# Semantics 1

July 17, 2012

Gerhard Jäger

EBERHARD KARLS
UNIVERSITÄT
TÜBINGEN

# Signaling games

- sequential game:
  1. **nature** chooses a world $w$
     - out of a pool of possible worlds $W$
     - according to a certain probability distribution $p^*$
  2. nature shows $w$ to sender **S**
  3. S chooses a message $m$ out of a set of possible signals $M$
  4. S transmits $m$ to the receiver **R**
  5. R chooses an action $a$, based on the sent message.
- Both S and R have preferences regarding R's action, depending on $w$.
- S might also have preferences regarding the choice of $m$ (to minimize signaling costs).

# Tea or coffee?

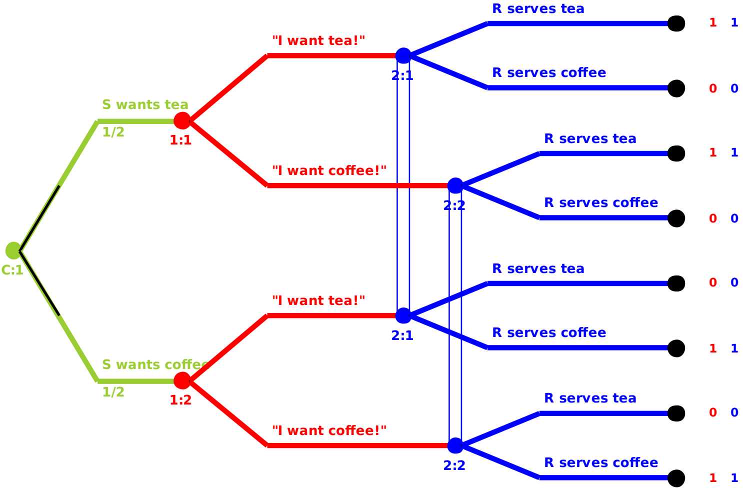
## An example

- Sally either prefers tea ($w_1$) or coffee ($w_2$), with $p^*(w_1) = p^*(w_2) = \frac{1}{2}$.
- Robin either serves tea ($a_1$) or coffee ($a_2$).
- Sally can send either of two messages:
  - $m_1$: *I prefer tea.*
  - $m_2$: *I prefer coffee.*
- Both messages are costless.

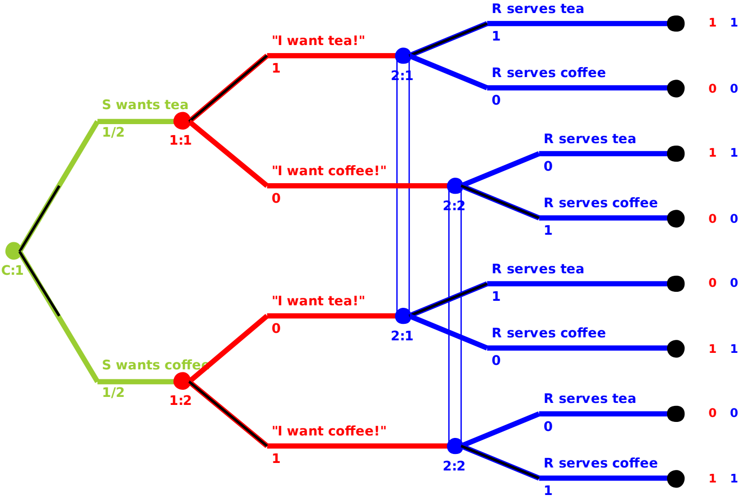
| | $a_1$ | $a_2$ |
|---|---|---|
| $w_1$ | $1, 1$ | $0, 0$ |
| $w_2$ | $0, 0$ | $1, 1$ |

