

# Formal and computational models of language evolution

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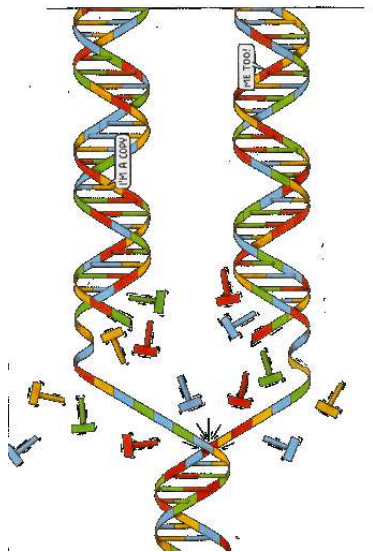
# Problems for classical GT

- multiple equilibria  $\Rightarrow$  no predictions possible
- “perfectly rational player” is too strong an idealization



# Evolutionary Game Theory

- populations of players
- individuals are (genetically) programmed for certain strategy
- individuals replicate and thereby pass on their strategy



# Utility and fitness

- number of offspring is monotonically related to average utility of a player
- high utility in a competition means the outcome improves reproductive chances (and vice versa)
- number of expected offspring (Darwinian “fitness”) corresponds to **expected utility** against a population of other players
- genes of individuals with high utility will spread

# Extinction of non-rationalizable strategies

- strictly dominated strategies always have less-than-average reproduction rate
- their proportion thus converges towards zero
- once a strictly dominated strategies dies out (or almost dies out), it can be ignored in the utility matrix
- corresponds to *elimination of a strictly dominated strategy*
- process gets iterated in evolutionary dynamics
- long-term effect:

## Theorem

If a strategy  $a_i$  is iteratively strictly dominated, then

$$\lim_{t \rightarrow \infty} p_t(a_i) = 0$$

# Evolutionary stability (cont.)

- replication sometimes unfaithful (mutation)
- population is **evolutionarily stable**  $\rightsquigarrow$  resistant against small amounts of mutation
- Maynard Smith (1982): static characterization of  
**Evolutionarily Stable Strategies**  
(ESS) in terms of utilities only
- related to Nash equilibria, but slightly different

## Rock-Paper-Scissor

	R	P	S
R	0	-1	1
P	1	0	-1
S	-1	1	0

- one symmetric Nash equilibrium:  $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$
- not evolutionarily stable though

# Evolutionary stability (cont.)

## Pigeon orientation game

- “players” are pigeons that go together on a journey
- *A*-pigeons can find their way back, *B*-pigeons cannot

	<i>A</i>	<i>B</i>
<i>A</i>	1	1
<i>B</i>	1	0



# Evolutionary stability (cont.)

- $A$  is a non-strict Nash equilibrium, but nevertheless evolutionarily stable
- to be evolutionarily stable, a population must be able either
  - to fight off invaders directly (strict Nash equilibrium)
  - to successfully invade the invaders (non-strict Nash equilibrium)

# Evolutionary Stable Strategy

## Definition

The mixed strategy  $\alpha$  is an **Evolutionarily Stable Strategy** in a symmetric two-person game iff

- $U(\alpha, \alpha) \geq U(\alpha', \alpha)$  for all  $\alpha'$ , and
- if  $U(\alpha, \alpha) = U(\alpha', \alpha)$  for some  $\alpha' \neq \alpha$ , then  $U(\alpha, \alpha') > U(\alpha', \alpha')$ .

*Strict Nash Equilibria*

⊂

*Evolutionarily Stable Strategies*

⊂

*Nash Equilibria*

# The Replicator Dynamics

- implicit assumption behind notion of ESS
  - Populations are (practically) infinite.
  - Each pair of individuals is equally likely to interact.
  - The expected number of offspring of an individual (i.e., its fitness in the Darwinian sense) is monotonically related to its average utility.
- can be made explicit in a dynamic model

# Replicator Dynamics (cont.)

- easiest correlation between utility and fitness:

$u(i, j)$  = *expected number of offspring  
of an individual of type  $i$   
in a  $j$ -population*

## Suppose

- time is discrete
- in each round, each pair of players is equally likely to interact

