Mathematical and computational models of language evolution

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The evolution of vowel spaces

- micro-variation in the inventory of vowels between languages: every language is different
- however, very strong tendencies:
  - most languages have five vowels
  - (almost) every language has [a], [i] and [u] like vowels
  - most vowel inventories are peripheral and symmetric etc.
- proposal (see for instance de Boer 2001):

Vowel inventories must be evolutionarily stable!

What is a vowel?

Articulation

- speech sound
- voiced
- no constriction of the vowel tract
- vowel quality depends on
  - position of tongue
  - gesture of the lips
  - ...

What is a vowel?

Acoustics

- periodic sonic wave

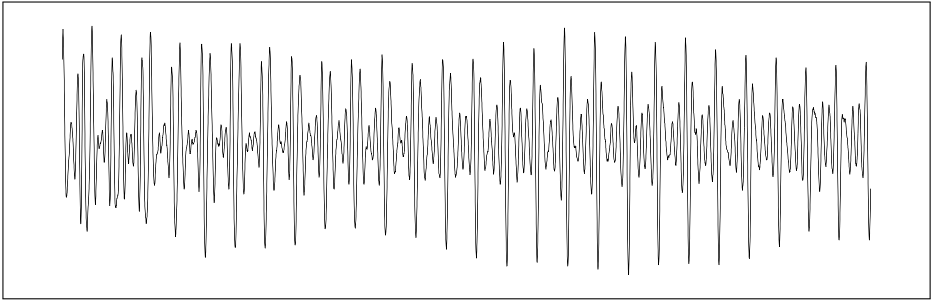


Figure: Amplitude of the vowel /u/

What is a vowel?

Acoustics

- spectral analysis:

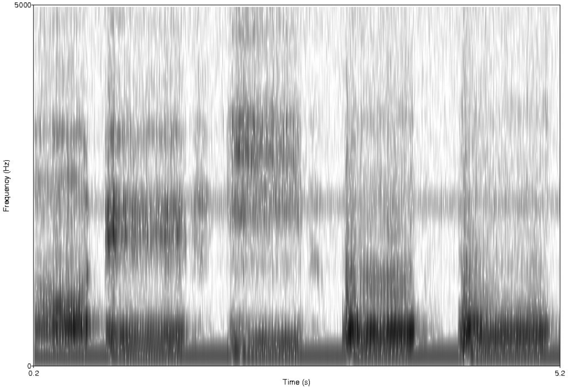


Figure: Spectrogramm of /a/-/e/-/i/-/o/-/u/

What is a vowel?

Acoustics

- vowel is superposition of discrete harmonic waves:
  - fundamental frequency
  - formants

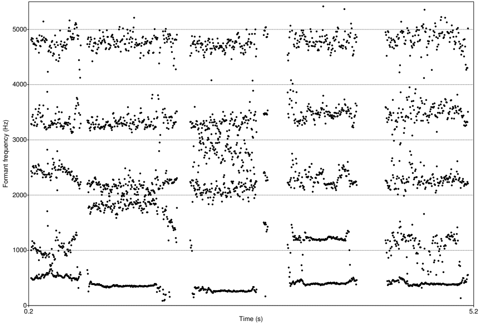
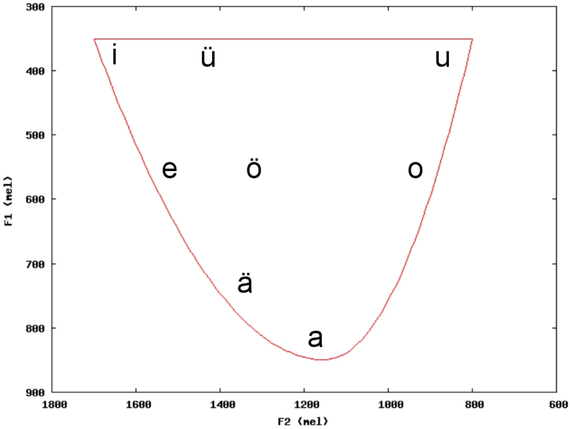


Figure: first five formants of /a-e-i-o-u/

What is a vowel?