Table: utility matrix

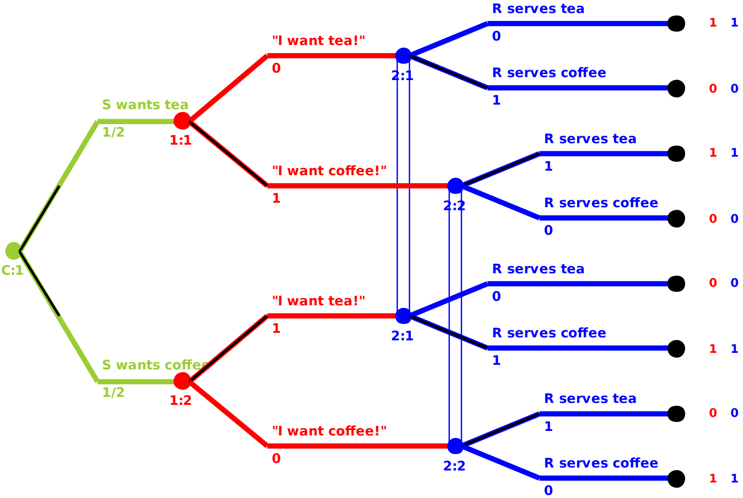
# Extensive form



# Extensive form



# Extensive form



# A coordination problem

- two strict Nash equilibria
  - S always says the truth and R always believes her.
  - S always says the opposite of the truth and R interprets everything ironically.
- Both equilibria are equally rational.
- Still, first equilibrium is more reasonable because it employs exogenous meanings of messages for equilibrium selection.
- Criterion for equilibrium selection:

> *Always say the truth, and always believe what you are told!*

- What happens if it is not always rational to be honest/credulous?

# Partially aligned interests

**Rabin's (1990) example**

- In $w_1$ and $w_2$, S and R have identical interests.
- In $w_3$, S would prefer R to believe in $w_2$.
- The propositions $\{w_1\}$ and $\{w_2, w_3\}$ are *credible*.
- The propositions $\{w_2\}$ and $\{w_3\}$ are *not credible*.

| | $a_1$ | $a_2$ | $a_3$ |
|---|---|---|---|
| $w_1$ | $10, 10$ | $0, 0$ | $0, 0$ |
| $w_2$ | $0, 0$ | $10, 10$ | $5, 7$ |
| $w_3$ | $0, 0$ | $10, 0$ | $5, 7$ |

Table: Partially aligned interests

# Partially aligned interests

### Rabin's (1990) example

- Suppose there are three messages:
  - $m_1$: We are in $w_1$.
  - $m_2$: We are in $w_2$.
  - $m_3$: We are in $w_3$.
- reasonable S will send $m_1$ if and only if $w_1$
- reasonable R will react to $m_1$ with $a_1$
- nothing else can be inferred

| | $a_1$ | $a_2$ | $a_3$ |
|---|---|---|---|
| $w_1$ | $10, 10$ | $0, 0$ | $0, 0$ |
| $w_2$ | $0, 0$ | $10, 10$ | $5, 7$ |
| $w_3$ | $0, 0$ | $10, 0$ | $5, 7$ |

Table: Partially aligned interests

# Revised maxim

**Always say the truth,
and always believe what you are told,
unless you have reasons to do otherwise!**

But what does this mean?

# IBR sequence for Rabin's example

| $\sigma_0$ | $m_1$ | $m_2$ | $m_3$ |
|---|---|---|---|
| $w_1$ | 1 | 0 | 0 |
| $w_2$ | 0 | 1 | 0 |
| $w_3$ | 0 | 0 | 1 |

| $\rho_0$ | $a_1$ | $a_2$ | $a_3$ |
|---|---|---|---|
| $m_1$ | 1 | 0 | 0 |
| $m_2$ | 0 | 1 | 0 |
| $m_3$ | 0 | 0 | 1 |

| $\sigma_1$ | $m_1$ | $m_2$ | $m_3$ |
|---|---|---|---|
| $w_1$ | 1 | 0 | 0 |
| $w_2$ | 0 | 1 | 0 |
| $w_3$ | 0 | 1 | 0 |

| $\rho_2$ | $a_1$ | $a_2$ | $a_3$ |
|---|---|---|---|
| $m_1$ | 1 | 0 | 0 |
| $m_2$ | 0 | 0 | 1 |
| $m_3$ | 0 | 0 | 1 |

| $\sigma_2$ | $m_1$ | $m_2$ | $m_3$ |
|---|---|---|---|
| $w_1$ | 1 | 0 | 0 |
| $w_2$ | 0 | $\frac{1}{2}$ | $\frac{1}{2}$ |
| $w_3$ | 0 | $\frac{1}{2}$ | $\frac{1}{2}$ |

| $\rho_1$ | $a_1$ | $a_2$ | $a_3$ |
|---|---|---|---|
| $m_1$ | 1 | 0 | 0 |
| $m_2$ | 0 | 0 | 1 |
| $m_3$ | 0 | 0 | 1 |

$$F = (\sigma_2, \rho_1)$$

# Interpretation games

- How does this relate to linguistic examples?
- There is a quasi-algorithmic procedure (due to Franke 2009) how to construct a game from an example sentence.

