Mathematical and computational models of language evolution

Gerhard Jäger

Institute of Linguistics, Tübingen University

DGfS Summer School

August 19, 2013

EGT and pragmatics

Horn strategies: prototypical meanings tend to go with simple expressions and less prototypical meanings with complex expressions.

(1) **a.** John went to church/jail. (prototypical interpretation)
b. John went to the church/jail. (literal interpretation)

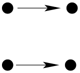
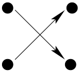
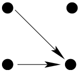
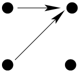
(2) **a.** I am going to marry you. (no indirect speech act)
b. I will marry you. (indirect speech act)

(3) **a.** I need a new driller/cooker.
b. I need a new drill/cook.

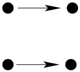
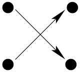
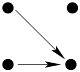
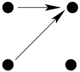
Horn strategies

- simple game:
  - players: speaker and hearer
  - two forms: f_0 (short) and f_1 (long)
  - two meanings: m_0 (frequent) and m_1 (rare)
  - speaker strategies: mappings from meanings to forms
  - hearer strategies: mappings from forms to meanings

Speaker strategies

- $S_1 : m_0 \mapsto f_0,\ m_1 \mapsto f_1$:  "Horn strategy"
- $S_2 : m_0 \mapsto f_1,\ m_1 \mapsto f_0$:  "anti-Horn strategy"
- $S_3 : m_0 \mapsto f_0,\ m_1 \mapsto f_0$:  "Smolensky strategy"
- $S_4 : m_0 \mapsto f_1,\ m_1 \mapsto f_1$:  "anti-Smolensky strategy"

Hearer strategies

- $H_1 : f_0 \mapsto m_0,\ f_1 \mapsto m_1$:  “Horn strategy’
- $H_2 : f_0 \mapsto m_1,\ f_1 \mapsto m_0$:  “anti-Horn strategy”
- $H_3 : f_0 \mapsto m_0,\ f_1 \mapsto m_0$:  “Smolensky strategy”
- $H_4 : f_0 \mapsto m_1,\ f_1 \mapsto m_1$:  “anti-Smolensky strategy”

Utility of Horn games

- whether communication works depends both on speaker strategy S and hearer strategy H
- two factors for functionality of communication
  - communicative success ("hearer economy")

$$\delta_m(S,H) = \begin{cases} 1 & \text{iff} \quad H(S(m)) = m \\ 0 & \text{else} \end{cases}$$

  - least effort ("speaker economy")

$$\mathit{cost}(f) \ldots \text{measure of complexity of expression}$$

Utility of Horn games

$$u_{s/h}(S,H) = \sum_m p_m \times (\delta_m(S,H) - \textit{cost}(S(m)))$$

$p \ldots$ probability distribution over meanings

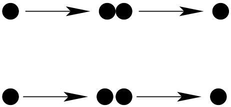
Utility of Horn game

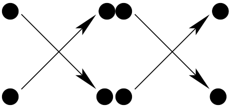
Let's make up some numbers:

- $p(m_0) = .75$
- $p(m_1) = .25$
- $\textit{cost}(f_0) = .1$
- $\textit{cost}(f_1) = .2$

Utility of Horn game

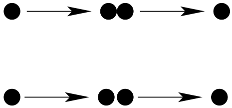
	H_1	H_2	H_3	H_4
S_1	.875	-.125	.625	.125
S_2	-.175	.825	.575	.25
S_3	.65	.15	.65	.15
S_4	.05	.55	.55	.05

