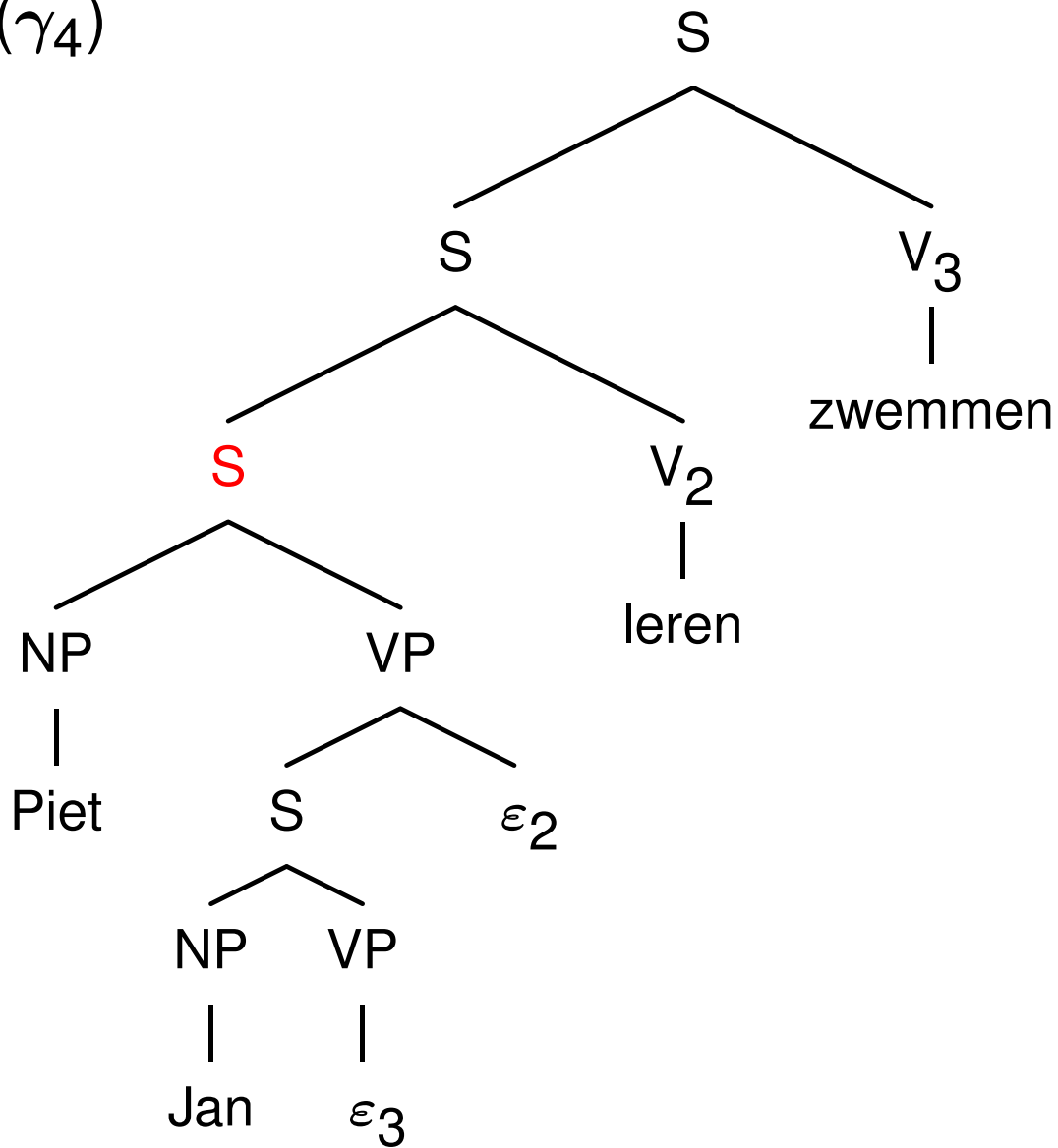


Unrestricted adjoining

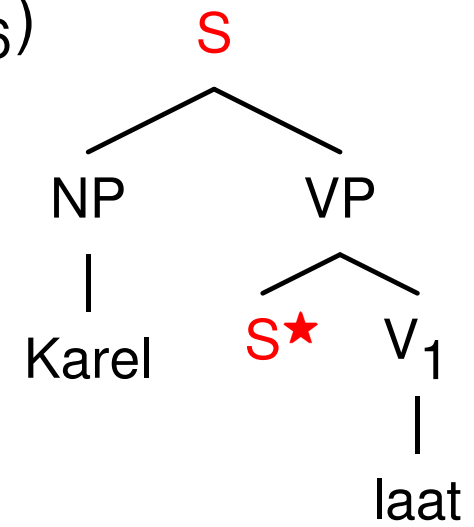


Unrestricted adjoining

(γ_4)

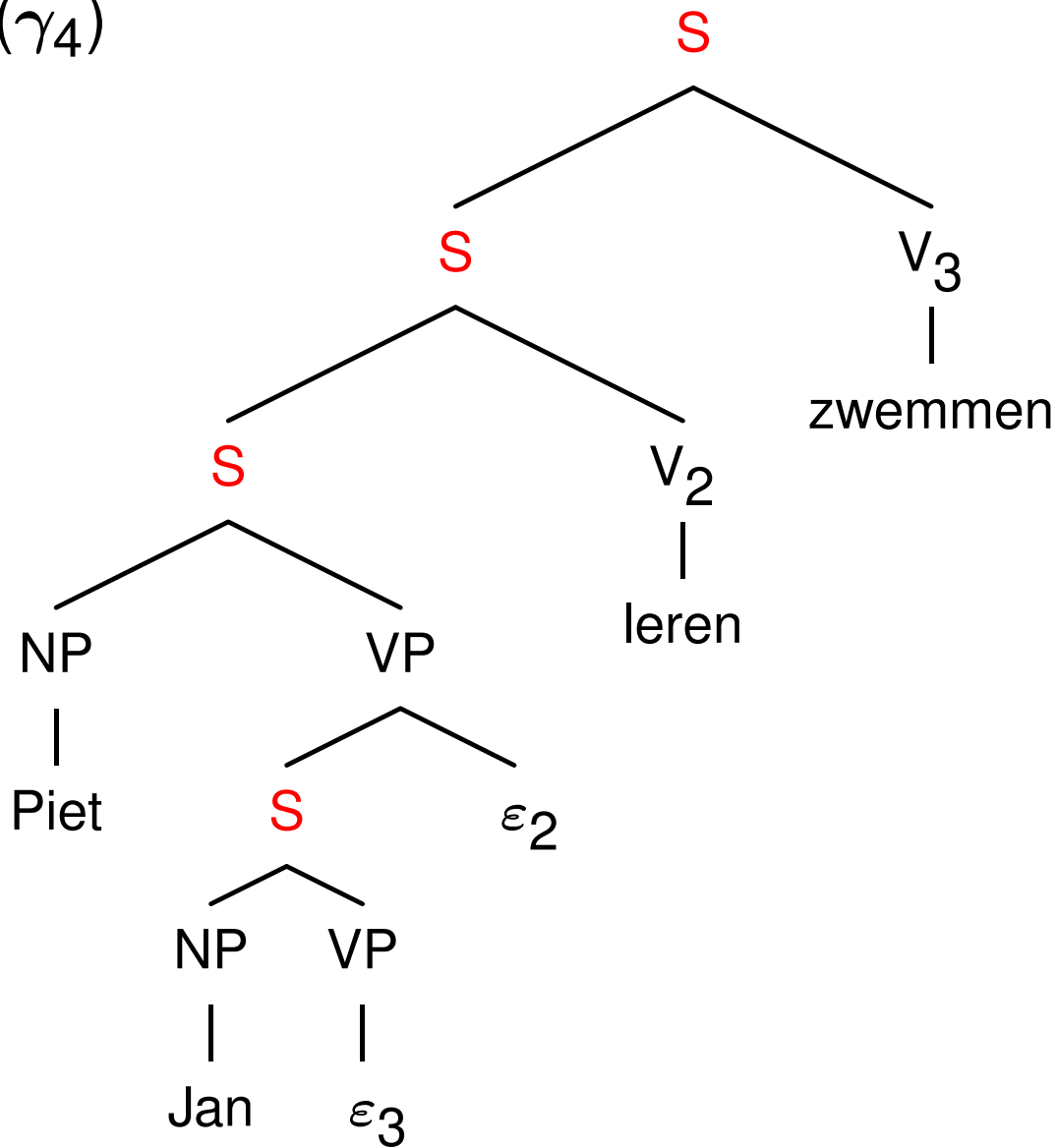


(β_6)

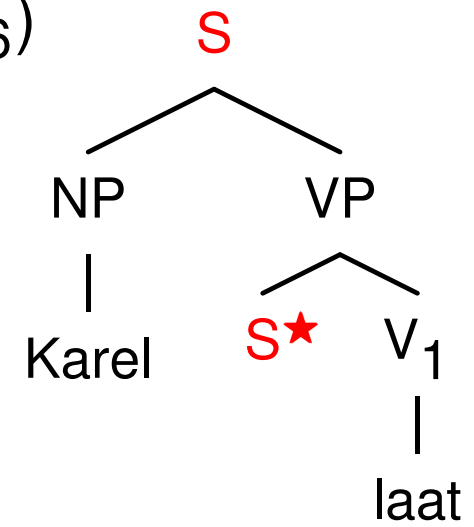


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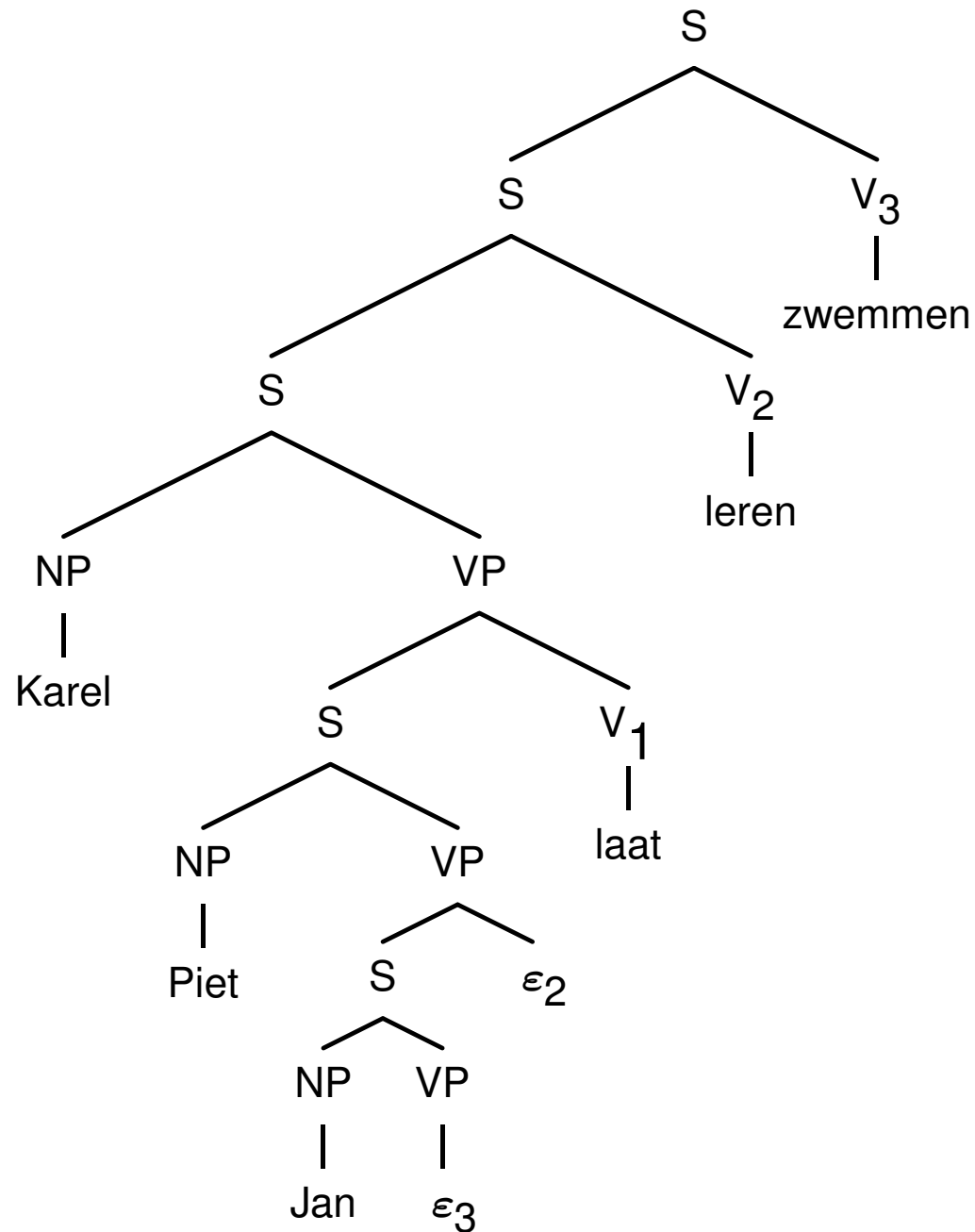