**What is given?**

- example sentence
- set of expression alternatives
- jointly form set of messages
- question under discussion QUD
- set of complete answers to QUD is the set of possible worlds

**What do we need?**

- interpretation function $\| \cdot \|$
- prior probability distribution $p^*$
- set of actions
- utility functions

# Interpretation games

## QUD

- often QUD is not given explicitly
- procedure to construct QUD from expression $m$ and its alternatives $ALT(m)$:
  - Let $ct$ be the context of utterances, i.e. the maximal set of statements that is common knowledge between Sally and Robin.
  - any subset $w$ of $ALT(m) \cup \{\neg m' | m' \in ALT(m)\}$ is a possible world iff
    - $w$ and $ct$ are consistent, i.e. $w \cup ct \not\vdash \bot$
    - for any set $X : w \subset X \subseteq ALT(m) \cup \{\neg m' | m' \in ALT(m)\}$, $ct \cup X$ is inconsistent

# Interpretation games

## Game construction

- interpretation function:

$$\|m'\| = \{w | w \vdash m\}$$

- $p^*$ is uniform distribution over W
- justified by principle of insufficient reason
- set of actions is $W$
- intuitive idea: Robin's task is to figure out which world Sally is in
- utility functions:

$$u_{s/r}(w,a) = \begin{cases} 1 & \text{iff } w = a \\ 0 & \text{else} \end{cases}$$

- both players want Robin to succeed

# Example: Quantity implicatures

(1)
a. Who came to the party?
b. SOME: Some boys came to the party.
c. NO: No boys came to the party.
d. ALL: All boys came to the party.

## Game construction

- $ct = \emptyset$
- $W = \{w_{\neg\exists}, w_{\exists\neg\forall}, w_{\forall}\}$
- $w_{\neg\exists} = \{\text{NO}\}, w_{\exists\neg\forall} = \{\text{SOME}\}, w_{\forall} = \{\text{SOME}, \text{ALL}\}$
- $p^* = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$

- interpretation function:

$$\begin{aligned} \|\text{SOME}\| &= \{w_{\exists\neg\forall}, w_{\forall}\} \\ \|\text{NO}\| &= \{w_{\neg\exists}\} \\ \|\text{ALL}\| &= \{w_{\forall}\} \end{aligned}$$

- utilities:

| | $a_{\neg\exists}$ | $a_{\exists\neg\forall}$ | $a_{\forall}$ |
|---|---|---|---|
| $w_{\neg\exists}$ | 1, 1 | 0, 0 | 0, 0 |
| $w_{\exists\neg\forall}$ | 0, 0 | 1, 1 | 0, 0 |
| $w_{\forall}$ | 0, 0 | 0, 0 | 1, 1 |

# Interpretation games

- utility functions are identity matrices
- therefore the step *multiply with utility matrix* can be omitted in best response computation
- also, restriction to uniform priors makes simplifies computation of posterior distribution
- simplified IBR computation:

# Interpretation games

## Sally

1. flip $\rho$ along diagonal
2. place a 0 in each cell that is non-maximal within its row
3. normalize each row

## Robin

1. flip $\sigma$ along diagonal
2. if a row contains only 0s, fill in a 1 in each cell corresponding to a true world-message association
3. place a 0 in each cell that is non-maximal within its row
4. normalize each row

# Example: Quantity implicatures

| $\sigma_0$ | NO | SOME | ALL |
|---|---|---|---|
| $w_{\neg\exists}$ | 1 | 0 | 0 |
| $w_{\exists\neg\forall}$ | 0 | 1 | 0 |
| $w_{\forall}$ | 0 | $\frac{1}{2}$ | $\frac{1}{2}$ |

| $\rho_0$ | $w_{\neg\exists}$ | $w_{\exists\neg\forall}$ | $w_{\forall}$ |
|---|---|---|---|
| NO | 1 | 0 | 0 |
| SOME | 0 | 1 | 0 |
| ALL | 0 | 0 | 1 |

| $\sigma_1$ | NO | SOME | ALL |
|---|---|---|---|
| $w_{\neg\exists}$ | 1 | 0 | 0 |
| $w_{\exists\neg\forall}$ | 0 | 1 | 0 |
| $w_{\forall}$ | 0 | 0 | 1 |

| $\rho_1$ | $w_{\neg\exists}$ | $w_{\exists\neg\forall}$ | $w_{\forall}$ |
|---|---|---|---|
| NO | 1 | 0 | 0 |
| SOME | 0 | 1 | 0 |
| ALL | 0 | 0 | 1 |

$$F = (\rho_0, \sigma_1)$$

In the fixed point, SOME is interpreted as entailing ¬ALL, i.e. exhaustively.