## Discrete time dynamics:

$$N_i(t+1) = N_i(t) + N_i(t) \left( \sum_{j=1}^n x_j u(i,j) - d \right)$$

$N(t)$  ... population size at time  $t$

$N_i(t)$  ... number of players playing strategy  $s_i$

$x_j(t)$  ...  $\frac{N_j(t)}{N(t)}$

$d$  ... death rate

# Replicator Dynamics (cont.)

generalizing to continuous time:

$$N_i(t + \Delta t) = N_i + \Delta t N_i \left( \sum_{j=1}^n x_j u(i, j) - d \right)$$

thus

$$\frac{\Delta N_i}{\Delta t} = N_i \left( \sum_{j=1}^n x_j u(i, j) - d \right)$$

# Replicator Dynamics (cont.)

if  $\Delta t \rightarrow 0$

$$\frac{dN_i}{dt} = N_i \left( \sum_{j=1}^n x_j u(i, j) - d \right)$$



# Replicator Dynamics (cont.)

size of entire population may also change:

$$\begin{aligned} N(t + \Delta t) &= \sum_{i=1}^n (N_i + \Delta t (N_i \sum_{j=1}^n x_j u(i, j) - d)) \\ &= N + \Delta t (N \sum_{i=1}^n x_i \sum_{j=1}^n x_j u(i, j)) \end{aligned}$$

hence

$$\frac{dN}{dt} = N \left( \sum_{i=1}^n x_i \left( \sum_{j=1}^n x_j u(i, j) - d \right) \right)$$

# Replicator Dynamics (cont.)

let

$$\sum_{j=1}^n x_j u(i, j) = \tilde{u}_i$$
$$\sum_{i=1}^n x_i \tilde{u}_i = \tilde{u}$$

then we have

$$\frac{dN_i}{dt} = N_i(\tilde{u}_i - d)$$
$$\frac{dN}{dt} = N(\tilde{u} - d)$$

# Replicator dynamics (cont.)

remember some calculus?

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$$

# Replicator dynamics (cont.)

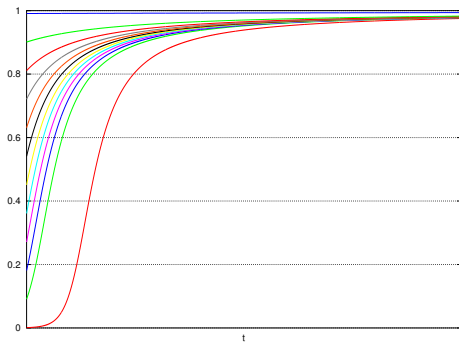
remember some calculus?

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$$

$$\begin{aligned}\frac{dx_i}{dt} &= \frac{(NN_i(\tilde{u}_i - d) - (N_iN(\tilde{u} - d)))}{N^2} \\ &= x_i(\tilde{u}_i - \tilde{u})\end{aligned}$$

# Pigeon orientation

- ESSs correspond to **asymptotically stable states**
- a.k.a. **point attractors**
- sample dynamics:

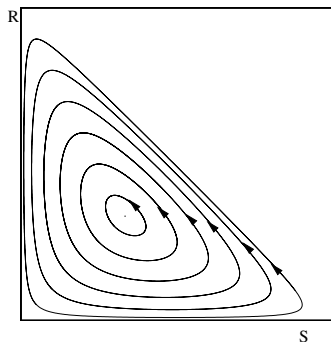


*x-axis: time*

*y-axis: proportion of A-players*

# Rock-Paper-Scissor again

- three-strategy game: two independent variables
  - number of R-players
  - number of P-players
- number of S-players follows because everything sums up to 1
- suppressing time dimension gives **orbits**



# Asymmetric games

- symmetric games:
  - same strategy set for both players
  - $u_A(i, j) = u_B(j, i)$  for all strategies  $s_i, s_j$
  - evolutionary interpretation: symmetric interaction *within one population*
- asymmetric games:
  - players have different strategy sets or utility matrices
  - evolutionary interpretation
    - different roles within one population (like incumbent vs. intruder, speaker vs. hearer, ...), or
    - interaction between disjoint populations
- evolutionary behavior differs significantly!