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'dat Karel Piet Jan laat leren zwemmen'

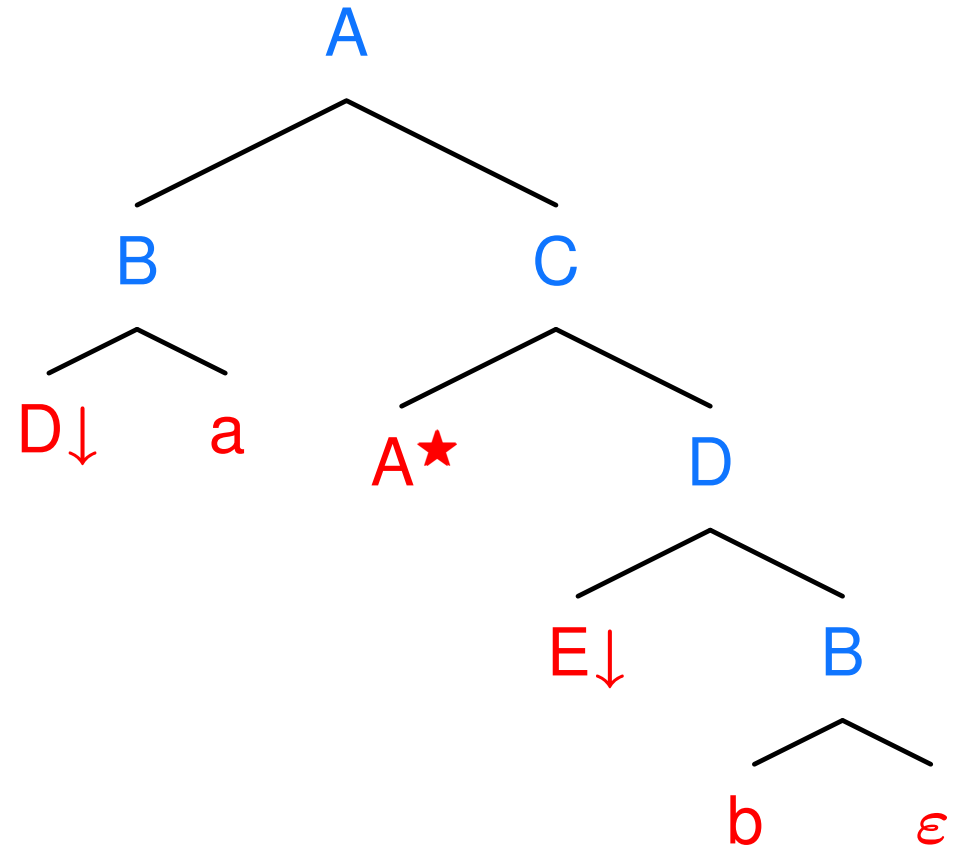
(76)



Adding adjoining constraints

V_N a set of nonterminals

V_T a set of terminals



$$t = \langle N_t, \triangleleft_t^*, \prec_t, \text{label}_t \rangle$$

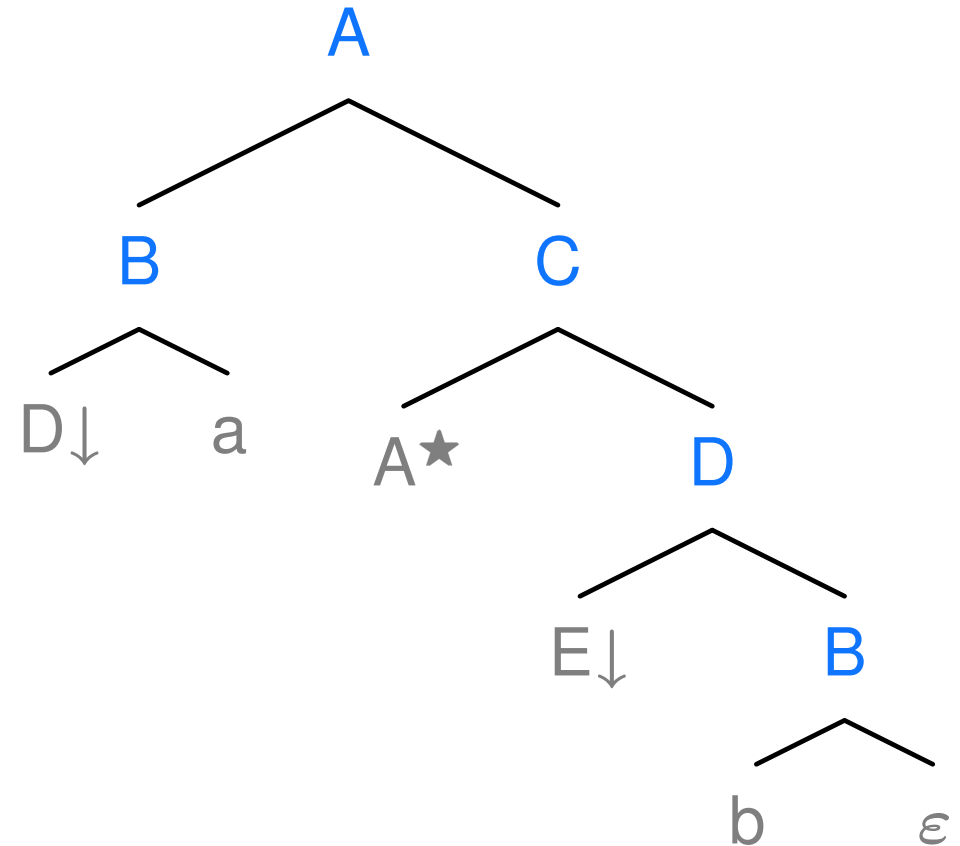
$\text{label}_t : \text{NonLeaves}_t \rightarrow V_N$

