# Phylogenetic trees II <br> Estimating distances, estimating trees from distances 

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## Background

- ideally, we could infer the historical time since the latest common ancestor for any pair of languages
- not possible - at least not in a purely data-driven way
- best we can hope for: estimate amount of linguistics change since latest common ancestor
- following the lead of bioinformatics, estimation is based on continuous time Markov process model
- basic idea:
- time is continuous
- language change involves mutations of discrete characters
- mutations can occur at any point in time
- mutations in different branches are stochastically independent


## Markov processes

## Discrete time Markov chains

Ewens and Grant (2005), 4.5-4.9, 11

## Definition

A discrete time Markov chain over a countable state space $S$ is a function from $\mathbb{N}$ into random variables $X$ over $S$ with the Markov property

$$
\mathbb{P}\left(X_{n+1}=x \mid X_{1}=x_{1}, X_{2}=x_{2}, \ldots, X_{n}=x_{n}\right)=\mathbb{P}\left(X_{n+1}=x \mid X_{n}=x_{n}\right)
$$

which is stationary:

$$
\forall m, n: \mathbb{P}\left(X_{n+1}=x_{i} \mid X_{n}=x_{j}\right)=\mathbb{P}\left(X_{m+1}=x_{i} \mid X_{m}=x_{j}\right)
$$

## Discrete time Markov chains

A dt Markov chain with finite state space is characterized by

- its initial distribution $X_{0}$, and
- its transition Matrix $P$, where

$$
p_{i j}=\mathbb{P}\left(X_{n+1}=x_{j} \mid X_{n}=x_{i}\right)
$$

$P$ is a stochastic matrix, i.e. $\forall i \sum_{j} p_{i, j}=1$.

## Definition

" $\operatorname{Markov}(\lambda, P)$ " is the dt Markov chain with initial distribution $\lambda$ and transition matrix $P$.

## Discrete time Markov chains

Transition matrices over a finite state space can conveniently be represented as weighted graphs.


## Discrete time Markov chains

- We say $i \rightarrow j$ if there is a path (with positive probabilities in each step) from $x_{i}$ to $x_{j}$.
- The symmetric closure of this relation, $i \leftrightarrow j$, is an equivalence relation. It partitions a Markov chain into communicating classes.
- A Markov chain is irreducible iff it consists of a single communicating class.
- A state $x_{i}$ is recurrent iff

$$
\forall n \exists m: \mathbb{P}\left(X_{n+m}=x_{i}\right)>0
$$

- A state is transient iff it is not recurrent.


## Discrete time Markov chains

- For each communicating class $C$ : Either all of its states are transient or all of its states are recurrent.


## Discrete time Markov chains

By convention, we assume that $\lambda$ is a row vector. The distribution at time $n$ is given by

$$
\mathbb{P}\left(X_{t}=x_{i}\right)=\left(\lambda P^{n}\right)_{i}
$$

## Discrete time Markov chains

For each stochastic matrix $P$ there is at least one distribution $\pi$ with

$$
\pi P=P
$$

( $\pi$ is a left eigenvector for $P$.) $\pi$ is called an invariant distribution.
$\pi$ need not be unique:

$$
P=\left(\begin{array}{ccc}
1-\alpha-\beta & \alpha & \beta \\
0 & 1 & 0 \\
0 & 0 & 1
\end{array}\right)
$$


$\pi=(0, \gamma, \delta)$ is a left eigenvector for $P$ for each $\gamma, \delta \in[0,1]$.

## Discrete time Markov chains

If an irreducible Markov chain converges, then it converges to an invariant distribution:

If $\lim _{n \rightarrow \infty} P^{n}=A$, then

- there is a distribution $\pi$ with $A_{i}=\pi$ for all $i$, and
- $\pi$ is invariant.
$\pi$ is called the equilibrium distribution. Not every Markov chain has an equilibrium:

$$
P=\left(\begin{array}{ll}
0 & 1 \\
1 & 0
\end{array}\right)
$$

## Discrete time Markov chains

## Definition

- The period $k$ of state $x_{i}$ is defined as

$$
k=\operatorname{gcd}\left\{n: \mathbb{P}\left(X_{n}=i \mid X_{0}=i\right)>0\right\}
$$

- A state is aperiodic iff its period $=1$.
- A Markov chain is aperiodic iff each of its states is aperiodic.


## Theorem

If a finite Markov chain is irreducible and aperiodic, then

- it has exactly one invariant distribution, $\pi$, and
- $\pi$ is its equilibrium.


## Discrete time Markov chains

## Theorem

If a finite Markov chain is irreducible and aperiodic, with equilibrium distribution $\pi$, then

$$
\lim _{n \rightarrow \infty} \frac{\left|\left\{k<n \mid X_{k}=x_{i}\right\}\right|}{n}=\pi_{i}
$$

Intuitively: the relative frequency of times spent in a state converges to the equilibrium probability of that state.