# Lifted games

- So far, it is hard-wired in the model that Sally has complete knowledge (or, rather, complete belief — whether or not she is right is inessential for IBR) about the world she is in.
- corresponds to strong version of **competence assumption**
- Sometimes this assumption is too strong:

# Lifted games

1.
   a. Ann or Bert showed up. (= OR)
   b. Ann showed up. (= A)
   c. Bert showed up. (= B)
   d. Ann and Bert showed up. (= AND)

- $w_a$: Only Ann showed up.
- $w_b$: Only Bert showed up.
- $w_{ab}$: Both showed up.

**Utility matrix**

| | $a_a$ | $a_b$ | $a_{ab}$ |
|---|---|---|---|
| $w_a$ | 1 | 0 | 0 |
| $w_b$ | 0 | 1 | 0 |
| $w_{ab}$ | 0 | 0 | 1 |

# Lifted games

## IBR sequence

| $\sigma_0$ | OR | A | B | AND |
|---|---|---|---|---|
| $w_a$ | $\frac{1}{2}$ | $\frac{1}{2}$ | 0 | 0 |
| $w_b$ | $\frac{1}{2}$ | 0 | $\frac{1}{2}$ | 0 |
| $w_{ab}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ |

| $\rho_0$ | $w_a$ | $w_b$ | $w_{ab}$ |
|---|---|---|---|
| OR | $\frac{1}{2}$ | $\frac{1}{2}$ | 0 |
| A | 1 | 0 | 0 |
| B | 0 | 1 | 0 |
| AND | 0 | 0 | 1 |

| $\sigma_1$ | OR | A | B | AND |
|---|---|---|---|---|
| $w_a$ | 0 | 1 | 0 | 0 |
| $w_b$ | 0 | 0 | 1 | 0 |
| $w_{ab}$ | 0 | 0 | 0 | 1 |

| $\rho_1$ | $w_a$ | $w_b$ | $w_{ab}$ |
|---|---|---|---|
| OR | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ |
| A | 1 | 0 | 0 |
| B | 0 | 1 | 0 |
| AND | 0 | 0 | 1 |

OR comes out as a message that would never be used!

# Lifted games

- full competence assumption is arguably too strong
- weaker assumption (Franke 2009):
  - Sally's information states are **partial answers to QUD**, ie. **sets** of possible worlds
  - Robin's task is to figure out which information state Sally is in.
  - *ceteris paribus*, Robin receives slightly higher utility for smaller (more informative) states

## Costs

- Preferences that are independent from correct information transmission are captured via *cost functions* for sender and receiver.
- For the sender this might be, *inter alia*, a preference for simpler expressions.
- For the receiver, the *Strongest Meaning Hypothesis* is a good candiate.

# Lifted games

## Formally

- cost functions $c_s, c_r$: $c_s : (POW(W) - \{\emptyset\}) \times M \mapsto \mathbb{R}^+$
- costs are **nominal**:

$$0 \leq c_s(i,m), c_r(i,m) < \min(\frac{1}{|POW(W) - \emptyset|^2}, \frac{1}{|ALT(m)|^2})$$

- guarantees that cost considerations never get in the way of information transmission considerations
- new utility functions:

$$\begin{aligned} u_s(i,m,a) &= -c_s(i,m) + \begin{cases} 1 & \text{if } i = a, \\ 0 & \text{else,} \end{cases} \\ u_r(i,m,a) &= -c_r(a,m) + \begin{cases} 1 & \text{if } i = a, \\ 0 & \text{else.} \end{cases} \end{aligned}$$

# Modified IBR procecure

## Sally

- flip $\rho$ along the diagonal
- subtract $c_s$
- place a 0 in each cell that is non-maximal within its row
- normalize each row