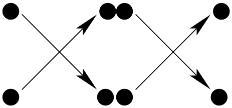




Utility of Horn game

	H_1	H_2	H_3	H_4
S_1	.875	-.125	.625	.125
S_2	-.175	.825	.575	.25
S_3	.65	.15	.65	.15
S_4	.05	.55	.55	.05





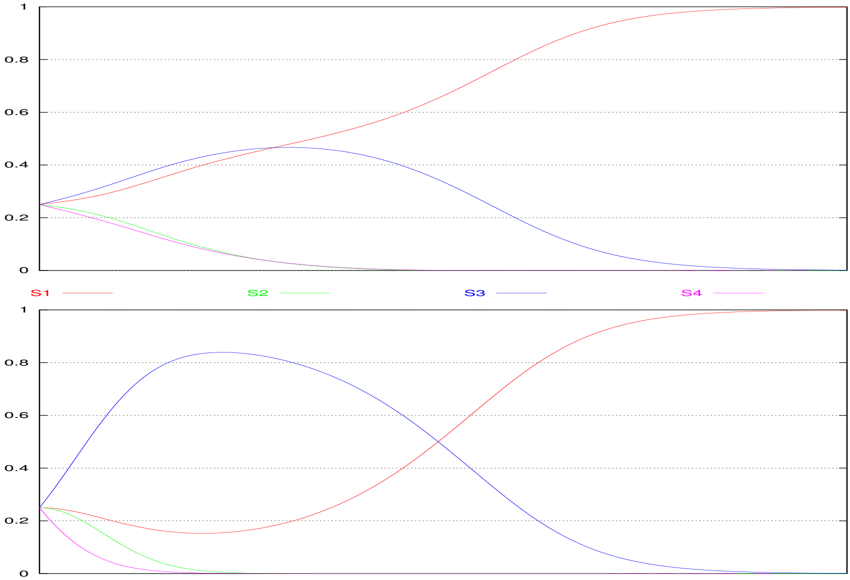
The problem of equilibrium selection

- both Horn and anti-Horn are evolutionarily stable
- EGT explains the aversion of natural languages against synonymy and ambiguity
- preference for Horn not directly explainable in standard EGT

The problem of equilibrium selection

- rationalistic considerations favor Horn over anti-Horn:
  - Horn strategy is **Pareto efficient** (nobody can do better in absolute terms)
  - Horn strategy **risk dominates** anti-Horn (if you know the population is in an equilibrium but you do not know in which one, going for Horn is less risky than anti-Horn)
- replicator dynamics favors Horn over anti-Horn:
  - complete random state evolves to Horn/Horn
  - basin of attraction of Horn is about 20 times as large as basin of attraction of anti-Horn (numerical approximation—does anybody know how to do this analytically?)

Dynamics starting from random state



The evolution of differential case marking

Ways of argument identification

- transitivity may lead to ambiguity

die Frau, die Maria kennt

the woman that Maria knows | the woman that knows Maria

- three ways out
  1. word order
  2. agreement
  3. **case**

die Frau, die er kennt

the woman that he knows

die Frau, die ihn kennt

the woman that knows him

- Suppose one argument is a pronoun and one is a noun (or a phrase)

  {I, BOOK, KNOW}

- both conversants have an interest in successful communication
- case marking (accusative or ergative) is usually more costly than zero-marking (nominative)
- speaker wants to avoid costs

speaker strategies	*hearer strategies*
always case mark the object (accusative)	ergative is agent and accusative object
always case mark the agent (ergative)	pronoun is agent
case mark the object if it is a pronoun	pronoun is object
	pronoun is agent unless it is accusative
⋮	⋮

Statistical patterns of language use

four possible clause types

	O/p	O/n
A/p	he knows it	he knows the book
A/n	the man knows it	the man knows the book

statistical distribution (from a corpus of spoken English)

	O/p	O/n
A/p	pp = 198	pn = 716
A/n	np = 16	nn = 75

pn $\gg$ np

- functionality of speaker strategies and hearer strategies depends on various factors:
  - How often will the hearer get the message right?
  - How many case markers does the speaker need per clause — on average?

speaker strategies that will be considered

agent is pronoun	*agent is noun*	*object is pronoun*	*object is noun*
e(rgative)	e(rgative)	a(ccusative)	a(ccusative)
e	e	a	z(ero)
e	e	z	a
e	e	z	z
e	z	a	a
...	...	...	...
z	e	z	z
z	z	a	a
z	z	a	z
z	z	z	a
z	z	z	z

- hearer strategies:
  - strict rule: ergative means "agent", and accusative means "object"
  - elsewhere rules:

1. SO: "The first phrase is always the agent."
2. pA: "Pronouns are agents, and nouns are objects."
3. pO: "Pronouns are objects, and nouns are agents."
4. OS: "The first phrase is always the object."