## Continuous time Markov chains

- If $P$ is the transition matrix of a discrete time Markov process, then so is $P^{n}$.
- In other words, $P^{n}$ give the transition probabilities for a time interval $n$.
- Generalization:
- $P(t)$ is transition matrix as a function of time $t$.
- For discrete time: $P(t)=P(1)^{t}$.
- How can this be generalized to continuous time?


## Matrix exponentials

## Definition

$$
e^{A} \doteq \sum_{k=0}^{\infty} \frac{A^{k}}{k!}
$$

Some properties:

- $e^{0}=I$
- If $A B=B A$, then $e^{A+B}=e^{A} e^{B}$
- $e^{n A}=\left(e^{A}\right)^{n}$
- If $Y$ is invertible, then $e^{Y A Y^{-1}}=Y e^{A} Y^{-1}$
- $e^{\operatorname{diag}\left(x_{1}, \ldots, x_{n}\right)}=\operatorname{diag}\left(e^{x_{1}}, \ldots, e^{x_{n}}\right)$


## Continuous time Markov chains

## Definition (Q-matrix)

A square matrix Q is a $\mathbf{Q}$-matrix or rate matrix iff

- $q_{i i} \leq 0$ for all $i$,
- $q_{i j} \geq 0$ iff $i \neq j$, and
- $\sum_{j} q_{i j}=0$ for all $i$.


## Theorem

If $P$ is a stochastic matrix, then there is exactly one $Q$-matrix $Q$ with

$$
e^{Q}=P
$$

## Continuous time Markov chains

## Definition

Let $Q$ be a Q-matrix and $\lambda$ the initial probability distribution. Then

$$
X(t) \doteq \lambda e^{t Q}
$$

is a continuous time Markov chain.

## Continuous time Markov chains

Q-matrices can be represented as graphs in the straightforward way (with loops being omitted).

$$
Q=\left(\begin{array}{ccc}
-2 & 1 & 1 \\
1 & -1 & 0 \\
2 & 1 & -3
\end{array}\right)
$$



## Description in terms of jump chain/holding times

Let $Q$ be a Q-matrix. The corresponding jump matrix $\Pi$ is defined as

$$
\begin{gathered}
\pi_{i j}= \begin{cases}-q_{i j} / q_{i i} & \text { if } j \neq i \text { and } q_{i i} \neq 0 \\
0 & \text { if } j \neq i \text { and } q_{i i}=0\end{cases} \\
\pi_{i i}= \begin{cases}0 & \text { if } q_{i i} \neq 0 \\
1 & \text { if } q_{i i}=0\end{cases} \\
Q=\left(\begin{array}{ccc}
-2 & 1 & 1 \\
1 & -1 & 0 \\
2 & 1 & -3
\end{array}\right) \quad \Pi=\left(\begin{array}{ccc}
0 & 1 / 2 & 1 / 2 \\
1 & 0 & 0 \\
2 / 3 & 1 / 3 & 0
\end{array}\right)
\end{gathered}
$$

## Description in terms of jump chain/holding times

Let $Q$ be a Q-matrix and $\Pi$ the corresponding jump matrix. The Markov process described by $\langle\lambda, Q\rangle$ can be conceived as:
(1) Choose an initial state according to distribution $\lambda$.
(2) If in state $i$, wait a time $t$ that is exponentially distributed with parameter $-q_{i i}$.
(3) Then jump into a new state $j$ chosen according to the distribution $\Pi_{i \text {. }}$.
(4) Goto 2.

## Continuous time Markov chains

Let $M=\langle\lambda, Q\rangle$ be a continuous time Markov chain and $\Pi$ be the corresponding jump matrix.

- A state is recurrent (transient) for $M$ if it is recurrent (transient) for a discrete time Markov chain with transition matrix $\Pi$.
- The communicating classes of $M$ are those defined by $\Pi$.
- $M$ is irreducible iff $\Pi$ is irreducible.


## Continuous time Markov chains

## Theorem

If $Q$ is irreducible and recurrent. Then there is a unique distribution $\pi$ with

- $\pi Q=0$
- $\pi e^{t Q}=\pi$
- $\lim _{t \rightarrow \infty}\left(e^{t Q}\right)_{i j}=\pi_{j}$


## Time reversibility

- Does not mean that $a \rightarrow b$ and $b \rightarrow a$ are equally likely.
- Rather, the condition is

$$
\begin{aligned}
\pi_{a} p(t)_{a b} & =\pi_{b} p(t)_{b a} \\
\pi_{a} q_{a b} & =\pi_{b} q_{b a}
\end{aligned}
$$

- This means that sampling an $a$ from the equilibrium distribution and observe a mutation to $b$ in some interval $t$ is as likely as sampling a $b$ in equilibrium and see it mutate into $a$ after time $t$.


