Formal and computational models of language evolution

Gerhard Jäger
Gerhard.Jaeger@uni-bielefeld.de

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The evolution of vowel spaces

- micro-variation in the inventory of vowels between languages: every language is different
- however, very strong tendencies:
  - most languages have five vowels
  - (almost) every language has [a], [i] and [u] like vowels
  - most vowel inventories are peripheral and symmetric etc.
- proposal (see for instance de Boer 2001):

Vowel inventories must be evolutionarily stable!

What is a vowel?

Articulation

- speech sound
- voiced
- no constriction of the vocal tract
- vowel quality depends on
  - position of tongue
  - gesture of the lips
  - ...

What is a vowel?

Acoustics

- periodic sonic wave

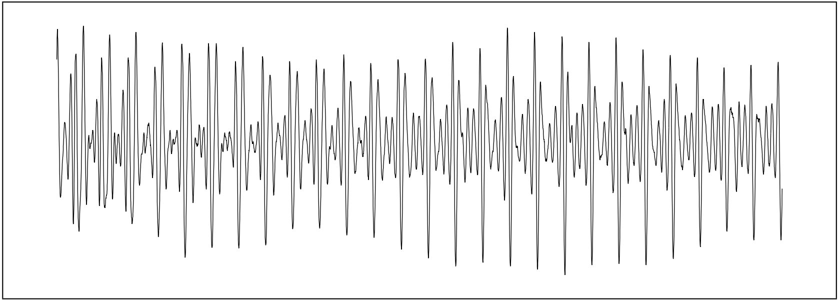


Figure: Amplitude of the vowel /u/

What is a vowel?

Acoustics

- spectral analysis:

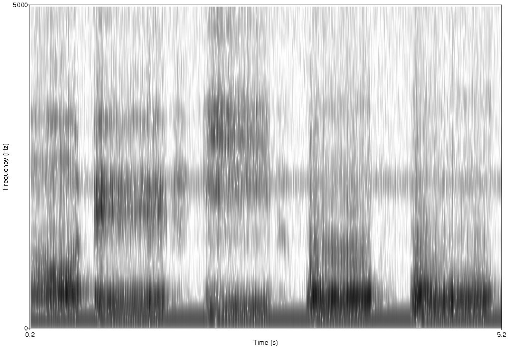


Figure: Spectrogramm of /a/-/e/-/i/-/o/-/u/

What is a vowel?

Acoustics

- vowel is superposition of discrete harmonic waves:
  - fundamental frequency
  - formants

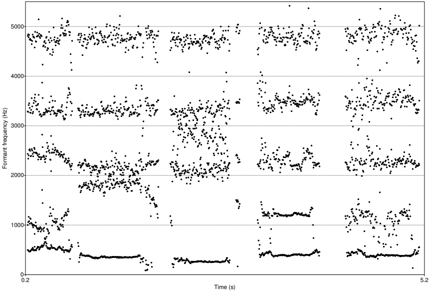


Figure: first five formants of /a-e-i-o-u/

What is a vowel?

Acoustics

- first two formants are crucial for identification of vowels

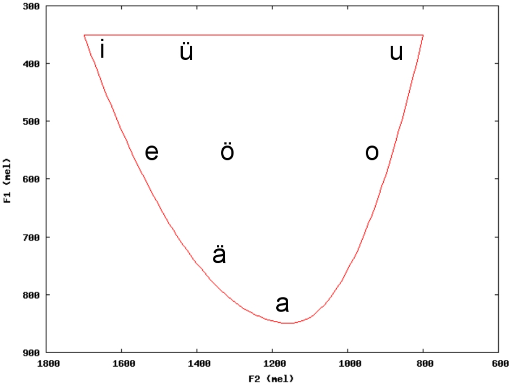


Figure: F1/F2-plane: German vowels

What is a vowel?

Acoustics

- more realistic picture:

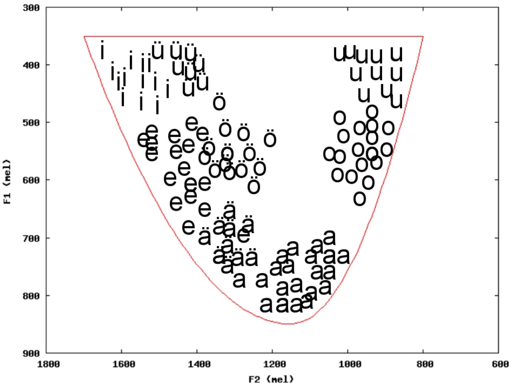


Figure: F1/F2-plane: German vowels

Universal tendencies of vowel inventories

- comparison of vowel inventories in hundreds of languages reveals
  - virtually all languages use the vowels [a], [i], [u]
  - almost all vowels in all languages are peripheral
  - vowel inventories tend to be symmetrical
  - ...