The game of case

- strategy space and utility function are known
- probability of meaning types can be estimated from corpus study
- hard to estimate how the complexity of a case morpheme compares to its benefit for disambiguation from the speaker perspective
- parameterized utility function

$$u(S,H) \quad = \quad \sum_m p_m \times (\delta_m(S,H) - k \times \mathit{cost}(S(m)))$$

Utility of case marking

- let us assume $k = .1$

Speaker strategies	Hearer strategies			
	SO	pA	pO	OS
$eezz$	0.90	0.90	0.90	0.90
$zzaa$	0.90	0.90	0.90	0.90
$ezaz$	0.85	0.85	0.85	0.85
$zeza$	0.81	0.81	0.81	0.81
$zeaz$	0.61	0.97	0.26	0.61
$ezzz$	0.86	0.86	0.87	0.86
$zezz$	0.54	0.89	0.54	0.54
$zzaz$	0.59	0.94	0.59	0.59
$zzza$	0.81	0.81	0.82	0.81
$zzzz$	0.50	0.85	0.15	0.50

Utility of case marking

- let us assume $k = .1$

Speaker strategies	Hearer strategies			
	SO	pA	pO	OS
$eezz$	0.90	0.90	0.90	0.90
$zzaa$	0.90	0.90	0.90	0.90
$ezaz$	0.85	0.85	0.85	0.85
$zeza$	0.81	0.81	0.81	0.81
$zeaz$	0.61	0.97	0.26	0.61
$ezzz$	0.86	0.86	0.87	0.86
$zezz$	0.54	0.89	0.54	0.54
$zzaz$	0.59	0.94	0.59	0.59
$zzza$	0.81	0.81	0.82	0.81
$zzzz$	0.50	0.85	0.15	0.50

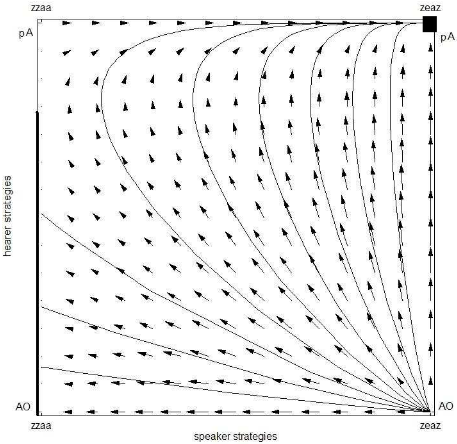
Utility of case marking

- only one evolutionarily stable state: $zeaz/pA$ (*split ergative*)
- very common among Australian aborigines languages

Non-strict Nash equilibria

Why are non-strict Nash Equilibria unstable?

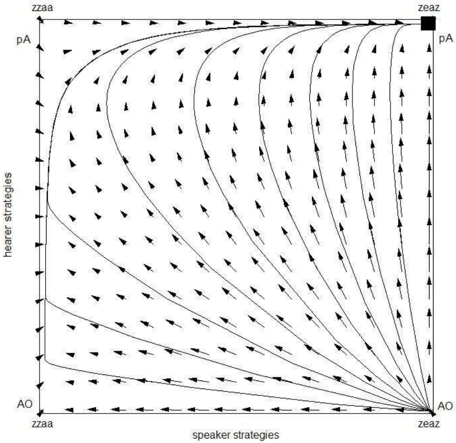
- Dynamics without mutation



Non-strict Nash equilibria

Why are non-strict Nash Equilibria unstable?

- Dynamics with mutation



Utility of case marking

If speakers get lazier...

- $k = 0.45$

Speaker strategies	Hearer strategies			
	SO	pA	pO	OS
$eezz$	0.550	0.550	0.550	0.550
$zzaa$	0.550	0.550	0.550	0.550
$ezaz$	0.458	0.458	0.458	0.458
$zeza$	0.507	0.507	0.507	0.507
$zeaz$	0.507	0.863	0.151	0.507
$ezzz$	0.545	0.538	0.553	0.545
$zezz$	0.505	0.861	0.148	0.505
$zzaz$	0.510	0.867	0.154	0.510
$zzza$	0.539	0.531	0.547	0.539
$zzzz$	0.500	0.849	0.152	0.500

Utility of case marking

If speakers get lazier...