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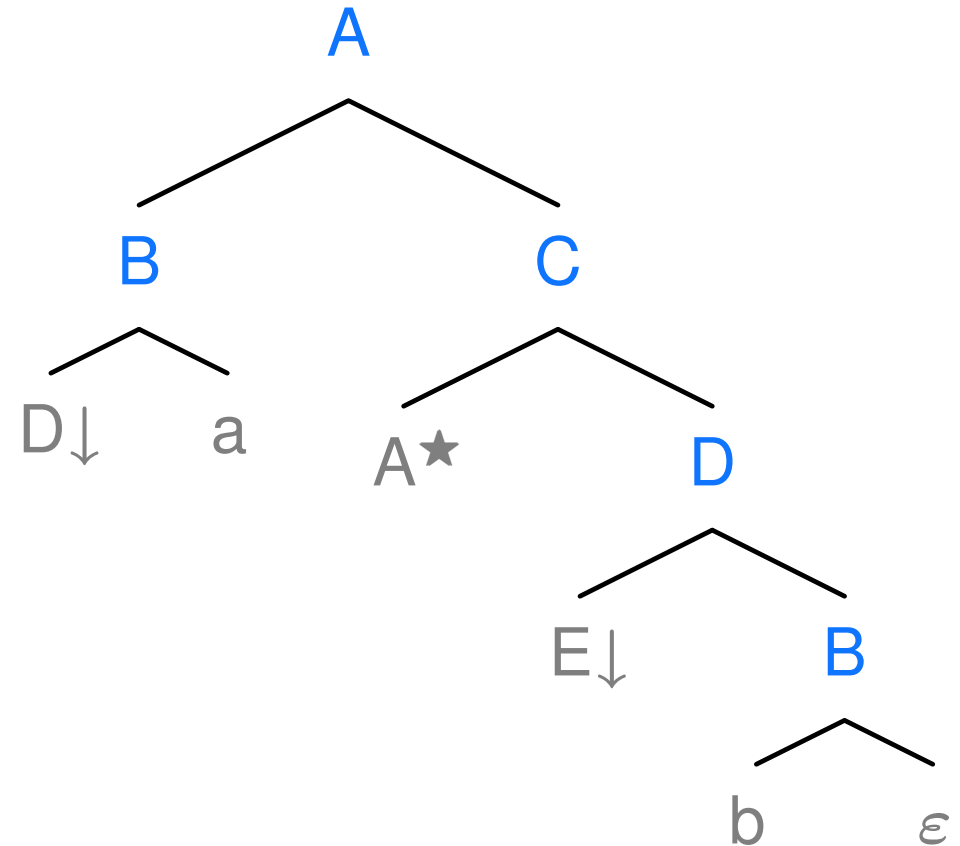
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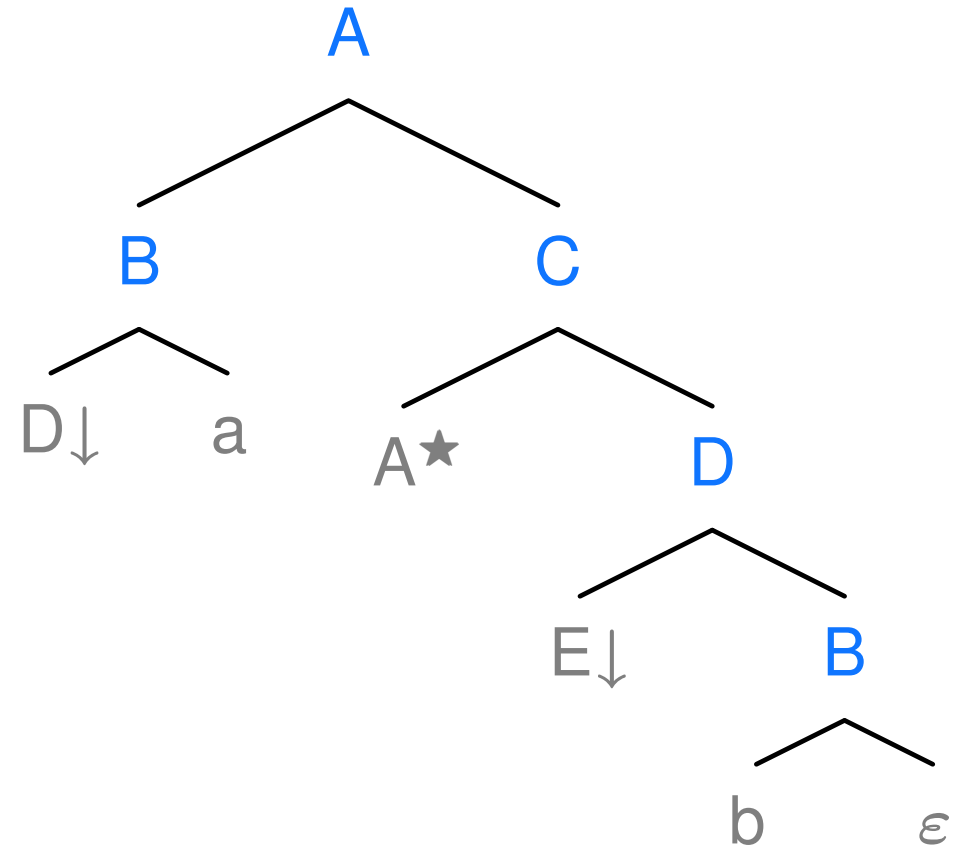
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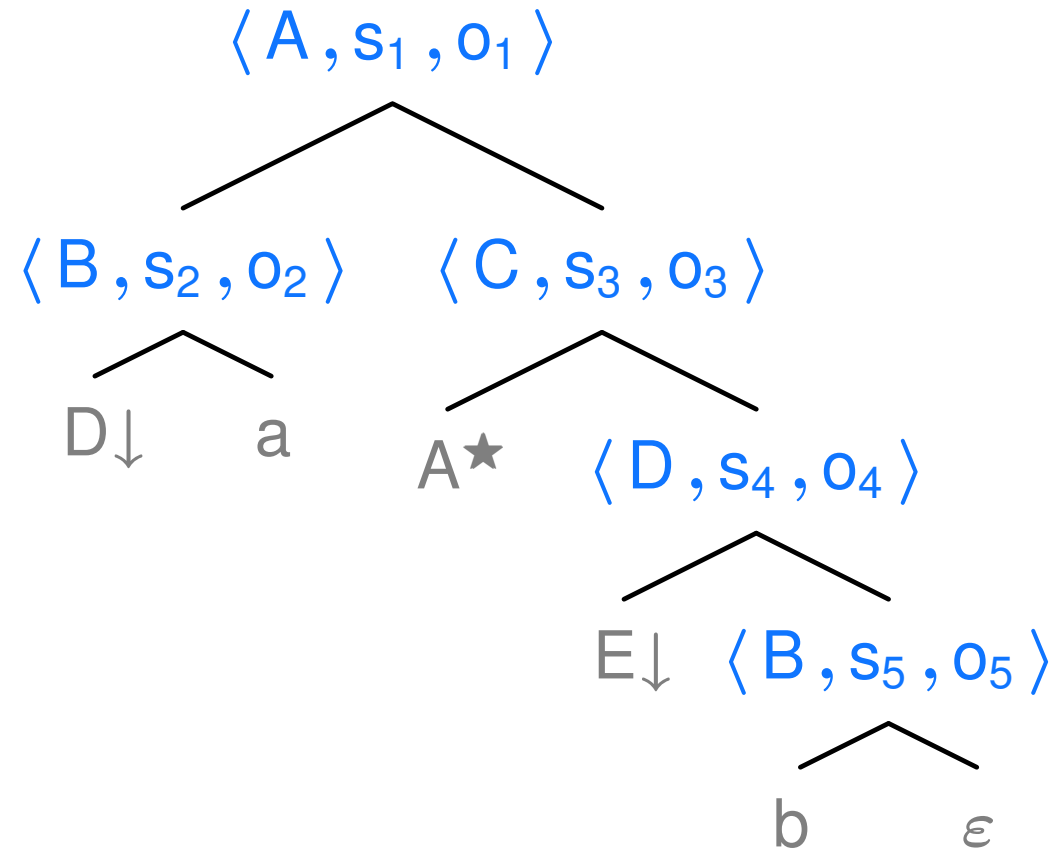
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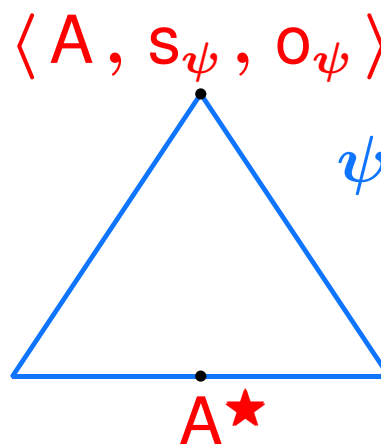
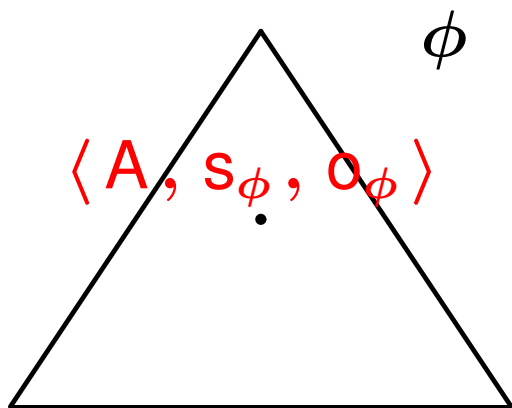
$$\text{Leaves}_t \rightarrow V_N \{\downarrow, \star\} \cup V_T \cup \{\epsilon\}$$

$$\text{adjoining} : \text{Trees}(V) \times \text{Trees}(V) \xrightarrow{\text{part}} 2^{\text{Trees}(V)}$$

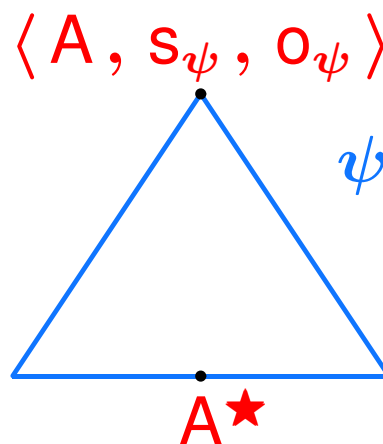
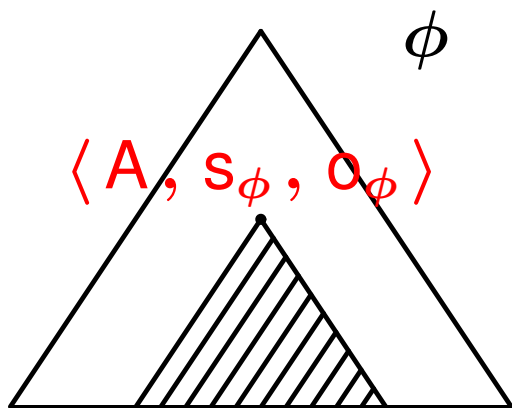
$\langle \phi, \psi \rangle \in \text{Domain}(\text{adjoining}) : \iff$

- ψ 's **root** is labeled $\langle A, s_\psi, o_\psi \rangle$ for some $s_\psi \subseteq T_{\text{Aux}}$ and $o_\psi \in \{+, -\}$, and ψ has a **leaf** labeled A^\star
- ϕ has a **node** labeled $\langle A, s_\phi, o_\phi \rangle$ for some $A \in V_N$, $s_\phi \subseteq T_{\text{Aux}}$ and $o_\phi \in \{+, -\}$ such that $\psi \in s_\phi$

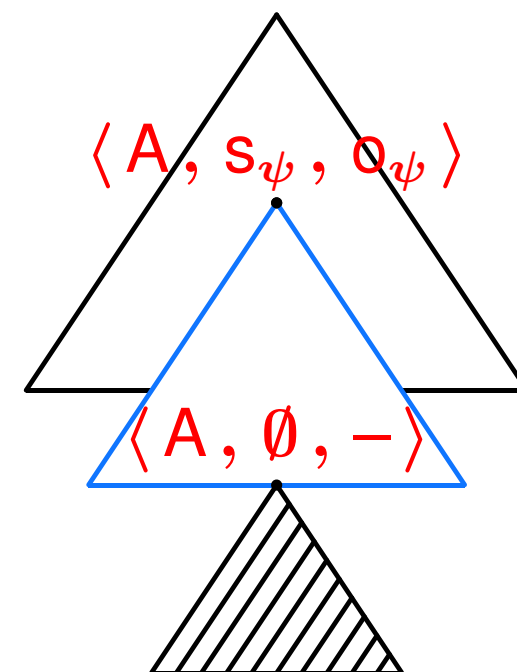
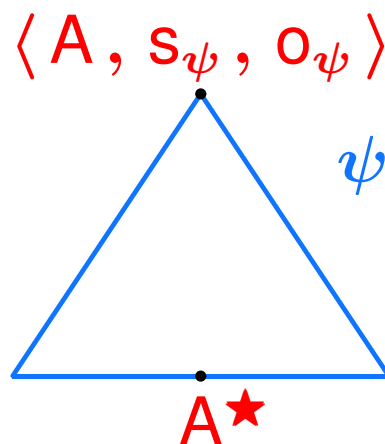
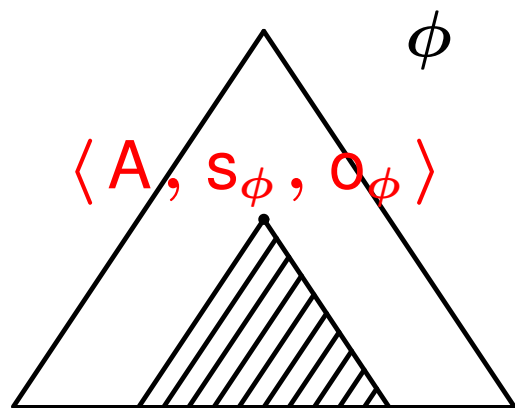
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Tree adjoining languages

Closure(G), the **closure** of a TAG $G = \langle V_N, V_T, T_{Ini}, T_{Aux}, S \rangle$,

is the closure of $T_{Ini} \cup T_{Aux}$ under substitution and adjoining.

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$t \in \text{Closure}(G)$ is **complete** : \iff

each **non-leaf-label** is of the form $\langle A, s, - \rangle$ for some $A \in V_N$ and $s \subseteq T_{Aux}$, $A = S$ in case of the **root**, and **yield(t) \in Strings(V_T)**.

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The **tree** and **string language** generated by G

$$T(G) = \{ t \mid t \in \text{Closure}(G) \text{ and complete} \}$$

$$L(G) = \{ \text{yield}(t) \mid t \in T(G) \}$$

Some formal properties

■ $CFL \subsetneq TACL \subsetneq CSL$

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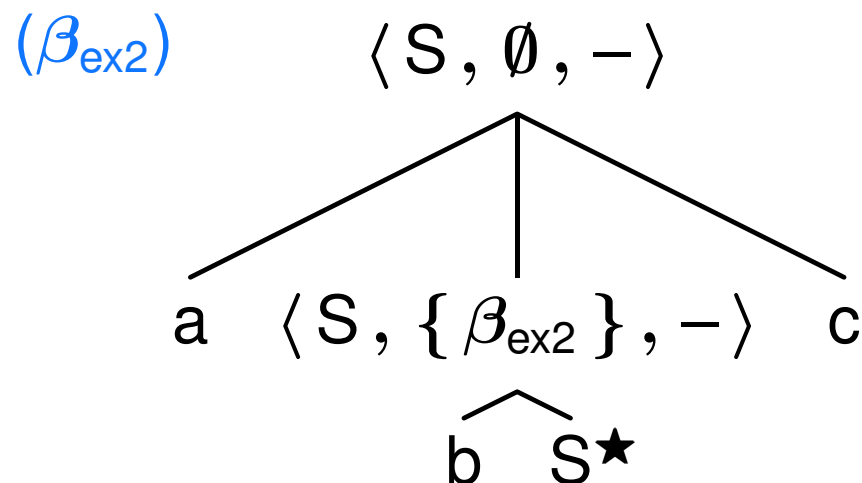
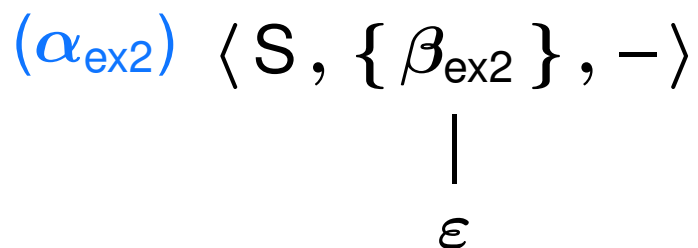
Now $TACL$ is a **substitution-closed full AFL**.