## Time reversibility

- Practical advantages of time reversibility:
- If $Q$ is time reversible, the lower triangle can be computed from the upper triangle, so we need only half the number of parameters.
- The likelihood of a tree does not depend on the location of the root.


## The Jukes-Cantor model

The Jukes-Cantor model of DNA evolution is defined by the rate matrix

$$
\begin{gathered}
Q=\left(\begin{array}{cccc}
-3 / 4 \mu & \mu / 4 & \mu / 4 & \mu / 4 \\
\mu / 4 & -3 / 4 \mu & \mu / 4 & \mu / 4 \\
\mu / 4 & \mu / 4 & -3 / 4 \mu & \mu / 4 \\
\mu / 4 & \mu / 4 & \mu / 4 & -3 / 4 \mu
\end{array}\right) \\
\Pi=\left(\begin{array}{cccc}
0 & 1 / 3 & 1 / 3 & 1 / 3 \\
1 / 3 & 0 & 1 / 3 & 1 / 3 \\
1 / 3 & 1 / 3 & 0 & 1 / 3 \\
1 / 3 & 1 / 3 & 1 / 3 & 0
\end{array}\right)
\end{gathered}
$$

## The Jukes-Cantor model

- $\pi=(1 / 4,1 / 4,1 / 4,1 / 4)$

$$
P(t)=\left(\begin{array}{llll}
1 / 4+3 / 4 e^{-t \mu} & 1 / 4-1 / 4 e^{-t \mu} & 1 / 4-1 / 4 e^{-t \mu} & 1 / 4-1 / 4 e^{-t \mu} \\
1 / 4-1 / 4 e^{-t \mu} & 1 / 4+3 / 4 e^{-t \mu} & 1 / 4-1 / 4 e^{-t \mu} & 1 / 4-1 / 4 e^{-t \mu} \\
1 / 4-1 / 4 e^{-t \mu} & 1 / 4-1 / 4 e^{-t \mu} & 1 / 4+3 / 4 e^{-t \mu} & 1 / 4-1 / 4 e^{-t \mu} \\
1 / 4-1 / 4 e^{-t \mu} & 1 / 4-1 / 4 e^{-t \mu} & 1 / 4-1 / 4 e^{-t \mu} & 1 / 4+3 / 4 e^{-t \mu}
\end{array}\right)
$$

## Two-states model, equal rates

$$
\begin{gathered}
Q=\left(\begin{array}{cc}
-r & r \\
r & -r
\end{array}\right) \quad P(t)=\frac{1}{2}\left(\begin{array}{cc}
1+e^{-2 r t} & 1-e^{-2 r t} \\
1-e^{-2 r t} & 1+e^{-2 r t}
\end{array}\right) \\
\pi=(1 / 2,1 / 2)
\end{gathered}
$$

## Two-states model, different rates

$$
\begin{aligned}
Q=\left(\begin{array}{cc}
-r & r \\
s & -s
\end{array}\right) \quad P(t) & =\frac{1}{r+s}\left(\begin{array}{cc}
s+r e^{-(r+s) t} & r-r e^{-(r+s) t} \\
s-s e^{-(r+s) t} & r+s e^{-(r+s) t}
\end{array}\right) \\
\pi & =(s / r+s, r / r+s)
\end{aligned}
$$

## Two-states model, different rates

- if we measure time in expected number of mutations, we have

$$
r+s=1
$$

- therefore:


## Two-state model

$$
\begin{gathered}
Q=\left(\begin{array}{cc}
-r & r \\
s & -s
\end{array}\right) \quad P(t)=\left(\begin{array}{cc}
s+r e^{-t} & r-r e^{-t} \\
s-s e^{-t} & r+s e^{-t}
\end{array}\right) \\
\pi=(s, r)
\end{gathered}
$$

The two-state model is always time reversible.

## Estimating distances

## Back to the running example

| language | iso_code | gloss | global_id | local_id | transcription | cognate_class |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| ELFDALIAN | qov | woman | 962 | woman | 'kèlıng | woman:Ag |
| DUTCH | nld | woman | 962 | woman | vrau | woman:B |
| GERMAN | deu | woman | 962 | woman | fraŭ | woman:B |
| DANISH | dan | woman | 962 | woman | 'h henə $_{\text {h }}$ wen | woman:D |
| DANISH_FJOLDE |  | woman | 962 | woman | kvin $^{\text {j }}$ | woman:D |
| GUTNISH_LAU |  | woman | 962 | woman | 'kvin:folk | woman:D |
| LATIN | lat | woman | 962 | woman | 'mulier | woman:E |
| LATIN | lat | woman | 962 | woman | fe:mina | woman:G |
| ENGLISH | eng | woman | 962 | woman | womən | woman:H |
| GERMAN | deu | woman | 962 | woman | vaĭp | woman:H |
| DANISH | dan | woman | 962 | woman | 'de:mə | woman:K |