- $k = 0.45$

Speaker strategies	Hearer strategies			
	SO	pA	pO	OS
$eezz$	0.550	0.550	0.550	0.550
$zzaa$	0.550	0.550	0.550	0.550
$ezaz$	0.458	0.458	0.458	0.458
$zeza$	0.507	0.507	0.507	0.507
$zeaz$	0.507	0.863	0.151	0.507
$ezzz$	0.545	0.538	0.553	0.545
$zezz$	0.505	0.861	0.148	0.505
$zzaz$	0.510	0.867	0.154	0.510
$zzza$	0.539	0.531	0.547	0.539
$zzzz$	0.500	0.849	0.152	0.500

Utility of case marking

... and lazier ...

- $k = 0.53$

Speaker strategies	Hearer strategies			
	SO	pA	pO	OS
$eezz$	0.470	0.470	0.470	0.470
$zzaa$	0.470	0.470	0.470	0.470
$ezaz$	0.368	0.368	0.368	0.368
$zeza$	0.436	0.436	0.436	0.436
$zeaz$	0.483	0.839	0.127	0.483
$ezzz$	0.473	0.465	0.480	0.473
$zezz$	0.497	0.854	0.141	0.497
$zzaz$	0.494	0.850	0.137	0.494
$zzza$	0.476	0.468	0.484	0.476
$zzzz$	0.500	0.848	0.152	0.500

Utility of case marking

... and lazier ...

- $k = 0.53$

Speaker strategies	Hearer strategies			
	SO	pA	pO	OS
$eezz$	0.470	0.470	0.470	0.470
$zzaa$	0.470	0.470	0.470	0.470
$ezaz$	0.368	0.368	0.368	0.368
$zeza$	0.436	0.436	0.436	0.436
$zeaz$	0.483	0.839	0.127	0.483
$ezzz$	0.473	0.465	0.480	0.473
$zezz$	0.497	0.854	0.141	0.497
$zzaz$	0.494	0.850	0.137	0.494
$zzza$	0.476	0.468	0.484	0.476
$zzzz$	0.500	0.848	0.152	0.500

Utility of case marking

... and lazier...

- $k = 0.7$

Speaker strategies	Hearer strategies			
	SO	pA	pO	OS
$eezz$	0.300	0.300	0.300	0.300
$zzaa$	0.300	0.300	0.300	0.300
$ezaz$	0.177	0.177	0.177	0.177
$zeza$	0.287	0.287	0.287	0.287
$zeaz$	0.431	0.788	0.075	0.431
$ezzz$	0.318	0.310	0.326	0.318
$zezz$	0.482	0.838	0.126	0.482
$zzaz$	0.457	0.814	0.101	0.457
$zzza$	0.343	0.335	0.350	0.343
$zzzz$	0.500	0.848	0.152	0.500

Utility of case marking

... and lazier...

- $k = 0.7$

Speaker strategies	Hearer strategies			
	SO	pA	pO	OS
$eezz$	0.300	0.300	0.300	0.300
$zzaa$	0.300	0.300	0.300	0.300
$ezaz$	0.177	0.177	0.177	0.177
$zeza$	0.287	0.287	0.287	0.287
$zeaz$	0.431	0.788	0.075	0.431
$ezzz$	0.318	0.310	0.326	0.318
$zezz$	0.482	0.838	0.126	0.482
$zzaz$	0.457	0.814	0.101	0.457
$zzza$	0.343	0.335	0.350	0.343
$zzzz$	0.500	0.848	0.152	0.500

Utility of case marking

...