Liljencrants and Lindblom 1972

- vowel systems tend to maximize perceptual distance between vowels
- can be modeled as minimizing potential energy of a vowel system
- energy is proportional to sum of inverse squared distances
- fairly good typological predictions

Survey of 500+ vowel inventories

number of vowels	vowel systems and their frequency of occurrence				
3	14				
4	14	5	4	2	
5	97	3			
6	26	12	12		
7	23	6	5	4	3
8	6	3	3	2	
9	7	7	3		

(from Schwartz et al. 1997, based on the UCLA Phonetic Segment Inventory Database)

Communication via the vowel space

Game theoretic model

- Signaling game
- types: between 3 and 9 vowel categories
- signals: each point within the two-dimensional (F1/F2) vowel space

Communication via the vowel space

One round of an evolutionary signaling game

- nature picks a vowel category v_S and shows it to S
- S picks a point p_{intend} in the vowel space
- a normally distributed random variable is added to p_{intend}, yielding p_{prod}
- another normally distributed random variable is added to p_{prod}, yielding p_{perc}
- R observes p_{perc} and picks a vowel category v_R
- if $v_S = v_R$, both players score a point

Exemplar dynamics

- empiricist view on language processing/language structure
- popular in functional linguistics (esp. phonology and morphology) and in computational linguistics (aka. "memory-based")

Basic idea

- large amounts of previously encountered instances ("exemplars") of linguems are stored in memory
- very detailed representation of exemplars
- little abstract categorization
- similarity metric between exemplars
- new linguemes are processed in a similarity-based way

Exemplar dynamics: implementation

Sender

- chooses p_{intend} at random from multiset $\{p | \langle v_S, p\rangle \in \text{ memory}\}$
- if communication succeeds ($v_S = v_R$), oldest item in memory is replaced with $\langle v_S, p_{prod}\rangle$
- otherwise memory remains unchanged

Receiver

- v_H is picked such that $\min\{d(p_{perc}, p) | \langle v_H, p\rangle \in \text{ memory}\}$ is minimized
- if communication succeeds ($v_S = v_R$), oldest item in memory is replaced by $\langle v_R, p_{perc}\rangle$
- otherwise memory remains unchanged

Simulations

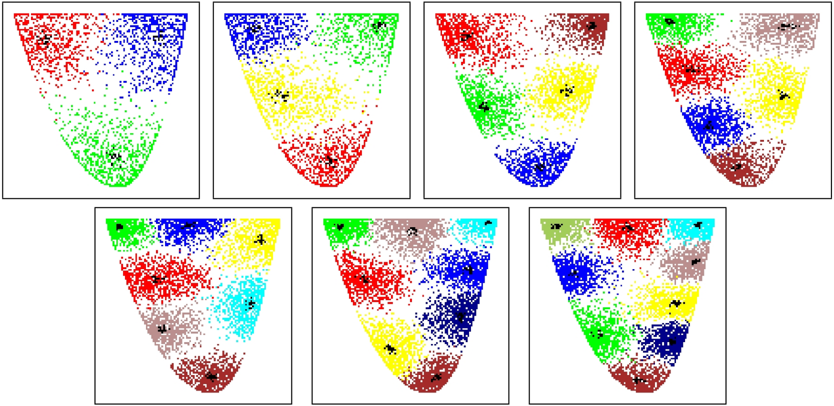
Setup

- population of 20 agents
- each agent has a memory of 4000 previous observations per vowel category (initialized with random values)
- 300k iterations of the signaling game
- sender and receiver are picked at random

Inspired by much more sophisticated simulations by Bart de Boer.