Acoustics

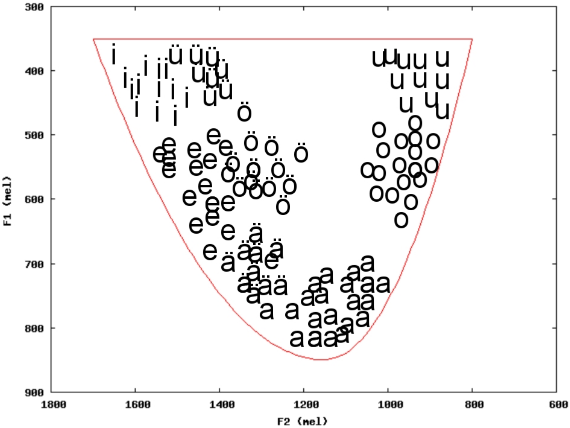
- first two formants are crucial for identification of vowels



What is a vowel?

Acoustics

- more realistic picture:



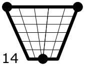
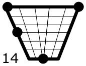
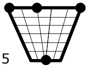
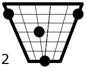
Universal tendencies of vowel inventories

- comparison of vowel inventories in hundreds of languages reveals
  - virtually all languages use the vowels [a], [i], [u]
  - almost all vowels in all languages are peripheral
  - vowel inventories tend to be symmetrical
  - ...

Liljencrants and Lindblom (1972)

- vowel systems tend to maximize perceptual distance between vowels
- can be modeled as minimizing potential energy of a vowel system
- energy is proportional to sum of inverse squared distances
- fairly good typological predictions

Survey of 500+ vowel inventories

number of vowels	vowel systems and their frequency of occurrence				
3	 14				
4	 14	 5	 4	 2	
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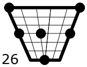
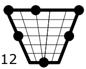
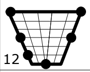
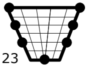
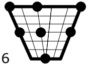
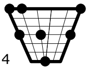
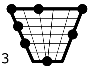
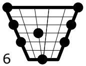
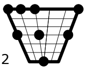
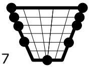
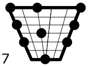
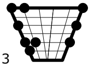
6	 26	 12	 12		
7	 23	 6	 5	 4	 3
8	 6	 3	 3	 2	
9	 7	 7	 3		

Communication via the vowel space

Game theoretic model

- Signaling game
- types: between 3 and 9 vowel categories
- signals: each point within the two-dimensional (F1/F2) vowel space

Communication via the vowel space

One round of an evolutionary signaling game

- nature picks a vowel category v_S and shows it to S
- S picks a point p_{intend} in the vowel space
- a normally distributed random variable is added to p_{intend}, yielding p_{prod}
- another normally distributed random variable is added to p_{prod}, yielding p_{perc}
- R observes p_{perc} and picks a vowel category v_R
- if $v_S = v_R$, both players score a point

Exemplar dynamics

- empiricist view on language processing/language structure
- popular in functional linguistics (esp. phonology and morphology) and in computational linguistics (aka. “memory-based”)

Basic idea

- large amounts of previously encountered instances (“exemplars”) of linguems are stored in memory
- very detailed representation of exemplars
- little abstract categorization
- similarity metric between exemplars
- new linguemes are processed in a similarity-based way

Exemplar dynamics: implementation

Sender

- chooses p_{intend} at random from multiset $\{p|\langle v_S, p\rangle \in$ memory$\}$
- if communication succeeds $(v_S = v_R)$, oldest item in memory is replaced with $\langle v_S, p_{prod}\rangle$
- otherwise memory remains unchanged

Receiver

- v_H is picked such that $\min\{d(p_{perc}, p)|\langle v_H, p\rangle \in$ memory$\}$ is minimized
- if communication succeeds $(v_S = v_R)$, oldest item in memory is replaced by $\langle v_R, p_{perc}\rangle$
- otherwise memory remains unchanged