- Let's focus on cognate classes for now.
- We transform the cognacy information into a binary character matrix


## Binary character matrices

| language | woman:Ag | woman: B | woman: D | woman: E | woman:G | woman:H | woman:K | $\cdots$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| DANISH | 0 | 0 | 1 | 0 | 0 | 0 | 1 | $\cdots$ |
| DANISH_FJOLDE | 0 | 0 | 1 | 0 | 0 | 0 | 0 | $\cdots$ |
| DUTCH | 0 | 1 | 0 | 0 | 0 | 0 | 0 | $\cdots$ |
| ELFDALIAN | 1 | 0 | 0 | 0 | 0 | 0 | 0 | $\cdots$ |
| ENGLISH | 0 | 0 | 0 | 0 | 0 | 1 | 0 | $\cdots$ |
| GERMAN | 0 | 1 | 0 | 0 | 0 | 1 | 0 | $\cdots$ |
| GUTNISH_LAU | 0 | 0 | 1 | 0 | 0 | 0 | 0 | $\cdots$ |
| LATIN | 0 | 0 | 0 | 1 | 1 | 0 | 0 | $\cdots$ |

## Binary character matrices

- We assume that gain/loss of cognate classes follows continuous time Markov process, and that characters a stochastically independent.
- Both assumptions are clearly false:
- Markov assumption is violated due to language contact $\rightarrow$ borrowings constitute mutations, but their probability depends on the state of the borrowing and the receiving language
- gaining a cognate class for a given concept increases likelihood for loss of different class and vice versa (avoidance of lexical gaps and synonymy)
- . . .
- For the time being, we will also assume that all cognate classes have the same mutation rate. (OMG!!!)
- Justification: Let's start with the simplest model possible and refine it step by step when necessary.


## Dollo model

- Ideally, each cognate class can be lost multiple times, but it can be gained only once.
- This amounts to a model with

```
r\approx0
s\approx1
```

- This goes by the name of Dollo model in theoretical biology.


## Dollo model

## Why the Dollo model is wrong

- Borrowings have the effect of introducing a cognate class into a lineage which originated elsewhere $\rightarrow$ multiple mutations $0 \rightarrow 1$
- Parallel semantic change:
- IELex cognate class leg: $Q$ derived from foot: $B$ independently in Greek, Indo-Iranian, Romanian, Swabian...
- Dollo model is still a good approximation


## Estimating distances

- Let's consider Italian and English
- contingeny matrix (ignoring all characters where one of the two languages is undefined)

|  | English : 0 | English : 1 |
| :--- | ---: | ---: |
| Italian : 0 | 1021 | 144 |
| Italian : | 129 | 62 |

- normalized

English: 0 English: 1

| Italian : 0 | 0.753 | 0.106 |
| :--- | :--- | :--- |
| Italian : 1 | 0.095 | 0.046 |

## Estimating distances

- model is time-reversible, so we can safely pretend that English is a direct descendant of Italian
- we also assume that Italian is in equilibrium
- note though: there are virtually infinitely possible cognate classes not covered, so the true frequency of 0 s is much higher than our counts
- expected values of normalized contingency table ( $t$ is the distance between Italian and English)

$$
P(t)\left(\begin{array}{ll}
s & 0 \\
0 & r
\end{array}\right)=\left(\begin{array}{cc}
s^{2}+r s e^{-t} & r s-r s e^{-t} \\
r s-r s e^{-t} & r^{2}+r s e^{-t}
\end{array}\right)
$$

## Dice distance

## Definition (Dice distance)

$$
\operatorname{dice}(A, B)=\frac{|A-B|+|B-A|}{|A|+|B|}
$$

- If time $t$ has passed between initial and final state, we expect the Dice distance between initial and final state to be (for positive $r$ )

$$
\operatorname{dice}(x, y)=s\left(1-e^{-t}\right)
$$

- If we have an estimate of dice $(\mathrm{x}, \mathrm{y})$, we can estimate $t$ as

$$
t=-\log \left(1-\frac{\operatorname{dice}(x, y)}{s}\right)
$$

## Dice distance

- According to Dollo assumption, $r$ converges to 0 and $s$ to 1

$$
\begin{aligned}
t & =-\log (1-\operatorname{dice}(x, y)) \\
\text { dice (Italian, English }) & =0.688 \\
t & =1.164
\end{aligned}
$$