Simulation results

- black dots display average sender strategy for each agent and vowel category)
- colored dots display receiver strategies (colors represent vowel categories)



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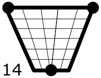
3					

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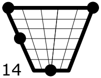
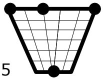
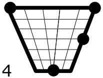
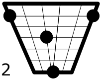
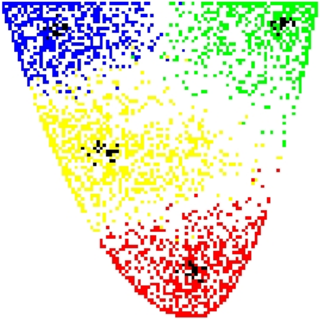
4	 14	 5	 4	 2	



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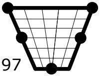
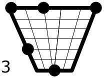
5	 97	 3			

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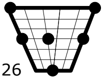
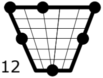
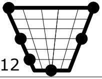
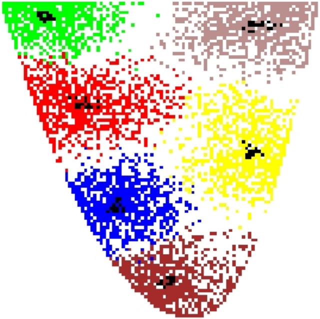
6	 26	 12	 12		



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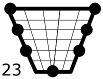
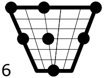
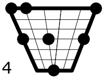
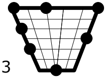
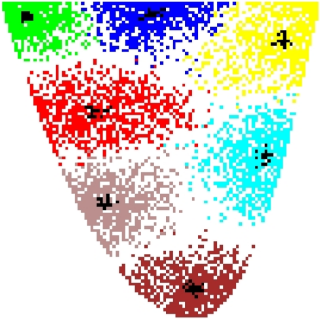
7	 23	 6	 5	 4	 3



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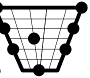
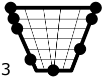
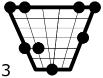
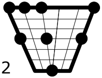
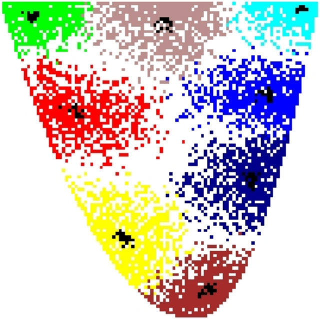
8	 6	 3	 3	 2	



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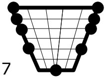
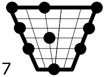
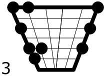
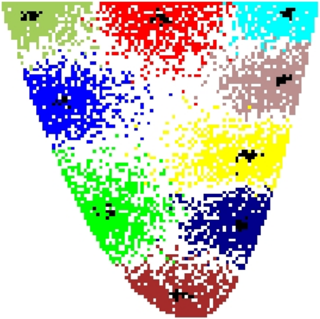
9	 7	 7	 3		



Evaluation

- more than half of the typologically dominant patterns correspond to (experimentally determined) ESSs (150 out of 264 in the database)
- five out of seven ESSs correspond to empirically attested vowel systems
- even the two outliers look natural (symmetric systems with peripheral prototypes)

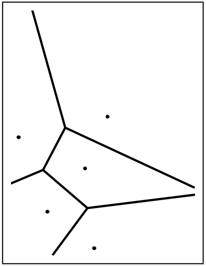
Theoretical considerations

ESS under replicator dynamics: strict Nash equilibria

- sender strategy: mapping from vowel categories to points in the vowel space
- receiver strategy: categorization of points

Voronoi tesselations

- suppose receiver strategy R is given and known to the sender: which sender strategy would be the best response to it?
  - every signal p has a "prototypical" interpretation: $R(p)$
  - for every vowel category v: S's best choice is to choose the p that minimizes the distance between p and $R(p)$
  - optimal S thus induces a **partition** of the meaning space
  - Voronoi tesselation, induced by the range of R



Open question

- numeric calculation of the ESSs for the human vowel space
- Exemplar Dynamics is similar but not identical to replicator dynamics
- conjecture: as the variance of the random variables goes to 0, the attractor states of the exemplar dynamics converges towards SNEs

Cognitive semantics

Gärdenfors (2000):

- meanings are arranged in **conceptual spaces**
- conceptual space has geometrical structure
- dimensions are founded in perception/cognition

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Convexity

A subset C of a conceptual space is said to be *convex* if, for all points x and y in C, all points between x and y are also in C.

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Criterion P

A *natural property* is a convex region of a domain in a conceptual space.