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Now TAL is a **substitution-closed full AFL**.

- Consider the TAG G_{ex2} whose elementary trees are the following :

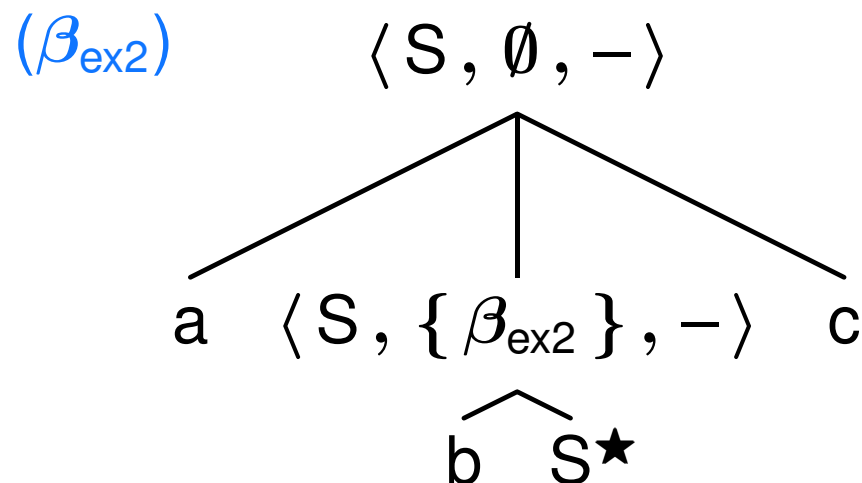
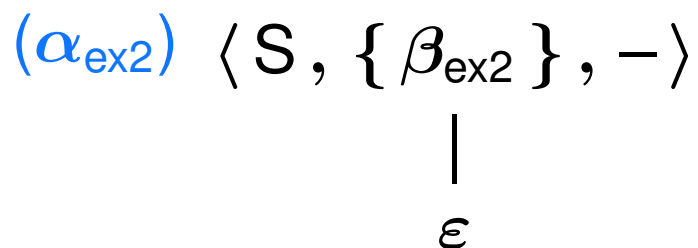


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$$L(G_{ex2}) = \{a^n b^n c^n \mid n \geq 0\}$$

Extensions of TAGs: multicomponent TAGs

$$G = \langle V_N, V_T, T_{Ini}, \text{Tuples}_{Aux}, S \rangle$$

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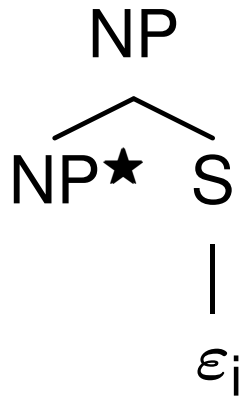
Extensions of TAGs: multicomponent TAGs

- Reasons for tree-local MCTAGs: “syntactic sugar”
- Reasons for set-local MCTAGs (?): “real” verb clusters

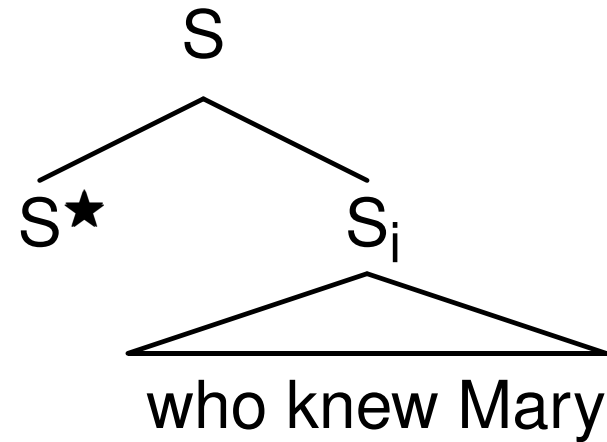
Elementary tree set: relative clause extraposed from subject

$$\beta_3 = \langle \beta_{31}, \beta_{32} \rangle$$

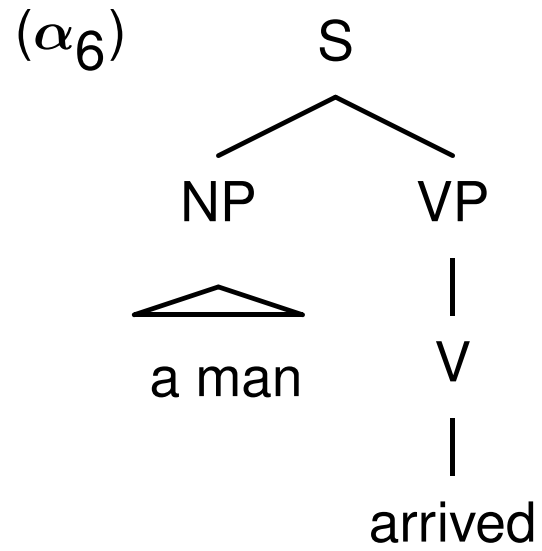
(β_{31})



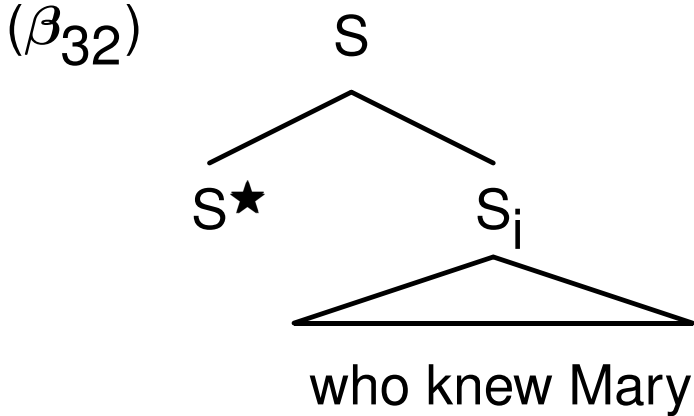
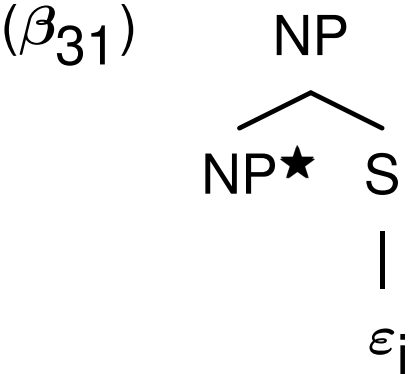
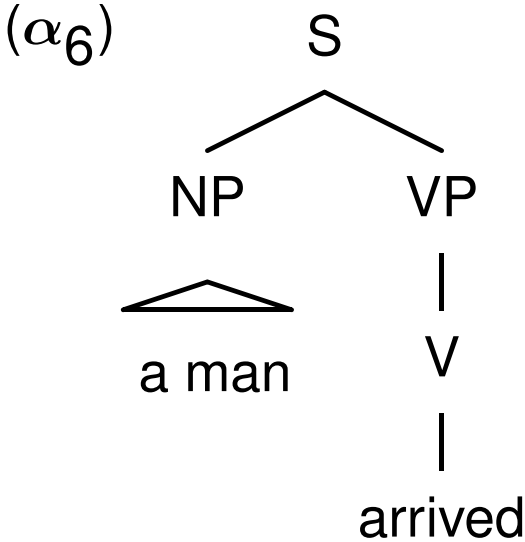
(β_{32})



Relative clause extraposed from subject NP

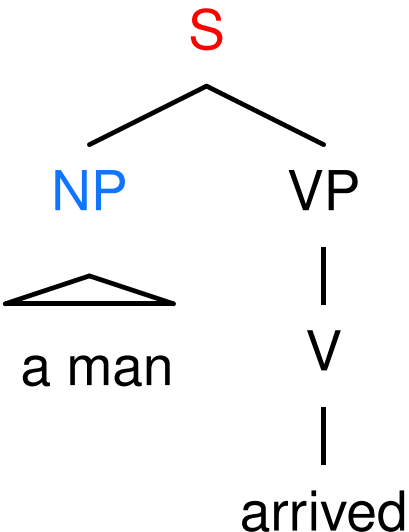


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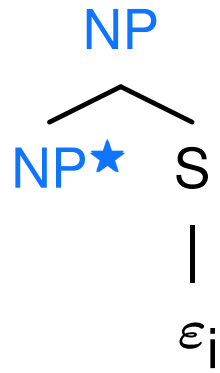


Relative clause extraposed from subject NP

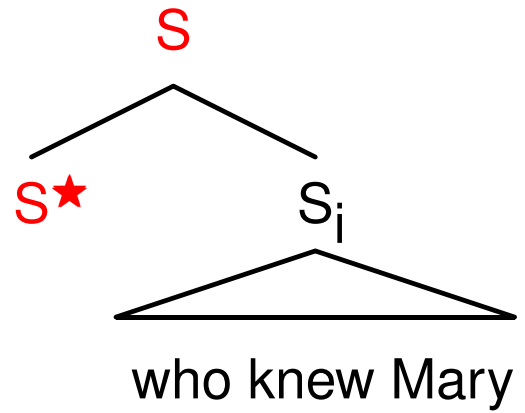
(α_6)