## Estimated distances

|  | Bengali | Breton | Bulgarian | Catalan | Czech | Danish | Dutch | English | French |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Bengali | - | 2.16 | 1.64 | 1.39 | 1.81 | 1.41 | 1.24 | 1.33 | 1.28 |
| Breton | 2.16 | - | 1.81 | 1.67 | 1.77 | 1.82 | 1.86 | 1.80 | 1.64 |
| Bulgarian | 1.64 | 1.81 | - | 1.55 | 0.34 | 1.44 | 1.52 | 1.31 | 1.56 |
| Catalan | 1.39 | 1.67 | 1.55 | - | 1.53 | 1.40 | 1.37 | 1.17 | 0.29 |
| Czech | 1.81 | 1.77 | 0.34 | 1.53 | - | 1.40 | 1.44 | 1.34 | 1.53 |
| Danish | 1.41 | 1.82 | 1.44 | 1.40 | 1.40 | - | 0.45 | 0.48 | 1.38 |
| Dutch | 1.24 | 1.86 | 1.52 | 1.37 | 1.44 | 0.45 | - | 0.51 | 1.31 |
| English | 1.33 | 1.80 | 1.31 | 1.17 | 1.34 | 0.48 | 0.51 | - | 1.09 |
| French | 1.28 | 1.64 | 1.56 | 0.29 | 1.53 | 1.38 | 1.31 | 1.09 | - |
| German | 1.25 | 1.72 | 1.45 | 1.39 | 1.40 | 0.43 | 0.27 | 0.49 | 1.28 |
| Greek | 1.57 | 2.09 | 1.74 | 1.72 | 1.85 | 1.64 | 1.69 | 1.64 | 1.71 |
| Hindi | 0.54 | 1.89 | 1.33 | 1.24 | 1.34 | 1.53 | 1.56 | 1.41 | 1.22 |
| Icelandic | 1.29 | 1.85 | 1.50 | 1.48 | 1.51 | 0.25 | 0.60 | 0.58 | 1.44 |
| Irish | 1.87 | 0.85 | 1.44 | 1.58 | 1.37 | 1.38 | 1.38 | 1.31 | 1.35 |
| Italian | 1.40 | 1.52 | 1.51 | 0.24 | 1.52 | 1.32 | 1.30 | 1.16 | 0.26 |
| Lithuanian | 2.22 | 1.66 | 0.84 | 1.22 | 0.83 | 1.34 | 1.41 | 1.25 | 1.19 |
| Nepali | 0.56 | 0.18 | 0.20 | 0.13 | 0.30 | 0.20 | 0.30 | 0.20 | 0.20 |
| Polish | 1.65 | 1.86 | 0.43 | 1.56 | 0.28 | 1.44 | 1.42 | 1.32 | 1.51 |
| Portuguese | 1.34 | 1.57 | 1.49 | 0.30 | 1.44 | 1.39 | 1.39 | 1.16 | 0.36 |
| Romanian | 1.32 | 1.05 | 1.19 | 0.32 | 1.19 | 1.12 | 1.09 | 1.00 | 0.28 |
| Russian | 1.64 | 1.73 | 0.34 | 1.49 | 0.29 | 1.38 | 1.45 | 1.26 | 1.44 |
| Spanish | 1.36 | 1.55 | 1.47 | 0.21 | 1.45 | 1.42 | 1.38 | 1.15 | 0.30 |
| Swedish | 1.43 | 1.87 | 1.49 | 1.41 | 1.44 | 0.15 | 0.49 | 0.57 | 1.43 |
| Ukrainian | 1.67 | 1.82 | 0.40 | 1.53 | 0.32 | 1.45 | 1.46 | 1.32 | 1.51 |
| Welsh | 2.08 | 0.38 | 1.39 | 1.19 | 1.41 | 1.00 | 1.08 | 1.15 | 1.02 |