- $k = 1$

Speaker strategies	Hearer strategies			
	SO	pA	pO	OS
$eezz$	0.000	0.000	0.000	0.000
$zzaa$	0.000	0.000	0.000	0.000
$ezaz$	−0.160	−0.160	−0.160	−0.160
$zeza$	0.024	0.024	0.024	0.024
$zeaz$	0.340	0.697	−0.016	0.340
$ezzz$	0.045	0.037	0.053	0.045
$zezz$	0.455	0.811	0.099	0.455
$zzaz$	0.394	0.750	0.037	0.394
$zzza$	0.106	0.098	0.144	0.106
$zzzz$	0.500	0.848	0.152	0.500

Utility of case marking

...

- $k = 1$

Speaker strategies	Hearer strategies			
	SO	pA	pO	OS
$eezz$	0.000	0.000	0.000	0.000
$zzaa$	0.000	0.000	0.000	0.000
$ezaz$	−0.160	−0.160	−0.160	−0.160
$zeza$	0.024	0.024	0.024	0.024
$zeaz$	0.340	0.697	−0.016	0.340
$ezzz$	0.045	0.037	0.053	0.045
$zezz$	0.455	0.811	0.099	0.455
$zzaz$	0.394	0.750	0.037	0.394
$zzza$	0.106	0.098	0.144	0.106
$zzzz$	0.500	0.848	0.152	0.500

Taking stock

$zeaz/pA$ split ergative	
$zzaz/pA$ differential object marking	$ezzz/pO$ inverse DOM —
$zezz/pA$ differential subject marking	$zzza/pO$ inverse DSM
$zzzz/pA$ no case marking	$zzza/pO$
$zzzz/pA$	

Taking stock

$zeaz/pA$ split ergative Australian languages	
$zzaz/pA$ differential object marking	$ezzz/pO$ inverse DOM —
$zezz/pA$ differential subject marking	$zzza/pO$ inverse DSM
$zzzz/pA$ no case marking	$zzza/pO$
$zzzz/pA$	

Taking stock

$zeaz/pA$
split ergative
Australian languages

$zzaz/pA$ differential object marking English, Dutch, ...	$ezzz/pO$ inverse DOM —
$zezz/pA$ differential subject marking	$zzza/pO$ inverse DSM
$zzzz/pA$ no case marking	$zzza/pO$
$zzzz/pA$	

Taking stock

$zeaz/pA$ split ergative Australian languages	
$zzaz/pA$ differential object marking English, Dutch, ...	$ezzz/pO$ inverse DOM —
$zezz/pA$ differential subject marking several caucasian languages	$zzza/pO$ inverse DSM
$zzzz/pA$ no case marking	$zzza/pO$

$zzzz/pA$

Taking stock

$zeaz/pA$ split ergative Australian languages	
$zzaz/pA$ differential object marking English, Dutch, ...	$ezzz/pO$ inverse DOM —
$zezz/pA$ differential subject marking several caucasian languages	$zzza/pO$ inverse DSM
$zzzz/pA$ no case marking Chinese, Thai	$zzza/pO$
$zzzz/pA$	

Taking stock

$zeaz/pA$ split ergative Australian languages	
$zzaz/pA$ differential object marking English, Dutch, ...	$ezzz/pO$ inverse DOM —
$zezz/pA$ differential subject marking several caucasian languages	$zzza/pO$ inverse DSM Nganasan
$zzzz/pA$ no case marking Chinese, Thai	$zzza/pO$
$zzzz/pA$	

Taking stock

- only very few languages are not evolutionary stable in this sense
  $zzaa$: *Hungarian*, $ezza$: *Parachi, Yazguljami (Iranian languages),*
  $eeaa$: *Wangkumara*
- curious asymmetry: if there are two competing stable states, one is common and the other one rare
- similar pattern as with Horn vs. anti-Horn

Alle equilibria are stable, but some equilibria are more stable than others.