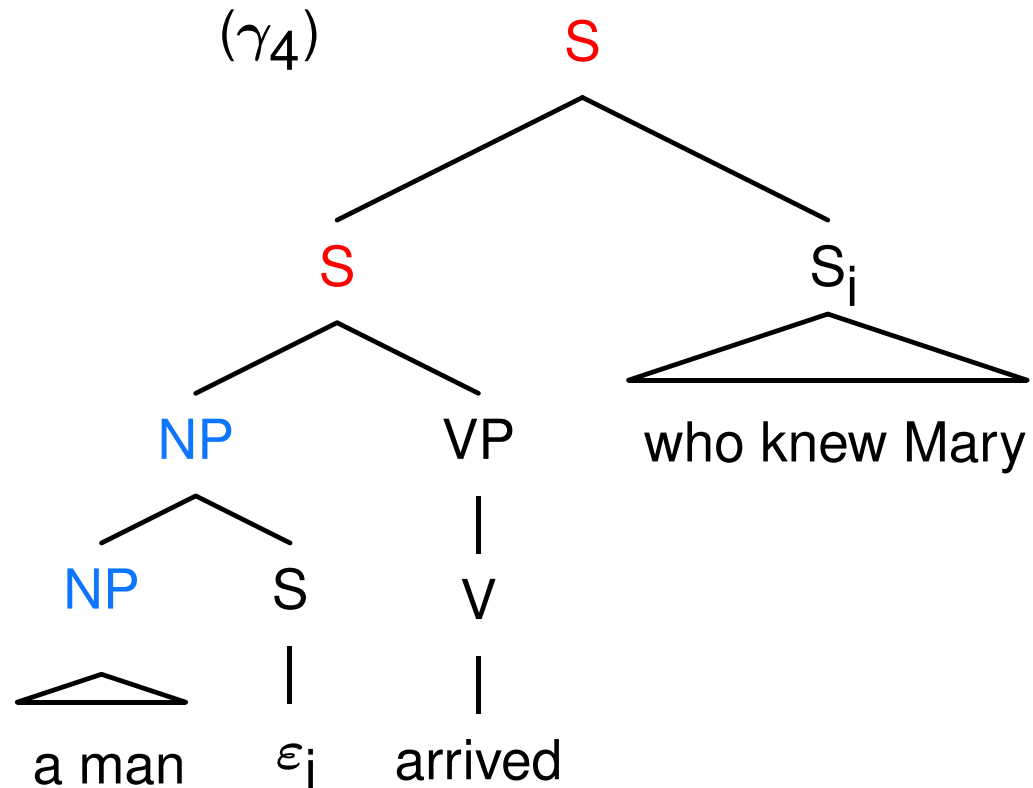
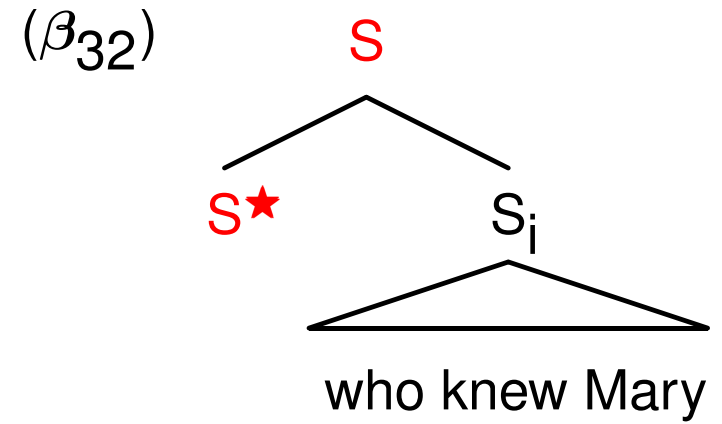
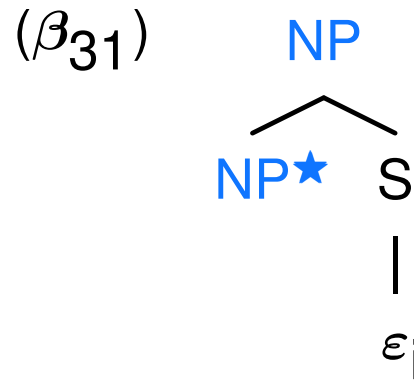
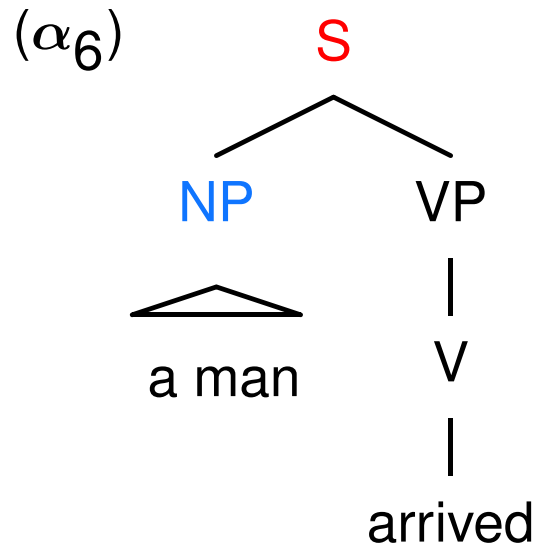
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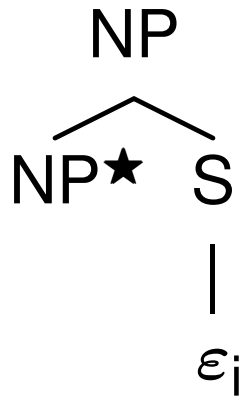
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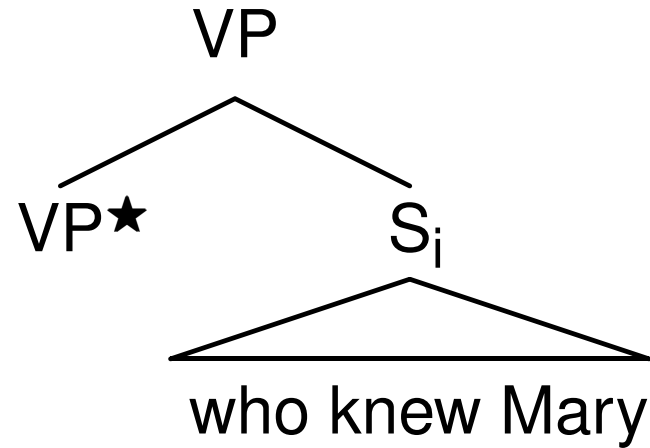
Elementary tree set: relative clause extraposed from object

$$\beta_4 = \langle \beta_{41}, \beta_{42} \rangle$$

(β_{41})



(β_{42})

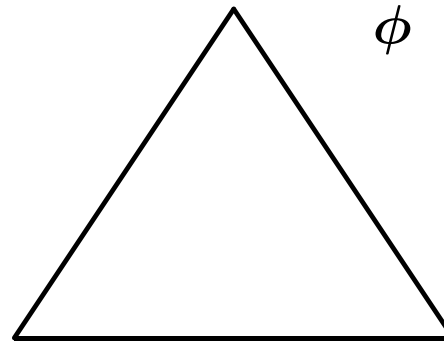


Tree-local multicomponent TAGs (MCTAGs)

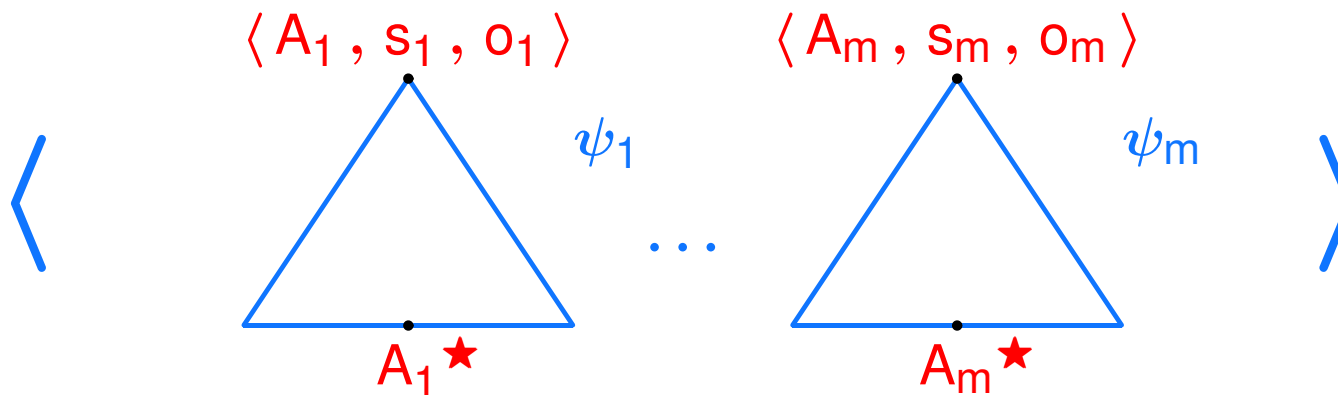
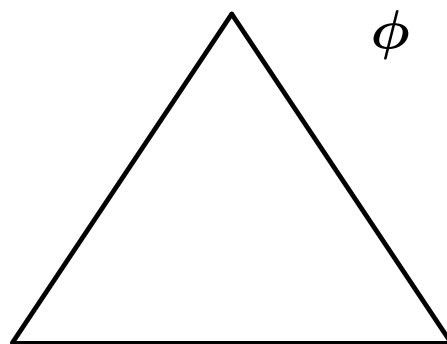
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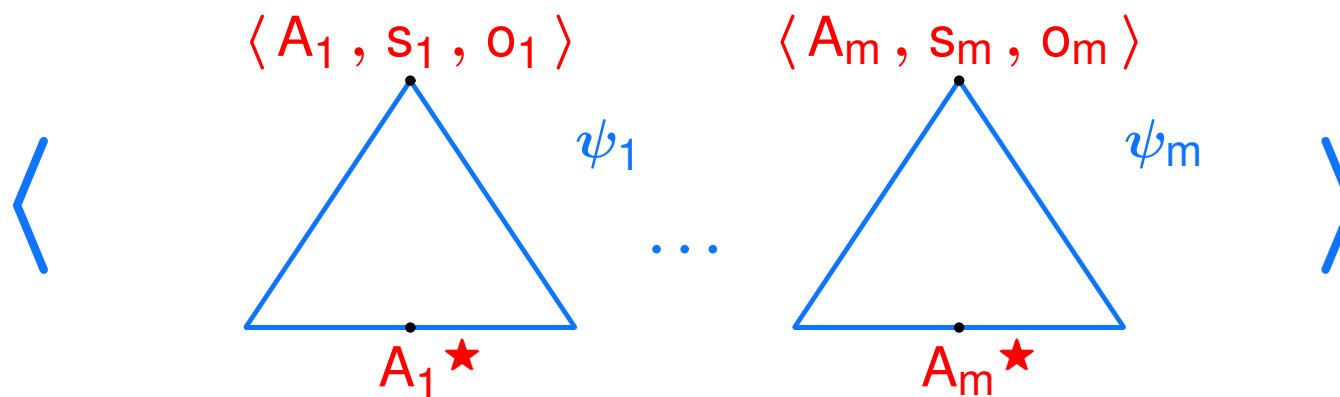
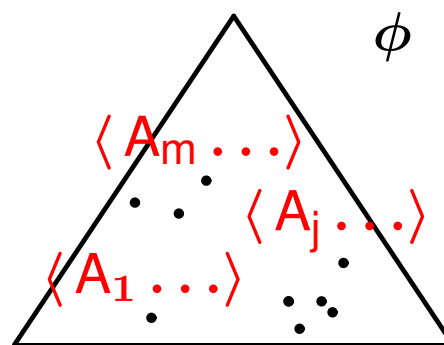
adjoining : $(T_{\text{Ini}} \cup \text{Tuples}_{\text{Aux}}) \times \text{Treetuples}(V) \xrightarrow{\text{part}} 2^{\text{Treetuples}(V)}$