## Robin

- flip $\sigma$ along diagonal
- if a row contains only 0s,
  - fill in a 1 in each cell corresponding to a true world-message association
- else
  - subtract $c_r^T$
- place a 0 in each cell that is non-maximal within its row
- normalize each row

# The Strongest Meaning Hypothesis

- if in doubt, Robin will assume that Sally is competent
- captured in following cost function:

$$c_r(a, m) = \frac{|a|}{\max(|M|, 2^{|W|})^2}$$

$$\begin{array}{lcllcl}
c_r(\{w_a\}, \cdot) & = & \frac{1}{49} & c_r(\{w_a, w_{ab}\}, \cdot) & = & \frac{2}{49} \\
c_r(\{w_b\}, \cdot) & = & \frac{1}{49} & c_r(\{w_b, w_{ab}\}, \cdot) & = & \frac{2}{49} \\
c_r(\{w_{ab}\}, \cdot) & = & \frac{1}{49} & c_r(\{w_a, w_b, w_{ab}\}, \cdot) & = & \frac{3}{49} \\
c_r(\{w_a, w_b\}, \cdot) & = & \frac{2}{49} & & &
\end{array}$$

# Lifted games

IBR sequence: 1

| $\sigma_0$ | OR | A | B | AND |
|---|---|---|---|---|
| $\{w_a\}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $0$ | $0$ |
| $\{w_b\}$ | $\frac{1}{2}$ | $0$ | $\frac{1}{2}$ | $0$ |
| $\{w_{ab}\}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ |
| $\{w_a, w_b\}$ | $1$ | $0$ | $0$ | $0$ |
| $\{w_a, w_{ab}\}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $0$ | $0$ |
| $\{w_b, w_{ab}\}$ | $\frac{1}{2}$ | $0$ | $\frac{1}{2}$ | $0$ |
| $\{w_a, w_b, w_{ab}\}$ | $1$ | $0$ | $0$ | $0$ |

# Lifted games

IBR sequence: flipping and subtracting costs

| $\rho_0$ | $\{w_a\}$ | $\{w_b\}$ | $\{w_{ab}\}$ | $\{w_a, w_b\}$ | $\{w_a, w_{ab}\}$ | $\{w_b, w_{ab}\}$ | $\{w_a, w_b, w_{ab}\}$ |
|---|---|---|---|---|---|---|---|
| OR | 0.48 | 0.48 | 0.23 | **0.96** | 0.46 | 0.46 | 0.94 |
| A | **0.48** | −0.02 | 0.23 | −0.04 | 0.46 | −0.04 | −0.06 |
| B | −0.02 | **0.48** | 0.23 | −0.04 | −0.04 | 0.46 | −0.06 |
| AND | −0.02 | −0.02 | **0.23** | −0.04 | −0.04 | −0.04 | −0.06 |

# Lifted games

IBR sequence: 2

| $\rho_0$ | $\{w_a\}$ | $\{w_b\}$ | $\{w_{ab}\}$ | $\{w_a, w_b\}$ | $\{w_a, w_{ab}\}$ | $\{w_b, w_{ab}\}$ | $\{w_a, w_b, w_{ab}\}$ |
|---|---|---|---|---|---|---|---|
| OR | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| A | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| B | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| AND | 0 | 0 | 1 | 0 | 0 | 0 | 0 |

# Lifted games

IBR sequence: 3

| $\sigma_1$ | OR | A | B | AND |
|---|---|---|---|---|
| $\{w_a\}$ | 0 | 1 | 0 | 0 |
| $\{w_b\}$ | 0 | 0 | 1 | 0 |
| $\{w_{ab}\}$ | 0 | 0 | 0 | 1 |
| $\{w_a, w_b\}$ | 1 | 0 | 0 | 0 |
| $\{w_a, w_{ab}\}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | 0 | 0 |
| $\{w_b, w_{ab}\}$ | $\frac{1}{2}$ | 0 | $\frac{1}{2}$ | 0 |
| $\{w_a, w_b, w_{ab}\}$ | 1 | 0 | 0 | 0 |

# Lifted games

- OR is only used in $\{w_a, w_b\}$ in the fixed point
- this means that it carries two implicatures:
  - exhaustivity: Ann and Bert did not both show up
  - ignorance: Sally does not know which one of the two disjuncts is true