Stochastic EGT

Random mutation and stability

- idealizations of standard Evolutionary Game Theory
  - populations are (practically) infinite
  - mutations rate is constant and low
- better model (Young 1993; Kandori, Mailath and Rob 1993)
  - finite population
  - mutation is noisy

Consequences of finite population model

- every mutation barrier will occasionally be taken
- no absolute stability
- if multiple Strict Nash Equilibria coexist, system will oscillate between them
- some equilibria are more stable than others
- system will spend most of the time in most robustly stable state
- stochastically stable states

A particular model

- discrete time/finite population version of replicator dynamics
- mutations occur rarely (most generations have no mutants at all)
- if mutation occurs, each individual in this generation has same probability to be a mutant
- mutation frequency and mutation rate equal for both populations
- each strategy is equally likely for a mutant (within its population)

The formulas

$$\frac{\Delta x_i}{\Delta t} = x_i(\tilde{u}_i - \tilde{u}^A) + \sum_j \frac{Z_{ji} - Z_{ij}}{n}$$

$$\frac{\Delta y_i}{\Delta t} = y_i(\tilde{u}_i - \tilde{u}^B) + \sum_j \frac{Z_{ji} - Z_{ij}}{n}$$

The formulas

$$\frac{\Delta x_i}{\Delta t} = x_i(\tilde{u}_i - \tilde{u}^A) + \sum_j \frac{Z_{ji} - Z_{ij}}{n}$$

$$\frac{\Delta y_i}{\Delta t} = y_i(\tilde{u}_i - \tilde{u}^B) + \sum_j \frac{Z_{ji} - Z_{ij}}{n}$$

- x_i: frequency of speaker strategy i

The formulas

$$\frac{\Delta x_i}{\Delta t} = x_i(\tilde{u}_i - \tilde{u}^A) + \sum_j \frac{Z_{ji} - Z_{ij}}{n}$$

$$\frac{\Delta y_i}{\Delta t} = y_i(\tilde{u}_i - \tilde{u}^B) + \sum_j \frac{Z_{ji} - Z_{ij}}{n}$$

- x_i: frequency of speaker strategy i
- y_i: frequency of hearer strategy i

The formulas

$$\frac{\Delta x_i}{\Delta t} = x_i(\tilde{u}_i - \tilde{u}^A) + \sum_j \frac{Z_{ji} - Z_{ij}}{n}$$

$$\frac{\Delta y_i}{\Delta t} = y_i(\tilde{u}_i - \tilde{u}^B) + \sum_j \frac{Z_{ji} - Z_{ij}}{n}$$

- x_i: frequency of speaker strategy i
- y_i: frequency of hearer strategy i
- $\tilde{u}_i$: expected utility of strategy i

The formulas

$$\frac{\Delta x_i}{\Delta t} = x_i(\tilde{u}_i - \tilde{u}^A) + \sum_j \frac{Z_{ji} - Z_{ij}}{n}$$

$$\frac{\Delta y_i}{\Delta t} = y_i(\tilde{u}_i - \tilde{u}^B) + \sum_j \frac{Z_{ji} - Z_{ij}}{n}$$

- x_i: frequency of speaker strategy i
- y_i: frequency of hearer strategy i
- $\tilde{u}_i$: expected utility of strategy i
- $\tilde{u}^R$: average utility of entire R-population

The formulas

$$\frac{\Delta x_i}{\Delta t} = x_i(\tilde{u}_i - \tilde{u}^A) + \sum_j \frac{Z_{ji} - Z_{ij}}{n}$$

$$\frac{\Delta y_i}{\Delta t} = y_i(\tilde{u}_i - \tilde{u}^B) + \sum_j \frac{Z_{ji} - Z_{ij}}{n}$$

- x_i: frequency of speaker strategy i
- y_i: frequency of hearer strategy i
- $\tilde{u}_i$: expected utility of strategy i
- $\tilde{u}^R$: average utility of entire R-population
- Z_{ij}: random variable; distributed according to the binomial distribution $b(p_{ij}, \lfloor x_i n \rfloor)$
- p_{ij}: probability that an i-individual mutates to strategy j

The formulas

$$\frac{\Delta x_i}{\Delta t} = x_i(\tilde{u}_i - \tilde{u}^A) + \sum_j \frac{Z_{ji} - Z_{ij}}{n}$$

$$\frac{\Delta y_i}{\Delta t} = y_i(\tilde{u}_i - \tilde{u}^B) + \sum_j \frac{Z_{ji} - Z_{ij}}{n}$$

- x_i: frequency of speaker strategy i
- y_i: frequency of hearer strategy i
- $\tilde{u}_i$: expected utility of strategy i
- $\tilde{u}^R$: average utility of entire R-population
- Z_{ij}: random variable; distributed according to the binomial distribution $b(p_{ij}, \lfloor x_i n \rfloor)$
- p_{ij}: probability that an i-individual mutates to strategy j
- n: population size

A simulation

Stochastic stability

- punctuated equilibria
- long periods of dynamic stability alternate with short transition periods
- in the long run, more time in Horn state (67% vs. 26% in anti-Horn)
- simulation suggests that Horn is stable while anti-Horn is not
- can this be proved?