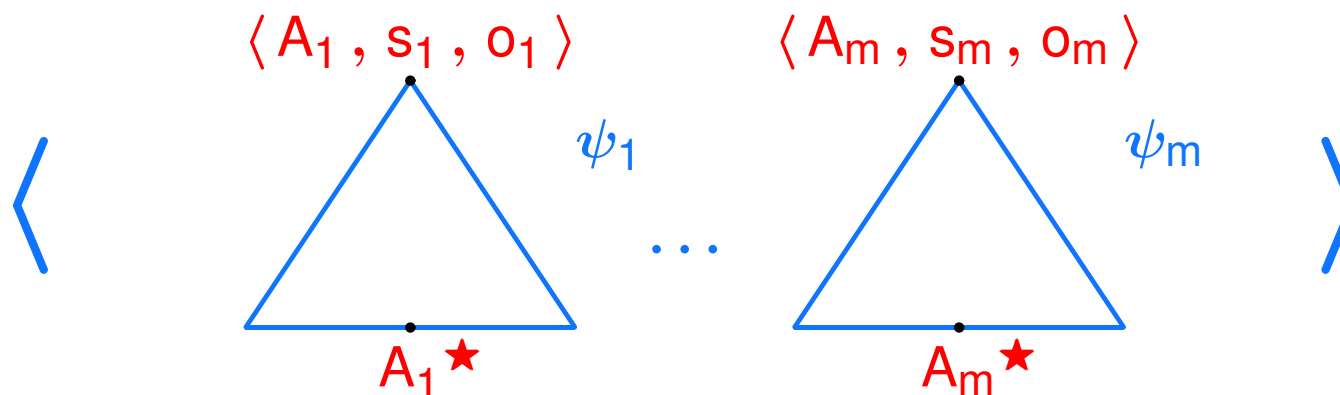
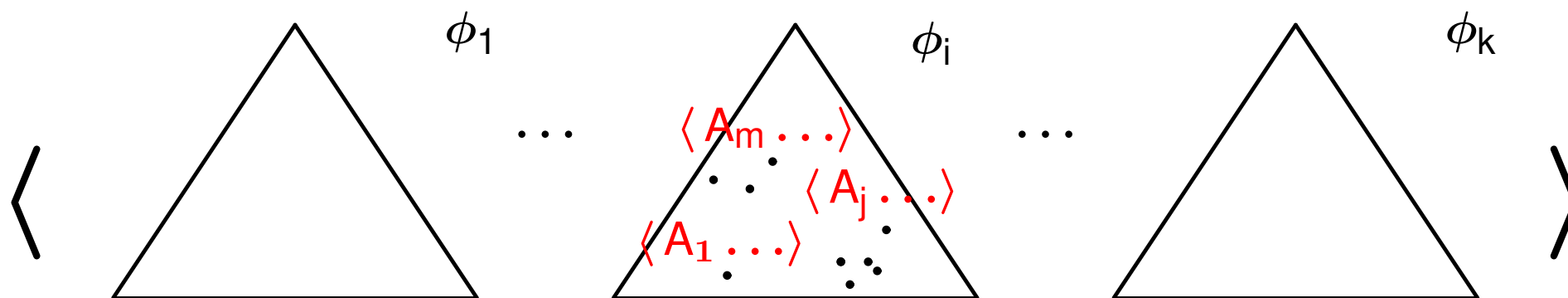
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adjoining : $(T_{\text{Ini}} \cup \text{Tuples}_{\text{Aux}}) \times \text{Treetuples}(V) \xrightarrow{\text{part}} {}_2\text{Treetuples}(V)$



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Some formal properties

TAGs and tree-local MCTAGs are strongly equivalent !

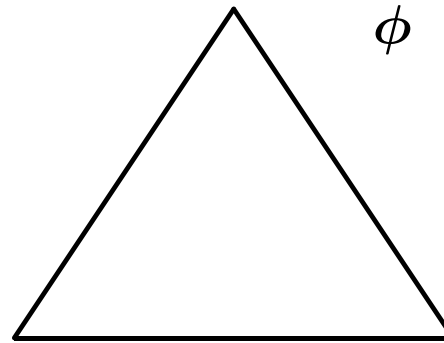
This does not hold for TAGs and set-local MCTAGs !

Set-local multicomponent TAGs (MCTAGs)

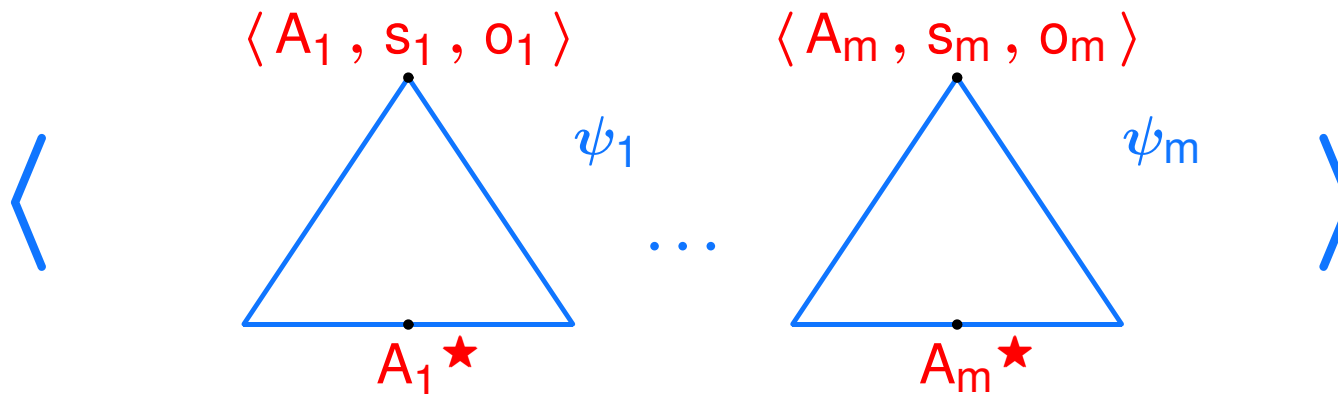
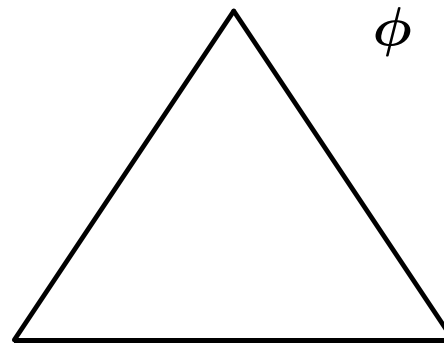
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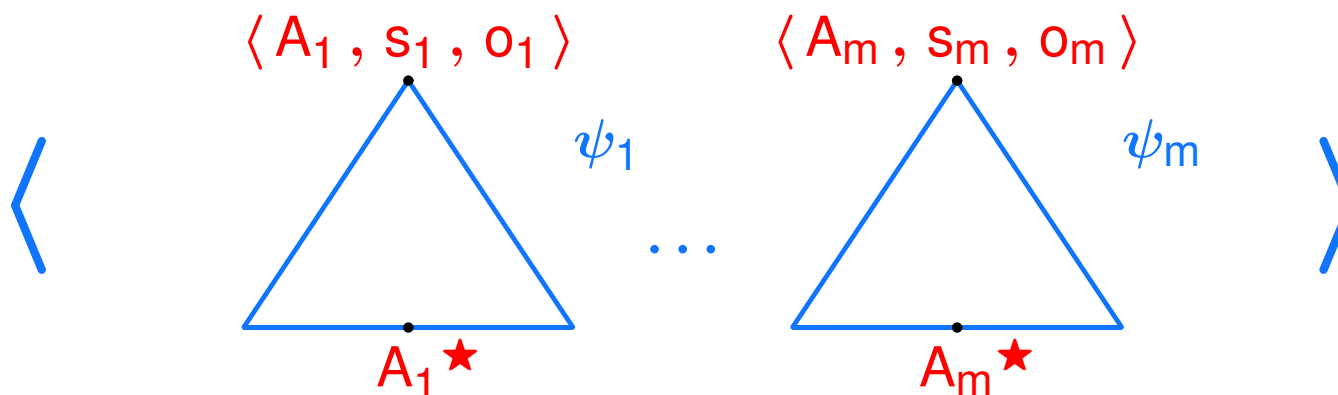
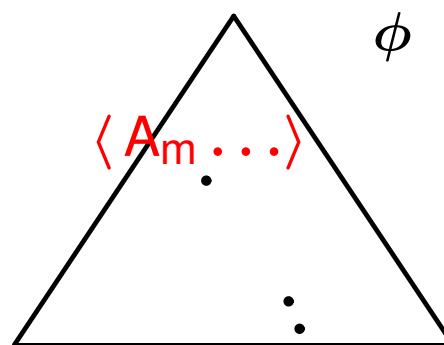
adjoining : $(T_{\text{Ini}} \cup \text{Tuples}_{\text{Aux}}) \times \text{Treetuples}(V) \xrightarrow{\text{part}} 2^{\text{Treetuples}(V)}$