# Sender costs

2
a. Ann or Bert or both showed up. $(= \text{AB-OR})$
b. Ann showed up. $(= \text{A})$
c. Bert showed up. $(= \text{B})$
d. Ann and Bert showed up. $(= \text{AND})$
e. Ann or Bert showed up. $(= \text{OR})$
f. Ann or both showed up. $(= \text{A-OR})$
g. Bert or both showed up. $(= \text{B-OR})$

- Message (e) is arguably more efficient for Sally than (a)
- Let us say that $c_s(\cdot, \text{AB-OR}) = \frac{1}{50}, c_s(\cdot, \text{A-OR}) = c_s(\cdot, \text{B-OR}) = \frac{1}{75}, c_s(\cdot, \text{OR}) = c_s(\cdot, \text{AND}) = \frac{1}{100})$, and $c_s(\cdot, \text{A}) = c_s(\cdot, \text{B}) = 0$.

# More ignorance implicatures

IBR sequence: 1

| $\sigma_0$ | AB-OR | A | B | AND | OR | A-OR | B-OR |
|---|---|---|---|---|---|---|---|
| $\{w_a\}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | 0 | 0 | $\frac{1}{4}$ | $\frac{1}{4}$ | 0 |
| $\{w_b\}$ | $\frac{1}{4}$ | 0 | $\frac{1}{4}$ | 0 | $\frac{1}{4}$ | 0 | $\frac{1}{4}$ |
| $\{w_{ab}\}$ | $\frac{1}{7}$ | $\frac{1}{7}$ | $\frac{1}{7}$ | $\frac{1}{7}$ | $\frac{1}{7}$ | $\frac{1}{7}$ | $\frac{1}{7}$ |
| $\{w_a, w_b\}$ | $\frac{1}{2}$ | 0 | 0 | 0 | $\frac{1}{2}$ | 0 | 0 |
| $\{w_a, w_{ab}\}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | 0 | 0 | $\frac{1}{4}$ | $\frac{1}{4}$ | 0 |
| $\{w_b, w_{ab}\}$ | $\frac{1}{4}$ | 0 | $\frac{1}{4}$ | 0 | $\frac{1}{4}$ | 0 | $\frac{1}{4}$ |
| $\{w_a, w_b, w_{ab}\}$ | $\frac{1}{2}$ | 0 | 0 | 0 | $\frac{1}{2}$ | 0 | 0 |

# More ignorance implicatures

IBR sequence: 1

| $\rho_0$ | $\{w_a\}$ | $\{w_b\}$ | $\{w_{ab}\}$ | $\{w_a, w_b\}$ | $\{w_a, w_{ab}\}$ | $\{w_b, w_{ab}\}$ | $\{w_a, w_b, w_{ab}\}$ |
|---|---|---|---|---|---|---|---|
| AB-OR | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| A | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| B | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| AND | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| OR | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| A-OR | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| B-OR | 0 | 1 | 0 | 0 | 0 | 0 | 0 |

# More ignorance implicatures

IBR sequence: 2

| $\sigma_1$ | AB-OR | A | B | AND | OR | A-OR | B-OR |
|---|---|---|---|---|---|---|---|
| $\{w_a\}$ | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| $\{w_b\}$ | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| $\{w_{ab}\}$ | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| $\{w_a, w_b\}$ | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| $\{w_a, w_{ab}\}$ | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| $\{w_b, w_{ab}\}$ | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| $\{w_a, w_b, w_{ab}\}$ | 0 | 0 | 0 | 0 | 1 | 0 | 0 |

# More ignorance implicatures

IBR sequence: 2

| $\rho_1$ | $\{w_a\}$ | $\{w_b\}$ | $\{w_{ab}\}$ | $\{w_a, w_b\}$ | $\{w_a, w_{ab}\}$ | $\{w_b, w_{ab}\}$ | $\{w_a, w_b, w_{ab}\}$ |
|---|---|---|---|---|---|---|---|
| ORBOTH | $\frac{1}{7}$ | $\frac{1}{7}$ | $\frac{1}{7}$ | $\frac{1}{7}$ | $\frac{1}{7}$ | $\frac{1}{7}$ | $\frac{1}{7}$ |
| A | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| B | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| AND | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| OR | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| A-OR | $\frac{1}{3}$ | 0 | $\frac{1}{3}$ | 0 | $\frac{1}{3}$ | 0 | 0 |
| B-OR | 0 | $\frac{1}{3}$ | $\frac{1}{3}$ | 0 | 0 | $\frac{1}{3}$ | 0 |