Analytic considerations

- Simple recipes for finding stochastically stable state in 2×2 games
- not easily extrapolated to larger games
- basic idea:
  - calculate the height of the invasion barrier of each ESS
  - the ESSs with maximal invasion barrier is stochastically stable

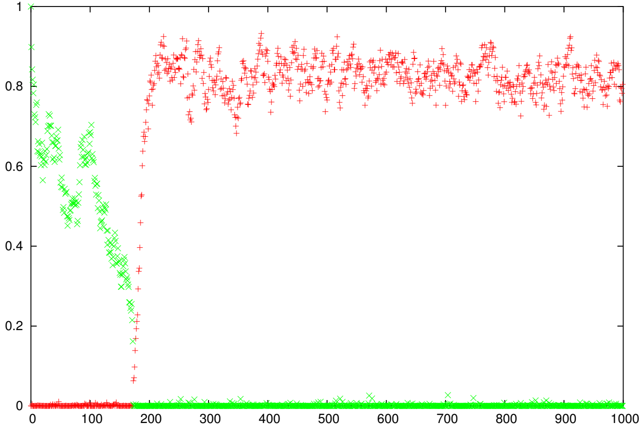
Analytic considerations

- invasion barrier = amount of mutations necessary to push the system into the basin of attraction of another ESS
- Horn $\Rightarrow$ anti-Horn: 50%
- anti-Horn $\Rightarrow$ Horn: 47.5%
- Hence:

Horn strategy is the only stochastically stable state

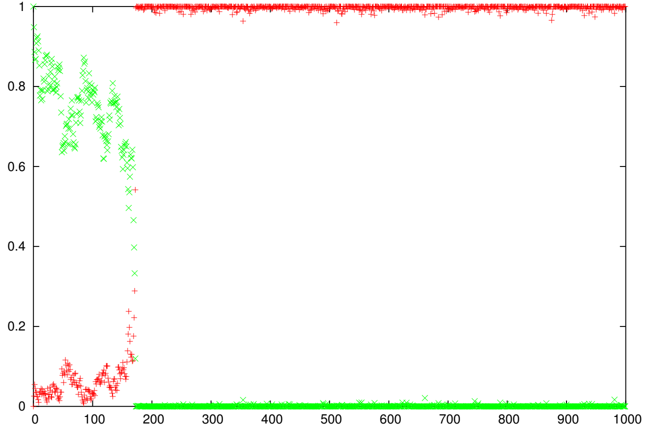
Stochastic evolution of case marking

- $k = 0.45$
- competition between $zzaz/pA$ and $ezzz/pO$
- evolution of speaker population:



Stochastic evolution of case marking

- $k = 0.45$
- competition between $zzaz/pA$ and $ezzz/pO$
- evolution of hearer population:



Analysis

- invasion barriers:
  - differential object marking: 45.2%
  - inverse differential subject marking: 2.06%

Differential object marking is stochastically stable; inverse differential subject marking is not.

- likewise, differential subject marking is stochastically stable while inverse differential object marking is not.

Stochastically stable states

$zeaz/pA$
split ergative
Australian languages

$zzaz/pA$
differential object marking
English, Dutch, ...

$zezz/pA$
differential subject marking
several caucasian languages

$zzzz/pA$
no case marking
Chinese, Thai

Conclusion

- out of $4 \times 16 = 64$ possible case marking patterns only four are stochastically stable
- vast majority of all languages that fit into this categorization are stochastically stable
- precise numbers are hard to come by though
- linguistic universals can be result of evolutionary pressure in the sense of cultural evolution