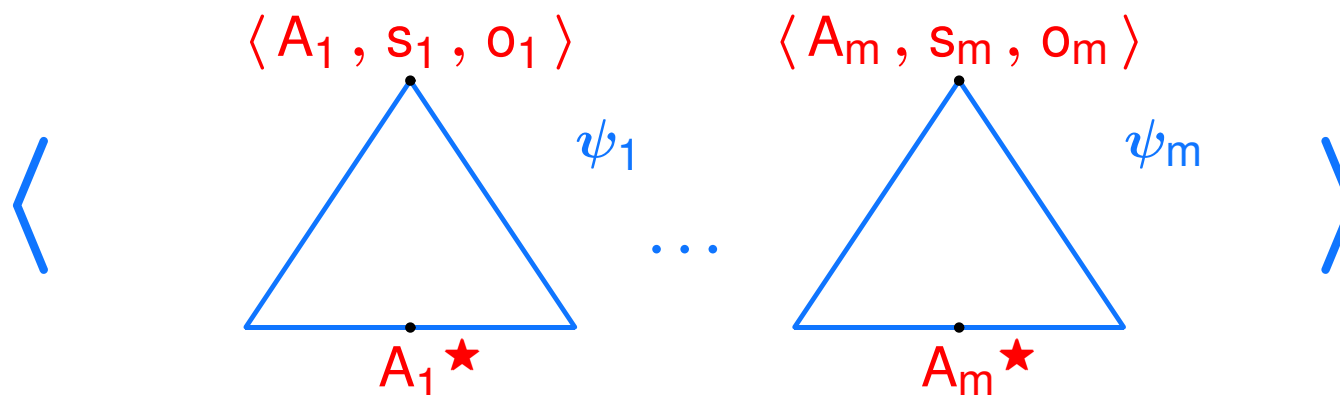
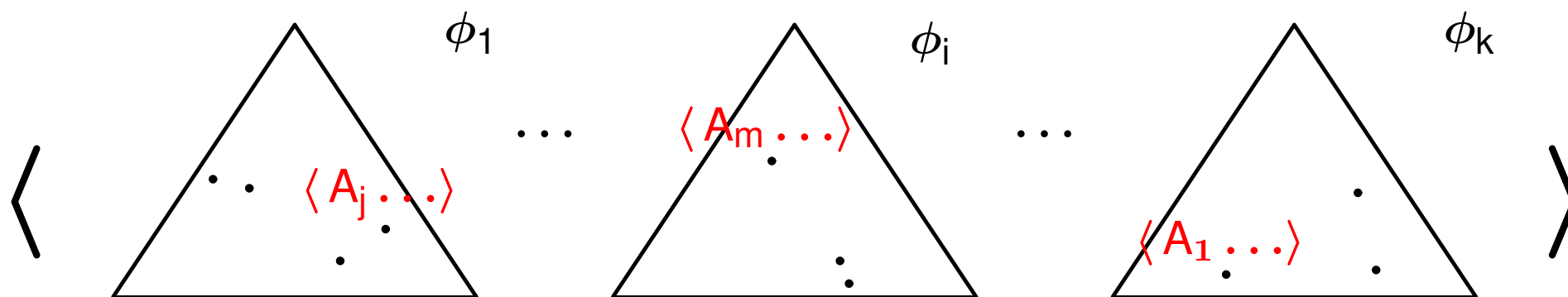
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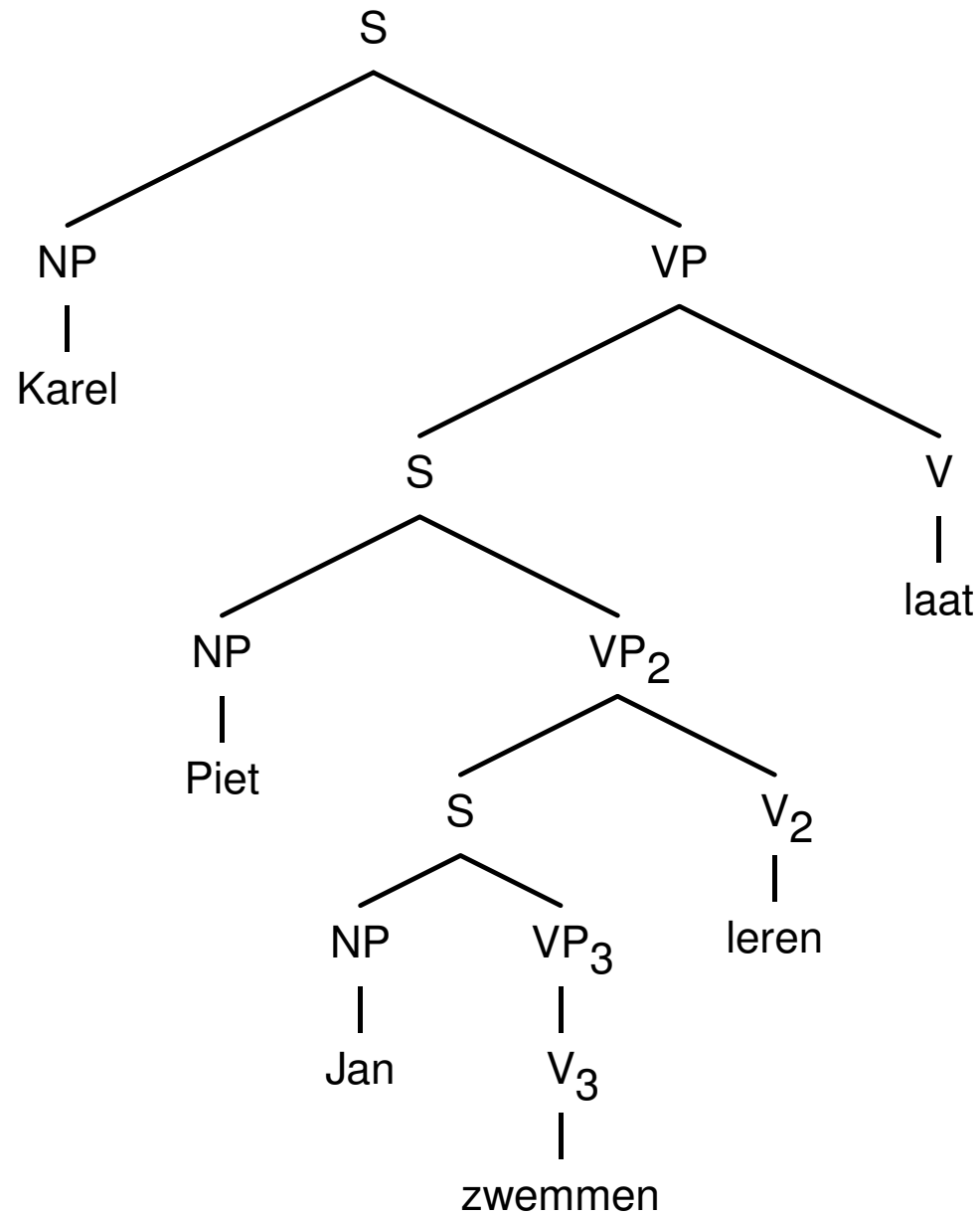
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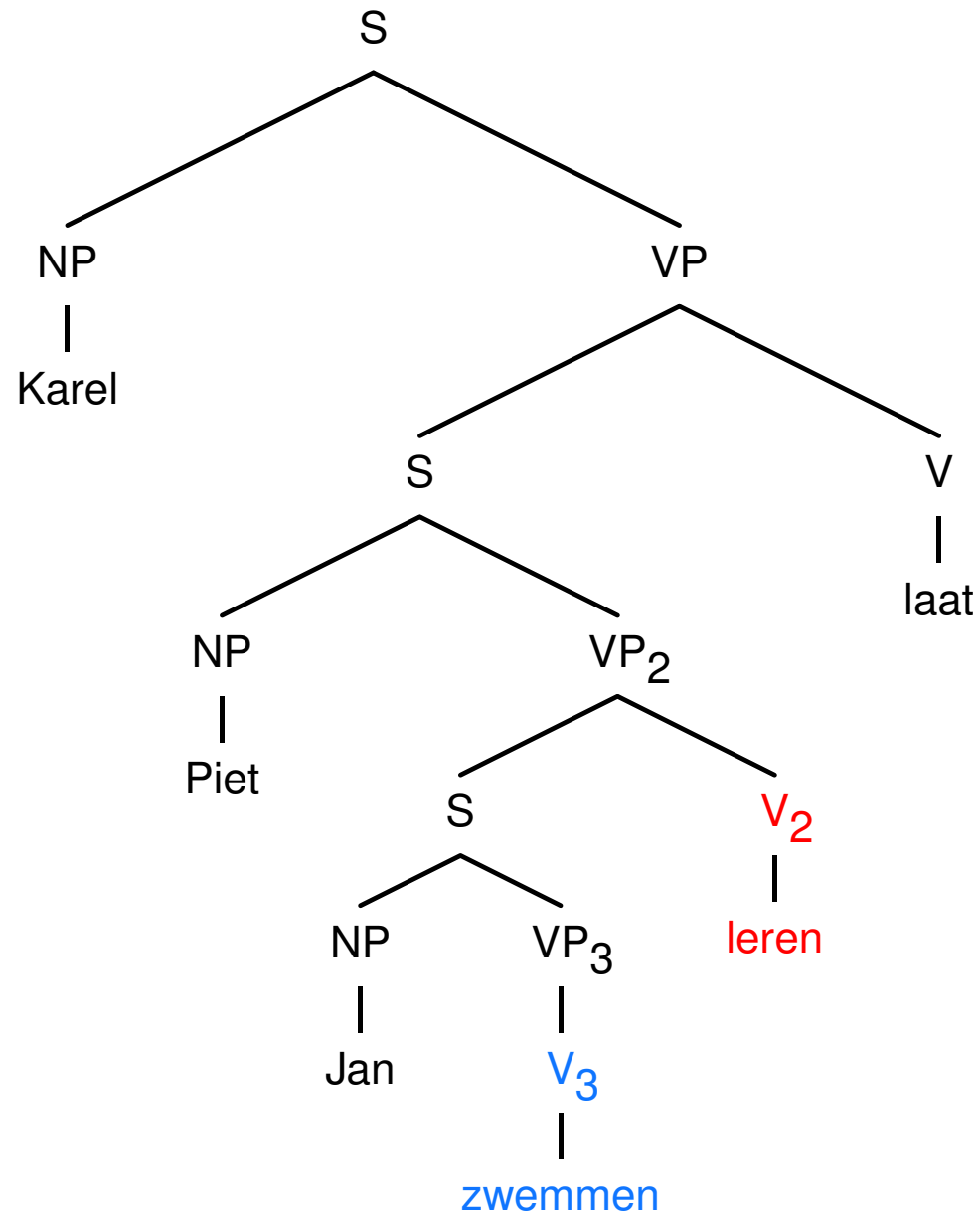
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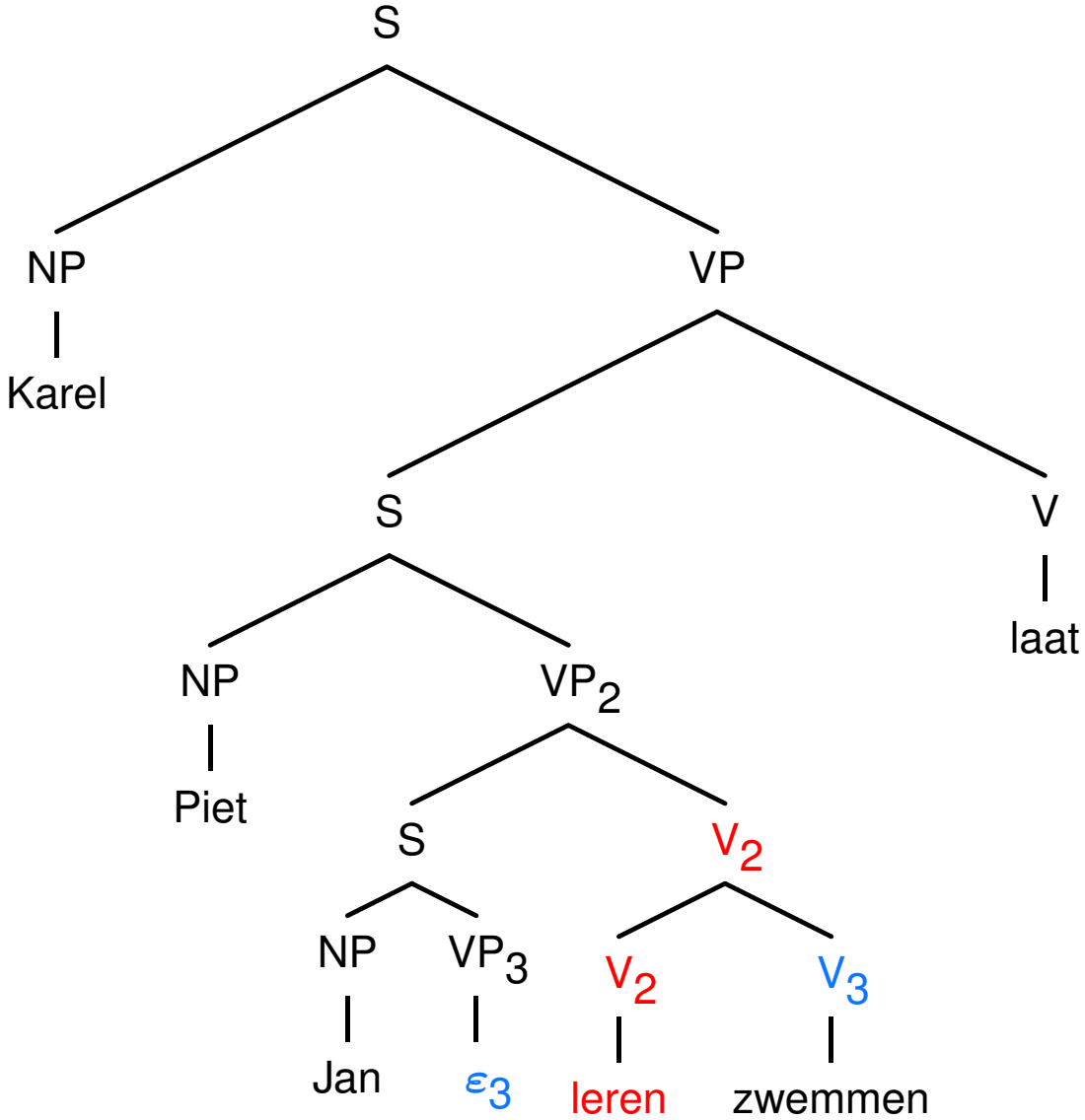
Basic structure: 'dat Karel Piet Jan laat leren zwemmen'