# More ignorance implicatures

IBR sequence: 3

| $\sigma_2$ | AB-OR | A | B | AND | OR | A-OR | B-OR |
|---|---|---|---|---|---|---|---|
| $\{w_a\}$ | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| $\{w_b\}$ | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| $\{w_{ab}\}$ | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| $\{w_a, w_b\}$ | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| $\{w_a, w_{ab}\}$ | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| $\{w_b, w_{ab}\}$ | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| $\{w_a, w_b, w_{ab}\}$ | 1 | 0 | 0 | 0 | 0 | 0 | 0 |

# More ignorance implicatures

IBR sequence: 3

| $\rho_2$ | $\{w_a\}$ | $\{w_b\}$ | $\{w_{ab}\}$ | $\{w_a, w_b\}$ | $\{w_a, w_{ab}\}$ | $\{w_b, w_{ab}\}$ | $\{w_a, w_b, w_{ab}\}$ |
|---|---|---|---|---|---|---|---|
| ORBOTH | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| A | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| B | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| AND | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| OR | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| A-OR | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| B-OR | 0 | 0 | 0 | 0 | 0 | 1 | 0 |

# I-implicatures

(2) a. John opened the door. (= OPEN)
b. John opened the door using the handle. (= OPEN-H)
c. John opened the door with an axe. (= OPEN-A)

**formally**

- $W = \{w_h, w_a\}$
- $p^*(w_1) = \frac{2}{3}, p^*(w_2) = \frac{1}{3}$
- $\|\text{OPEN-H}\| = \{w_h\}$, $\|\text{OPEN-A}\| = \{w_a\}$, and $\|\text{OPEN}\| = \{w_h, w_a\}$
- $c(m_1) = c(m_2) \in \frac{1}{20}$, $c(m_3) = 0$

| | $a_h$ | $a_a$ |
|---|---|---|
| $w_h$ | $1, 1$ | $0, 0$ |
| $w_a$ | $0, 0$ | $1, 1$ |

# I-implicatures

| $\sigma_0$ | OPEN | OPEN-H | OPEN-A |
|---|---|---|---|
| $w_h$ | $\frac{1}{2}$ | $\frac{1}{2}$ | 0 |
| $w_a$ | $\frac{1}{2}$ | 0 | $\frac{1}{2}$ |

| $\rho_0$ | $w_h$ | $w_a$ |
|---|---|---|
| OPEN | 1 | 0 |
| OPEN-H | 1 | 0 |
| OPEN-A | 0 | 1 |

| $\sigma_1$ | OPEN | OPEN-H | OPEN-A |
|---|---|---|---|
| $w_h$ | 1 | 0 | 0 |
| $w_a$ | 0 | 0 | 1 |

| $\rho_1$ | $w_h$ | $w_a$ |
|---|---|---|
| OPEN | 1 | 0 |
| OPEN-H | 1 | 0 |
| OPEN-A | 0 | 1 |

$$F = (\sigma_1, \rho_0)$$

# Measure terms

Krifka (2002,2007) notes that measure terms can be used in a precise or in a vague way, and that more complex expressions are less likely to be used in a vague way. Here is a schematic analysis:

- $w_1, w_3$: 100 meter, $w_2, w_4$: 101 meter
- $m_{100}$: "one hundred meter"
  $m_{101}$: "one hundred and one meter"
  $m_{ex100}$: "exactly one hundred meter"
- $\|m_{100}\| = \|m_{ex100}\| = \{w_1, w_3\}$,
  $\|m_{101}\| = \{w_2, w_4\}$
- $c(m_{100}) = 0$,
  $c(m_{101}) = c(m_{ex100}) = 0.15$
- $a_1, a_3$: 100, $a_2, a_4$: 101

- in $w_1, w_2$ precision is important
- in $w_3, w_4$ precision is not important

| | $a_1$ | $a_2$ | $a_3$ | $a_4$ |
|---|---|---|---|---|
| $w_1$ | 1 | 0.5 | 1 | 0.5 |
| $w_2$ | 0.5 | 1 | 0.5 | 1 |
| $w_3$ | 1 | 0.9 | 1 | 0.9 |
| $w_4$ | 0.9 | 1 | 0.9 | 1 |