Examples

- spatial dimensions: *above, below, in front of, behind, left, right, over, under, between ...*
- temporal dimension: *early, late, now, in 2005, after, ...*
- sensual dimenstions: *loud, faint, salty, light, dark, ...*
- abstract dimensions: *cheap, expensive, important, ...*

Signaling game with continuous meaning space

- two players:
  - **S**peaker
  - **H**earer
- infinite set of **M**eanings, arranged in a finite metrical space *distance is measured by function* $d : M^2 \mapsto R$
- finite set of **F**orms
- sequential game:
  1. nature picks out $m \in M$ according to some probability distribution p and reveals m to S
  2. S maps m to a form f and reveals f to H
  3. H maps f to a meaning m'

Signaling game with continuous meaning space

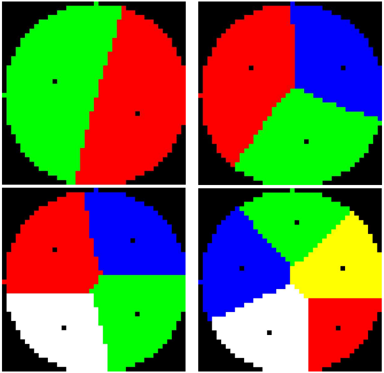
- **Goal:**
  - optimal communication
  - both want to minimize the distance between m and m'
- **Strategies:**
  - speaker: mapping S from M to F
  - hearer: mapping H from F to M
- **Average utility:** (identical for both players)

$$u(S,H) = \int_M p_m \times \exp(-d(m, H(S(m)))^2)dm$$

vulgo: average similarity between speaker's meaning and hearer's meaning

Simulations

- two-dimensional circular meaning space
- discrete approximation
- uniform distribution over meanings
- initial stratgies are randomized
- update rule according to (discrete time version of) replicator dynamics



Voronoi tesselations

- suppose H is given and known to the speaker: which speaker strategy would be the best response to it?
  - every form f has a "prototypical" interpretation: $H(f)$
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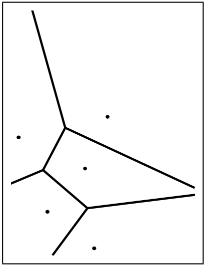

Voronoi tesselation

Lemma

The Voronoi tessellation based on a Euclidean metric always results in a partioning of the space into convex regions.

ESSs of the naming game

- best response of H to a given speaker strategy S not as easy to characterize
- general formula

$$H(f) = \arg\max_m \int_{S^{-1}(f)} p_{m'} \times \exp(-d(m, m')^2) dm'$$

- such a hearer strategy always exists
- linguistic interpretation: H maps every form f to the **prototype** of the property $S^{-1}(f)$

ESSs of the game

Lemma

In every ESS $\langle S, H\rangle$ *of the naming game, the partition that is induced by* S^{-1} *on* M *is the Voronoi tesselation induced by* $H[F]$.

ESSs of the game

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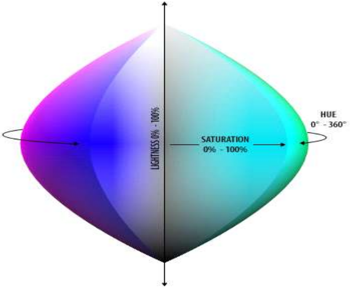
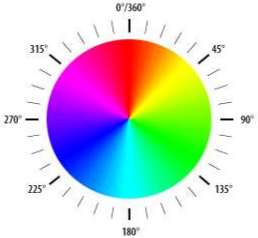
Theorem

For every form f, $S^{-1}(f)$ *is a convex region of* M.

The color space

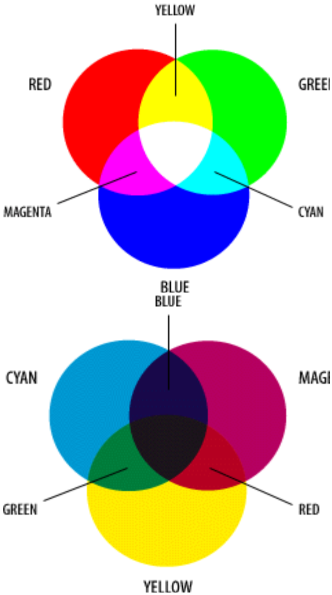
- physical color space is of infinite dimensionality
- psychological color space has only three dimensions:
  1. brightness
  2. hue
  3. saturation