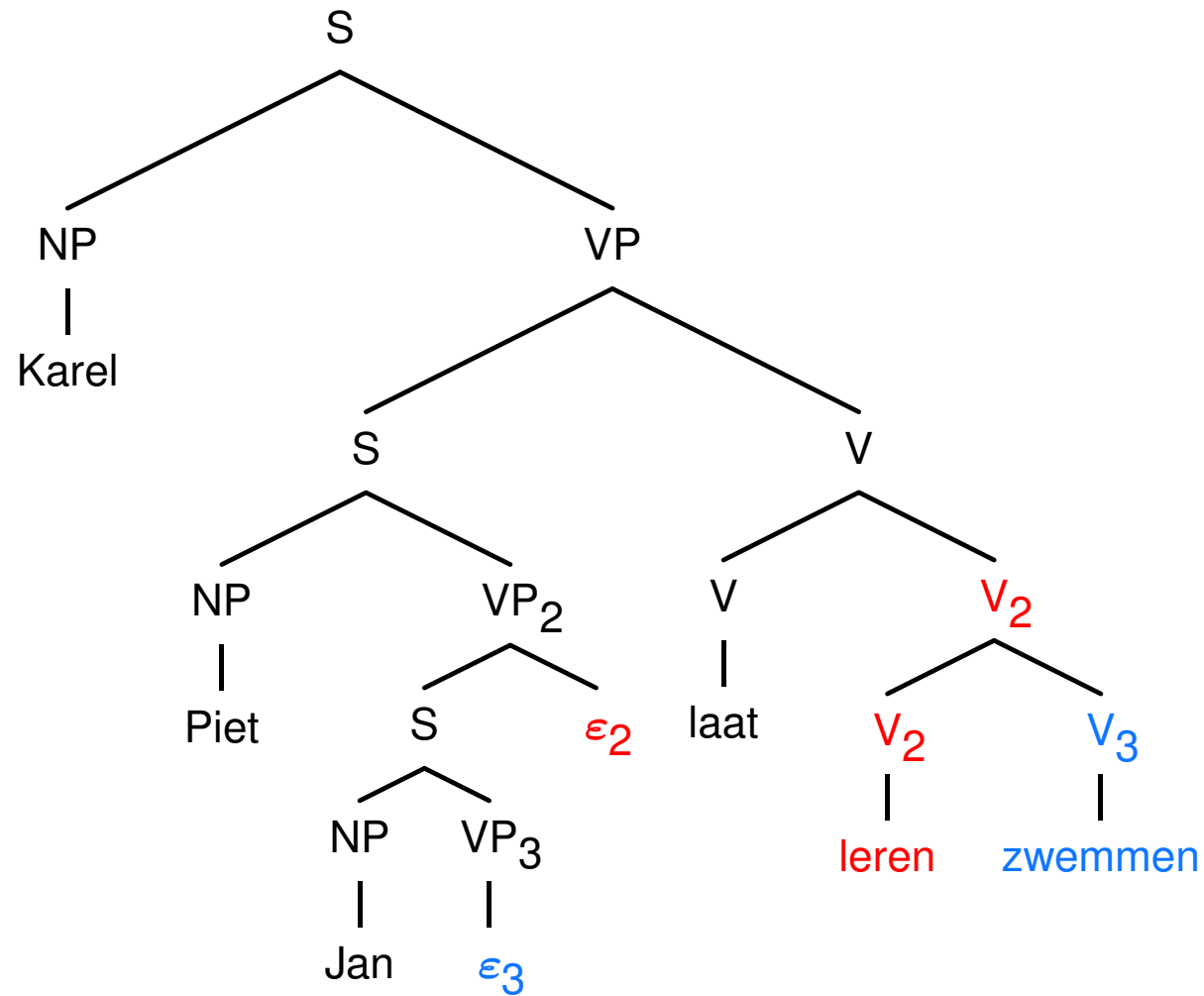
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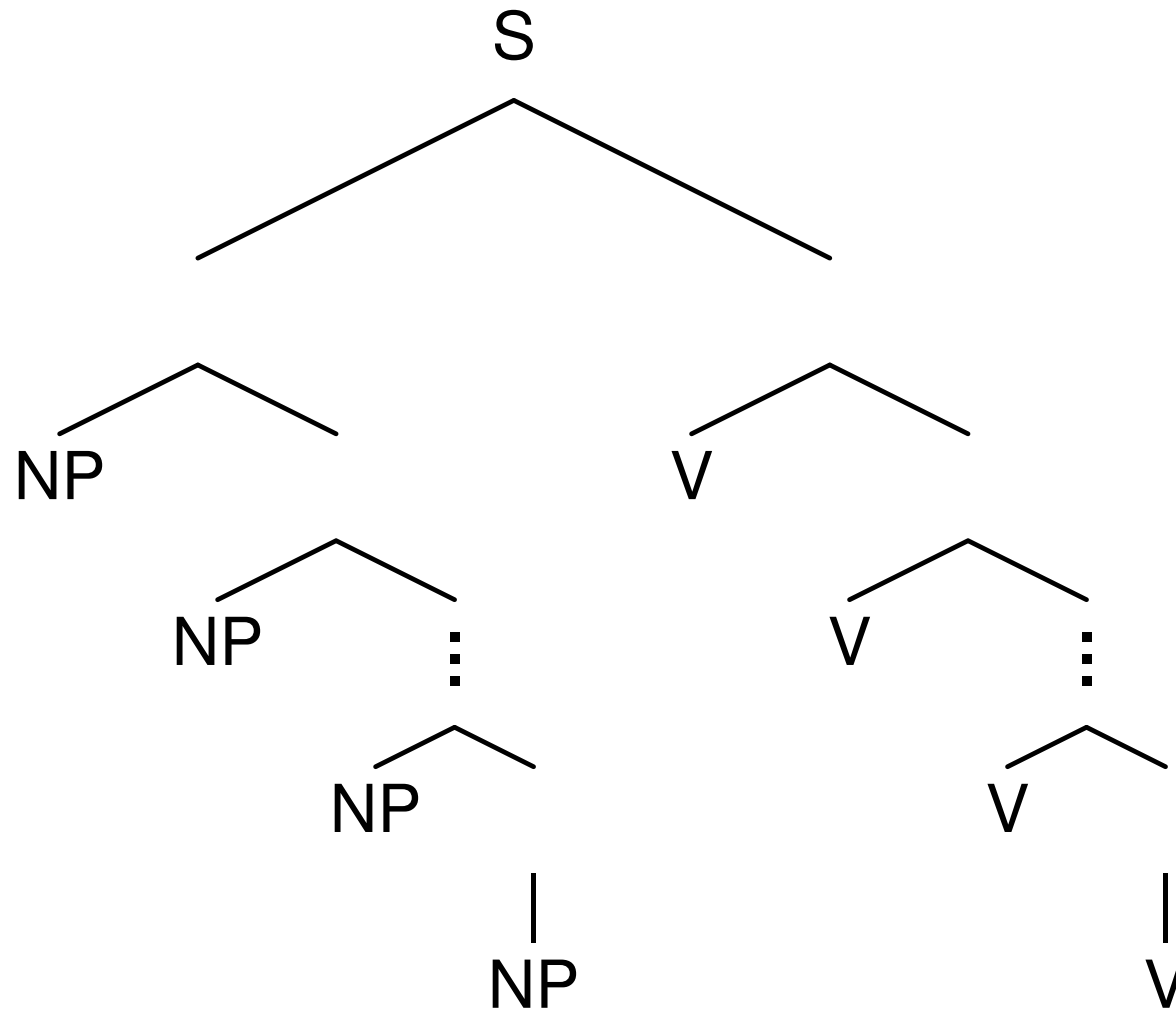
Successive cyclic head movement: 'dat Karel Piet Jan laat leren zwemmen



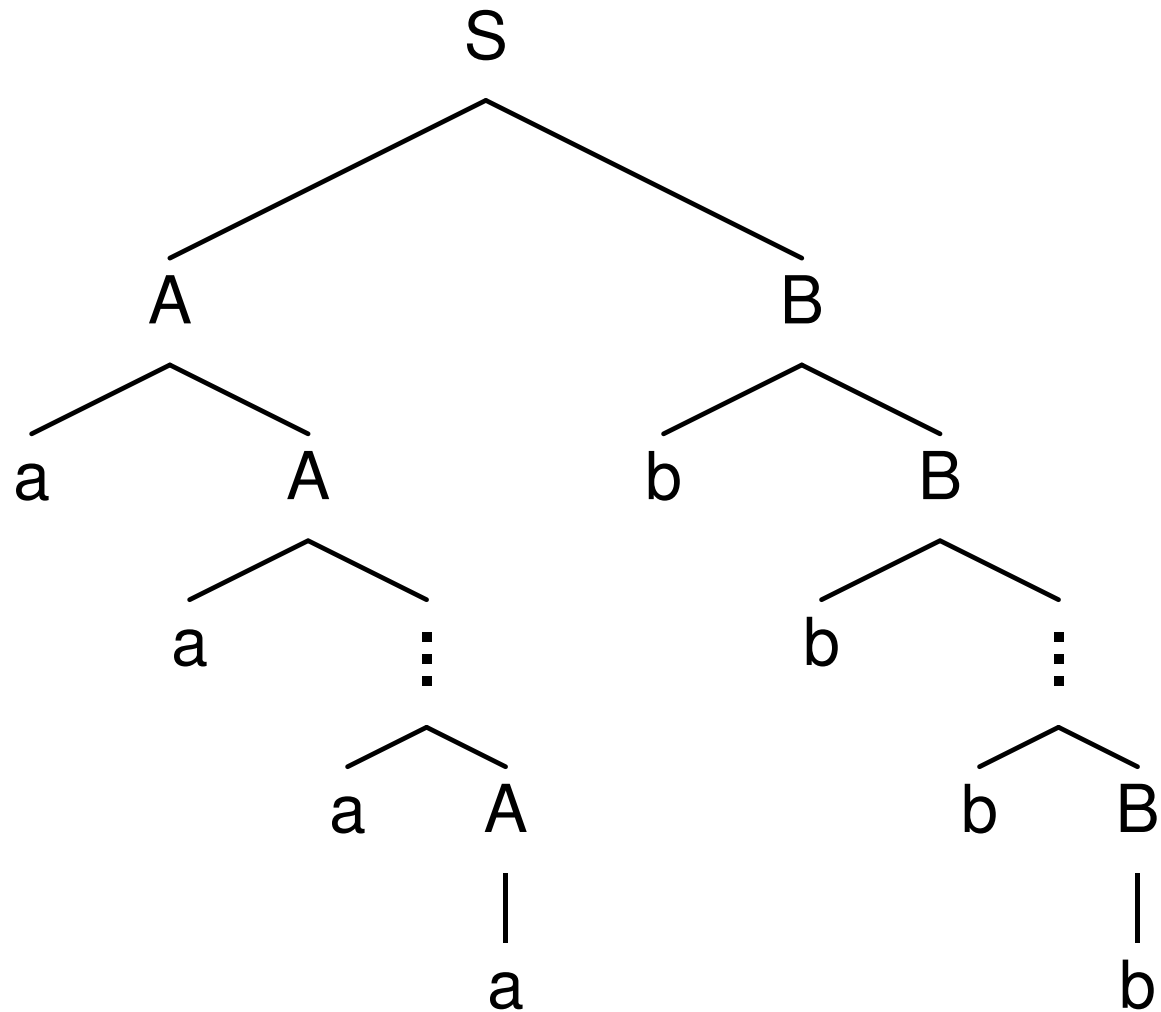
“Real” verb cluster analysis: ‘dat Karel Piet Jan laat leren zwemmen’



“Real” verb cluster analysis: essentially reduces to ...



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References

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- Gerald Gazdar. 1988. Applicability of indexed grammars to natural languages. In: U. Reyle and C. Rohrer (eds.), *Natural Language Parsing and Linguistic Theories*, pp. 69–94. D. Reidel, Dordrecht.
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