# Measure terms

| $\sigma_0$ | $m_{100}$ | $m_{101}$ | $m_{ex100}$ |
|---|---|---|---|
| $w_1$ | $\frac{1}{2}$ | 0 | $\frac{1}{2}$ |
| $w_2$ | 0 | 1 | 0 |
| $w_3$ | $\frac{1}{2}$ | 0 | $\frac{1}{2}$ |
| $w_4$ | 0 | 1 | 0 |

| $\rho_0$ | $a_1$ | $a_2$ | $a_3$ | $a_4$ |
|---|---|---|---|---|
| $m_{100}$ | $\frac{1}{2}$ | 0 | $\frac{1}{2}$ | 0 |
| $m_{101}$ | 0 | $\frac{1}{2}$ | 0 | $\frac{1}{2}$ |
| $m_{ex100}$ | $\frac{1}{2}$ | 0 | $\frac{1}{2}$ | 0 |

| $\sigma_1$ | $m_{100}$ | $m_{101}$ | $m_{ex100}$ |
|---|---|---|---|
| $w_1$ | 1 | 0 | 0 |
| $w_2$ | 0 | 1 | 0 |
| $w_3$ | 1 | 0 | 0 |
| $w_4$ | 1 | 0 | 0 |

| $\rho_1$ | $a_1$ | $a_2$ | $a_3$ | $a_4$ |
|---|---|---|---|---|
| $m_{100}$ | $\frac{1}{3}$ | 0 | $\frac{1}{3}$ | $\frac{1}{3}$ |
| $m_{101}$ | 0 | 1 | 0 | 0 |
| $m_{ex100}$ | $\frac{1}{2}$ | 0 | $\frac{1}{2}$ | 0 |

| $\sigma_2$ | $m_{100}$ | $m_{101}$ | $m_{ex100}$ |
|---|---|---|---|
| $w_1$ | 0 | 0 | 1 |
| $w_2$ | 0 | 1 | 0 |
| $w_3$ | 1 | 0 | 0 |
| $w_4$ | 1 | 0 | 0 |

| $\rho_2$ | $a_1$ | $wa_2$ | $a_3$ | $a_4$ |
|---|---|---|---|---|
| $m_{100}$ | 0 | 0 | $\frac{1}{2}$ | $\frac{1}{2}$ |
| $m_{101}$ | 0 | 1 | 0 | 0 |
| $m_{ex100}$ | 1 | 0 | 0 | 0 |

# M-implicatures

3. a. John stopped the car. (= STOP)
   b. John made the car stop. (= MAKE-STOP)

- $w_1$: John used the foot brake.
- $w_2$: John drove the car against a wall.
- $\|\text{STOP}\| = \|\text{MAKE-STOP}\| = \{w_1, w_2\}$
- $c(\text{STOP}) = 0$; $c(\text{MAKE-STOP} = 0.1$
- $p^*(w_1) = .8$; $p^*(w_2) = .2$.

**Utility matrix**

| | $a_1$ | $a_2$ |
|---|---|---|
| $w_1$ | 1 | 0 |
| $w_2$ | 0 | 1 |

# M-implicatures

## IBR sequence

| $\sigma_0$ | STOP | MAKE-STOP |
|---|---|---|
| $w_1$ | $\frac{1}{2}$ | $\frac{1}{2}$ |
| $w_2$ | $\frac{1}{2}$ | $\frac{1}{2}$ |

| $\rho_0$ | $a_1$ | $a_2$ |
|---|---|---|
| STOP | 1 | 0 |
| MAKE-STOP | 1 | 0 |

| $\sigma_1$ | STOP | MAKE-STOP |
|---|---|---|
| $w_1$ | 1 | 0 |
| $w_2$ | 1 | 0 |

| $\rho_1$ | $a_1$ | $a_2$ |
|---|---|---|
| STOP | 1 | 0 |
| MAKE-STOP | $\frac{1}{2}$ | $\frac{1}{2}$ |

| $\sigma_2$ | STOP | MAKE-STOP |
|---|---|---|
| $w_1$ | 1 | 0 |
| $w_2$ | 0 | 1 |

| $\rho_2$ | $a_1$ | $a_2$ |
|---|---|---|
| STOP | 1 | 0 |
| MAKE-STOP | 0 | 1 |