## Hawks and Doves

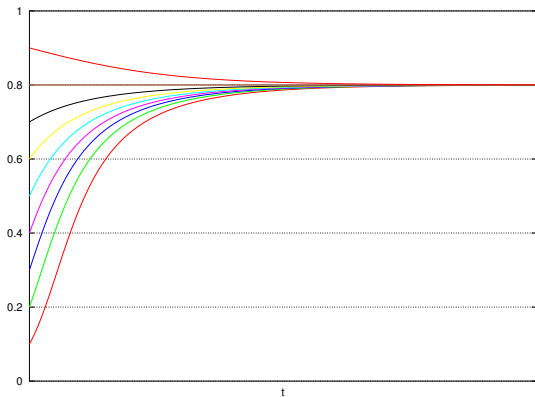
	H	D
H	1,1	7,2
D	2,7	3,3

- can be interpreted symmetrically or asymmetrically
- symmetric interpretation:
  - hawks prefer to interact with doves and vice versa
  - ESS: 80% hawks / 20% doves
  - both strategies have average utility of 2.2
  - dynamics:



# Symmetric Hawk-and-doves

- if hawks exceed 80%, doves thrive, and vice versa
- 80:20 ratio is only attractor state

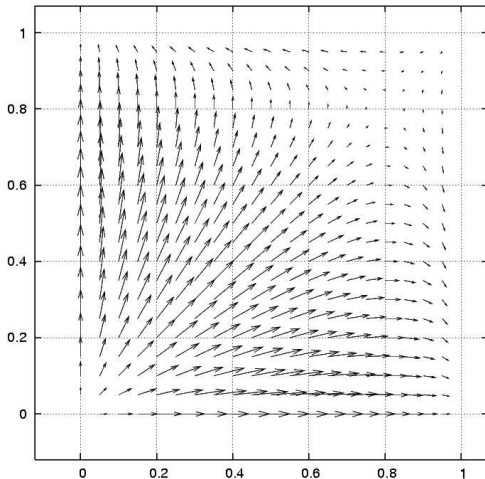


# Asymmetric Hawks-and-doves

- suppose two-population setting:
  - both  $A$  and  $B$  come in hawkish and dovish variant
  - everybody only interacts with individuals from opposite “species”
  - excess of  $A$ -hawks helps  $B$ -doves and vice versa
  - population push each other into opposite directions

# Hawks and doves

- 80:20 ratio in both populations is stationary
- not an attractor, but repeller



# Asymmetric stability

- crucial difference to symmetric games:  
*mutants do not play against themselves*
- makes second clause of the symmetric ESS superfluous

## Theorem (Selten 1980)

*In asymmetric games, a configuration is an ESS iff it is a strict Nash equilibrium.*

# Asymmetric replicator dynamic

$$\frac{dx_i}{dt} = x_i \left( \sum_{j=1}^n y_j u_A(i, j) - \sum_{k=1}^n x_k \sum_{j=1}^n y_j u_A(k, j) \right)$$
$$\frac{dy_i}{dt} = y_i \left( \sum_{j=1}^m x_j u_B(i, j) - \sum_{k=1}^n y_k \sum_{j=1}^m x_j u_B(k, j) \right)$$

$x_i$  ... proportion of  $s_i^A$  within the  $A$ -population

$y_i$  ... proportion of  $s_i^B$  within the  $B$ -population

# Symmetrizing asymmetric games

- asymmetric games can be “symmetrized”
- corresponding symmetric game shares Nash equilibria and ESSs
- new strategy set:

$$S^{AB} = S^A \times S^B$$

- new utility function

$$u^{AB}(\langle i, j \rangle, \langle k, l \rangle) = u^A(i, l) + u^B(j, k)$$

- 1 Find the symmetric ESSs of the following games (provided they exist):
  - Prisoner's dilemma
  - Stag hunt
- 2 Find the asymmetric ESSs of the following games (again, provided they exist):
  - Bach or Stravinsky
  - Matching pennies
- 3 Symmetrize the asymmetric version of Hawks and Doves and find the symmetric ESSs of the result. Which configuration in the original game do they correspond to?