## Estimated distances

|  | German | Greek | Hindi | Icelandic | Irish | Italian | Lithuanian | Nepali | Polish |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Bengali | 1.25 | 1.57 | 0.54 | 1.29 | 1.87 | 1.40 | 2.22 | 0.56 | 1.65 |
| Breton | 1.72 | 2.09 | 1.89 | 1.85 | 0.85 | 1.52 | 1.66 | 0.18 | 1.86 |
| Bulgarian | 1.45 | 1.74 | 1.33 | 1.50 | 1.44 | 1.51 | 0.84 | 0.20 | 0.43 |
| Catalan | 1.39 | 1.72 | 1.24 | 1.48 | 1.58 | 0.24 | 1.22 | 0.13 | 1.56 |
| Czech | 1.40 | 1.85 | 1.34 | 1.51 | 1.37 | 1.52 | 0.83 | 0.30 | 0.28 |
| Danish | 0.43 | 1.64 | 1.53 | 0.25 | 1.38 | 1.32 | 1.34 | 0.20 | 1.44 |
| Dutch | 0.27 | 1.69 | 1.56 | 0.60 | 1.38 | 1.30 | 1.41 | 0.30 | 1.42 |
| English | 0.49 | 1.64 | 1.41 | 0.58 | 1.31 | 1.16 | 1.25 | 0.20 | 1.32 |
| French | 1.28 | 1.71 | 1.22 | 1.44 | 1.35 | 0.26 | 1.19 | 0.20 | 1.51 |
| German | - | 1.65 | 1.46 | 0.61 | 1.30 | 1.28 | 1.30 | 0.20 | 1.38 |
| Greek | 1.65 | - | 1.53 | 1.68 | 1.70 | 1.60 | 1.74 | 0.41 | 1.85 |
| Hindi | 1.46 | 1.53 | - | 1.64 | 1.40 | 1.28 | 1.37 | 0.08 | 1.35 |
| Icelandic | 0.61 | 1.68 | 1.64 | - | 1.43 | 1.44 | 1.34 | 0.30 | 1.55 |
| Irish | 1.30 | 1.70 | 1.40 | 1.43 | - | 1.30 | 1.32 | 0.46 | 1.41 |
| Italian | 1.28 | 1.60 | 1.28 | 1.44 | 1.30 | - | 1.18 | 0.24 | 1.55 |
| Lithuanian | 1.30 | 1.74 | 1.37 | 1.34 | 1.32 | 1.18 | - | 0.81 | 0.78 |
| Nepali | 0.20 | 0.41 | 0.08 | 0.30 | 0.46 | 0.24 | 0.81 | - | 0.30 |
| Polish | 1.38 | 1.85 | 1.35 | 1.55 | 1.41 | 1.55 | 0.78 | 0.30 | - |
| Portuguese | 1.30 | 1.63 | 1.27 | 1.44 | 1.47 | 0.32 | 1.25 | 0.20 | 1.44 |
| Romanian | 1.00 | 1.36 | 0.96 | 1.18 | 1.00 | 0.26 | 1.20 | 0.22 | 1.19 |
| Russian | 1.36 | 1.78 | 1.34 | 1.46 | 1.41 | 1.48 | 0.84 | 0.20 | 0.32 |
| Spanish | 1.32 | 1.67 | 1.21 | 1.50 | 1.37 | 0.28 | 1.18 | 0.20 | 1.46 |
| Swedish | 0.50 | 1.68 | 1.60 | 0.30 | 1.38 | 1.36 | 1.41 | 0.20 | 1.46 |
| Ukrainian | 1.42 | 1.88 | 1.31 | 1.51 | 1.41 | 1.52 | 0.79 | 0.30 | 0.27 |
| Welsh | 0.94 | 1.12 | 0.96 | 1.20 | 0.54 | 1.02 | 0.69 | 0.69 | 1.39 |

## Estimated distances

|  | Portuguese | Romanian | Russian | Spanish | Swedish | Ukrainian | Welsh |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Bengali | 1.34 | 1.32 | 1.64 | 1.36 | 1.43 | 1.67 | 2.08 |
| Breton | 1.57 | 1.05 | 1.73 | 1.55 | 1.87 | 1.82 | 0.38 |
| Bulgarian | 1.49 | 1.19 | 0.34 | 1.47 | 1.49 | 0.40 | 1.39 |
| Catalan | 0.30 | 0.32 | 1.49 | 0.21 | 1.41 | 1.53 | 1.19 |
| Czech | 1.44 | 1.19 | 0.29 | 1.45 | 1.44 | 0.32 | 1.41 |
| Danish | 1.39 | 1.12 | 1.38 | 1.42 | 0.15 | 1.45 | 1.00 |
| Dutch | 1.39 | 1.09 | 1.45 | 1.38 | 0.49 | 1.46 | 1.08 |
| English | 1.16 | 1.00 | 1.26 | 1.15 | 0.57 | 1.32 | 1.15 |
| French | 0.36 | 0.28 | 1.44 | 0.30 | 1.43 | 1.51 | 1.02 |
| German | 1.30 | 1.00 | 1.36 | 1.32 | 0.50 | 1.42 | 0.94 |
| Greek | 1.63 | 1.36 | 1.78 | 1.67 | 1.68 | 1.88 | 1.12 |
| Hindi | 1.27 | 0.96 | 1.34 | 1.21 | 1.60 | 1.31 | 0.96 |
| Icelandic | 1.44 | 1.18 | 1.46 | 1.50 | 0.30 | 1.51 | 1.20 |
| Irish | 1.47 | 1.00 | 1.41 | 1.37 | 1.38 | 1.41 | 0.54 |
| Italian | 0.32 | 0.26 | 1.48 | 0.28 | 1.36 | 1.52 | 1.02 |
| Lithuanian | 1.25 | 1.20 | 0.84 | 1.18 | 1.41 | 0.79 | 0.69 |
| Nepali | 0.20 | 0.22 | 0.20 | 0.20 | 0.20 | 0.30 | 0.69 |
| Polish | 1.44 | 1.19 | 0.32 | 1.46 | 1.46 | 0.27 | 1.39 |
| Portuguese | - | 0.28 | 1.39 | 0.17 | 1.43 | 1.44 | 0.96 |
| Romanian | 0.28 | - | 1.13 | 0.24 | 1.13 | 1.20 | 0.69 |
| Russian | 1.39 | 1.13 | - | 1.41 | 1.43 | 0.22 | 1.23 |
| Spanish | 0.17 | 0.24 | 1.41 | - | 1.45 | 1.48 | 1.03 |
| Swedish | 1.43 | 1.13 | 1.43 | 1.45 | - | 1.46 | 1.06 |
| Ukrainian | 1.44 | 1.20 | 0.22 | 1.48 | 1.46 | - | 1.25 |
| Welsh | 0.96 | 0.69 | 1.23 | 1.03 | 1.06 | 1.25 | - |