The color space

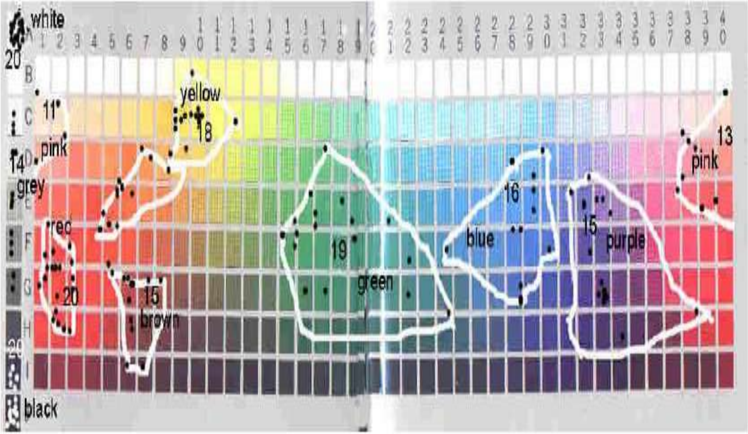
- alternative axes (but maintaining dimensionality of three)
  1. black-white
  2. red-green
  3. yellow-blue
- yet another triple of dimensions ("additive"):
  1. red
  2. green
  3. blue
- "subtractive" color space:
  1. cyan
  2. magenta
  3. yellow



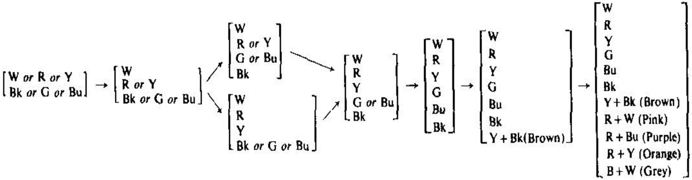
Color words

- Berlin and Kay (1969): study of the typology of color words
- subjects with typologically distant native languages
- subjects were asked about prototype and extension of the basic color words of their native language
- English: 11 basic colors

Berlin and Kay's study



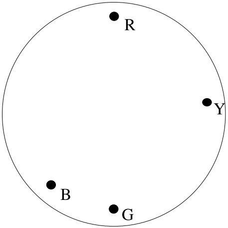
Implicational hierarchies



A toy example

- suppose
  - circular two-dimensional meaning space
  - four meanings are highly frequent
  - all other meanings are negligibly rare
- let's call the frequent meanings Red, Green, Blue and Yellow

$$p_i(\text{Red}) > p_i(\text{Green}) > p_i(\text{Blue}) > p_i(\text{Yellow})$$



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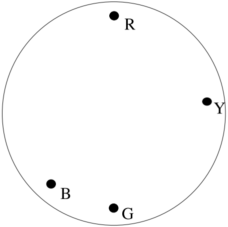
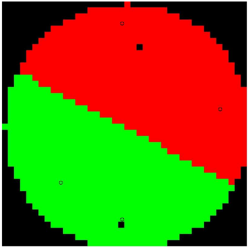


$$p_i(\text{Red}) > p_i(\text{Green}) > p_i(\text{Blue}) > p_i(\text{Yellow})$$

Yes, I made this up without empirical justification.

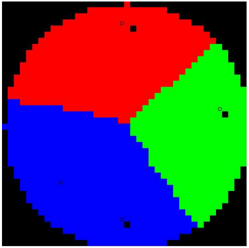
Two forms

- suppose there are just two forms
- only one Strict Nash equilibrium (up to permuation of the forms)
- induces the partition {**Red, Blue**}/{**Yellow, Green**}



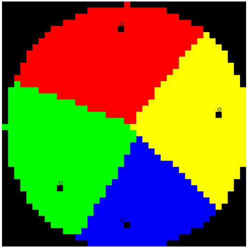
Three forms

- if there are three forms
- two Strict Nash equilibria (up to permuation of the forms)
- partitions {Red}/{Yellow}/{Green, Blue} and {Green}/{Blue}/{Red, Yellow}
- only the former is **stochastically stable** (resistent against random noise)



Four forms

- if there are four forms
- one Strict Nash equilibrium (up to permuation of the forms)
- partitions {Red}/{Yellow}/{Green}/{Blue}



Conclusion

Meaning spaces

- assumption: utility is correlated with similarity between speaker's meaning and hearer's meaning
- consequences:
  - convexity of meanings
  - prototype effects
  - uneven probability distribution over meanings leads to the kind of implicational universals that are known from typology of color terms