## Neighbor Joining tree



## Neighbor Joining tree

- data sparseness for Nepali (only 31 characters are defined) $\rightarrow$ all distances come out as way too small
- note that root was determined by midpoint rooting to make it look nicer
- Neighbor Joining does not tell us anything about the location of the root
- tree structure is largely consistent with received opinion (except that Italian and French should swap places, and English is too high within Germanic)


## UPGMA tree



## UPGMA tree

- tree structure largely recognizes the major sub-groupings
- fine structure of Romance is a bit of a mess


## WALS features

- WALS features are binarized $\rightarrow$ binary character matrix

| language | SVO | SOV | VSO | no dominant order | $\cdots$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| DANISH | 1 | 0 | 0 | 0 | $\cdots$ |
| DUTCH | 0 | 0 | 0 | 1 | $\cdots$ |
| ENGLISH | 1 | 0 | 0 | 0 | $\cdots$ |
| GERMAN | 0 | 0 | 0 | 1 | $\cdots$ |
| GREEK | 0 | 0 | 0 | 1 | $\cdots$ |
| HINDI | 0 | 1 | 0 | 0 | $\cdots$ |
| ICELANDIC | 1 | 0 | 0 | 0 | $\cdots$ |
| WELCH | 0 | 0 | 1 | 0 | $\cdots$ |

## WALS features

- Dollo assumption is too far off the mark here to apply it
- We need an estimate for $(r, s)$ !
- Null assumption: for each WALS feature, all values are equally likely in equilibrium
- leads to estimate

$$
\begin{aligned}
r & =\frac{\text { number of WALS features }}{\text { number of binary characters }} \\
& \approx 0.14 \\
s & =1-r \approx 0.86
\end{aligned}
$$

## Neighbor Joining tree



## Neighbor Joining tree

- clearly worse than cognacy tree
- some oddities
- Polish and Lithuanian have swapped places
- Celtic comes out as sub-group of Romance
- Bulgarian far removed from the rest of Slavic; it is sister-taxon of Greek


## UPGMA tree



## UPGMA tree

- somewhat better, but still pretty bad
- some oddities
- Greek as Slavic language
- Czech as Baltic language
- Romanian and Catalan are much too close
$\Rightarrow$ typological features are ill-suited for phylogenetic estimation
- strong influence of language contact
- non-independence of features
- data sparseness


## Working with phonetic strings

## Phonetic characters

- cognacy data and grammatical/typological classifications rely on expert judgments:
- labor intensive
- subjective, hard to replicate
- sound change, a very conspicuous aspect of language change, is ignored
- information on sound change does not come in nicely packaged discrete characters though


## Working with phonetic strings

- quick-and-dirty method to extract binary characters from phonetic strings:
(1) convert phonetic entries into ASJP format
(2) presence-absence characters for each sound class/concept combination
(3) character changes can
represent sound shift or lexical replacement
Latin puer $\rightarrow$ Italian bambino
child/p:1 $\rightarrow$ child/p:0
Latin oculus $\rightarrow$ Italian occhio
eye/u:1 $\rightarrow$ eye/u:0

| language | phonological form (IELex) | ASJP representation |
| :---: | :---: | :---: |
| Bengali | - | - |
| Breton | - | - |
| Bulgarian | mu're | murE |
| Catalan | mar; mar; ma | mar; mar; ma |
| Czech | 'more | morE |
| Danish | haw; ¢ $^{\text {? }}$ | how; se |
| Dutch | ze | ze |
| English | si: | si |
| French | mer | mEr |
| German | ze:;'o:tsea:n;me:e | ze; otsean; mea |
| Greek |  | 8alasa |
| Hindi | - | - |
| Icelandic | ha:v/sjou:r | hav; syour |
| Irish | 'fræer ${ }^{\text {ry }}$ I | fErCi |
| Italian | 'mare | mare |
| Lithuanian | 'ju:re | yura |
| Nepali | - | - |
| Polish | 'moze | moZE |
| Portuguese | mar | mar |
| Romanian | 'mare | mare |
| Russian | 'moriz | morE |
| Spanish | mar | mar |
| Swedish | ha:v; fø: | hov; Se |
| Ukrainian | 'more | morE |
| Welsh | - | - |

## Working with phonetic strings

|  | see:m | see:r | see:a | see:s | $\cdots$ | see:Z |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Bengali | - | - | - | - | $\cdots$ | - |
| Bulgarian | 1 | 1 | 0 | 0 | $\cdots$ | 0 |
| Catalan | 1 | 1 | 1 | 0 | $\cdots$ | 0 |
| Czech | 1 | 1 | 0 | 0 | $\cdots$ | 0 |
| Danish | 0 | 0 | 0 | 1 | $\cdots$ | 0 |
| Italian | 1 | 1 | 1 | 0 | $\cdots$ | 0 |
| Ukrainian | 1 | 1 | 0 | 0 | $\cdots$ | 0 |
| $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\ddots$ | $\vdots$ |

- estimating $r$ as

$$
\frac{\sum_{s \in \text { Sound classes }} \frac{\mid\{w \in \text { Words } \mid s \in w\} \mid}{\mid \text { words } \mid}}{\mid \text { sound classes } \mid} \approx 0.105
$$

## Neighbor Joining tree



## Neighbor Joining tree

- almost fully consistent with expert opinion
- two deviations
- Russian should be next two Ukrainian rather than next to Polish (language contact?)
- Italian and Romanian shouldn't be neighbors


## UPGMA tree



## UPGMA tree

- somewhat worse than NJ tree
- some oddities
- English too high within Germanic
- position of Russian is correct, but Czech comes out as East Slavic
- Italian and French at wrong positions within Romance


## Hands-on

## Data formats

## Newick format for trees

- see Wikipedia entry for details
- bracketed string
- labels of internal nodes (optional) after closing bracket
- edge lengths (optional) after node name, separated by ":"
- example:
(("Ancient Greek": 2,Latin:3):1,
((Dutch:2.5, "Old Norse":1):3, ("Old Church Slavonic":0.2, Russian:1.7):3.8):0.5);



## Data formats

## Character matrices as Nexus files

- Nexus (suffix .nex): versatile file format for phylogenetic information
- Structure of a Nexus file for a binary character matrix:
(1) header (ntax $=$ number of rows, nchar=number of columns): \#NEXUS

BEGIN DATA;
DIMENSIONS ntax=25 NCHAR=1481;
FORMAT DATATYPE=STANDARD GAP=? MISSING=- interleave=yes; MATRIX

## Data formats

## Character matrices as Nexus files

(2) matrix: each row consists of the taxon name, followed by white space, followed by matrix entries; undefined values are represented by "-"

| Greek | $0001000010000000000 \ldots$ |
| :--- | :---: |
| Bulgarian | $0010000010000000010 \ldots$ |
| Russian | $0010000010000000010 \ldots$ |
| Romanian | $-----010000------1$ |
| $\vdots$ | $\vdots$ |

(3) footer:
;
END;

## Loading Nexus files into $\mathbf{R}$

- phangorn is geared towards biomolecular data
- some workaround needed to handle binary matrices

```
library(ape)
library(phangorn)
contrasts <- matrix(data=c(1,0,
    0,1,
    1,1),
        ncol=2,byrow=T)
dimnames(contrasts) <- list(c('0','1','-'),
    c('0','1'))
cognacy.data <- phyDat(read.nexus.data('ielex.bin.nex'),
    'USER',
    levels=c('0','1','-'),
    contrast=contrasts,
    ambiguity='-')
cognacy.matrix <- as.character(cognacy.data)
```


## Exercise

- run the script loadNexusFiles.r in an interactive session
- implement the Dice distance. Note that all characters with value "-" in either of the vectors compared have to be ignored
- computed the distance matrices for the three Nexus files, using the estimates for $s$ from the slides
- compute the Neighbor Joining trees, using the function nj()
- display the tree with the plot() command
- experiment with different values for $s$ to get a feel for how sensitive the result is for this parameter


## Ewens, W. and G. Grant (2005). Statistical Methods in Bioinformatics: An Introduction. Springer, New York.