Simulations

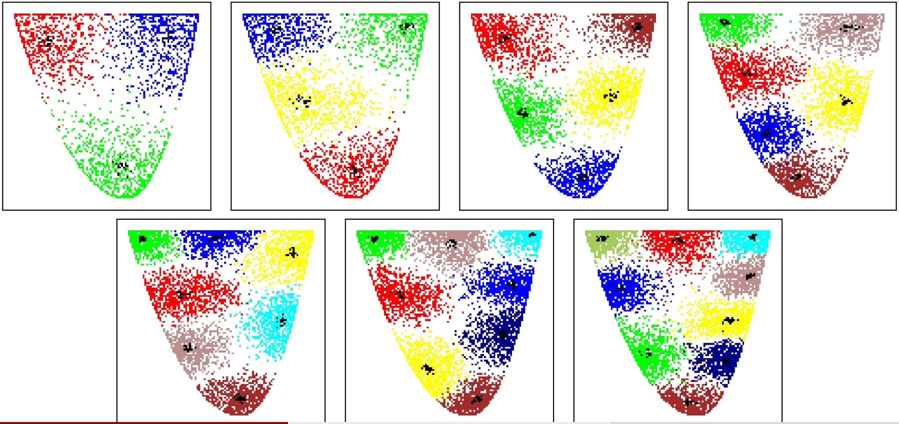
Setup

- population of 20 agents
- each agent has a memory of 4000 previous observations per vowel category (initialized with random values)
- 300k iterations of the signaling game
- sender and receiver are picked at random

Inspired by much more sophisticated simulations by Bart de Boer.

Simulation results

- black dots display average sender strategy for each agent and vowel category)
- colored dots display receiver strategies (colors represent vowel categories)



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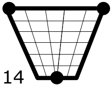
3					

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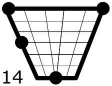
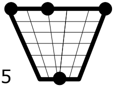
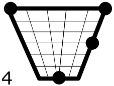
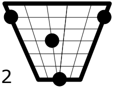
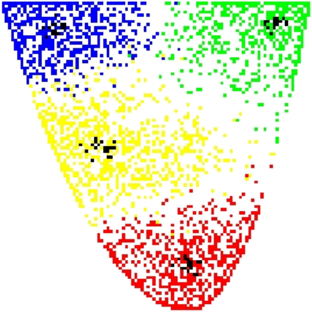
4	 14	 5	 4	 2	



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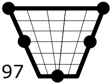
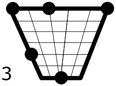
5	 97	 3			

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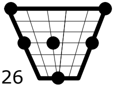
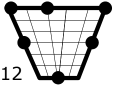
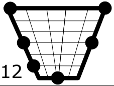
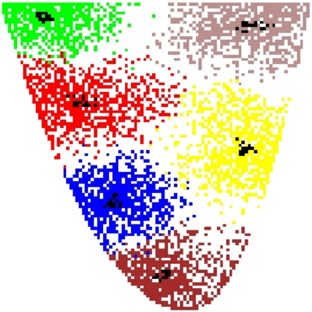
6	 26	 12	 12		



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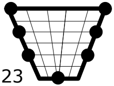
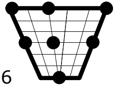
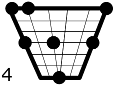
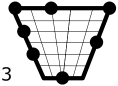
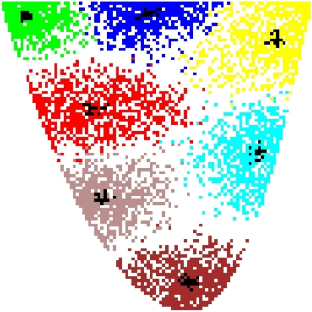
7	 23	 6	 5	 4	 3



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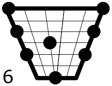
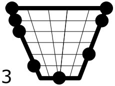
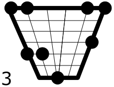
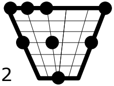
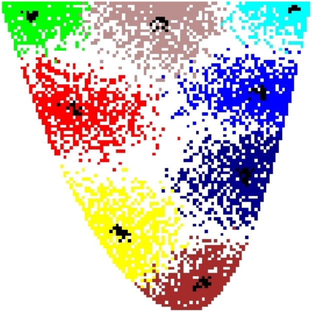
8	 6	 3	 3	 2	



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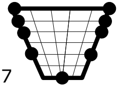
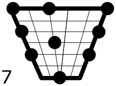
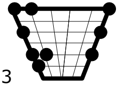
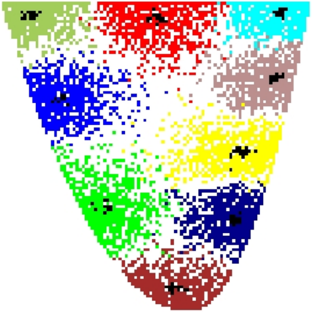
9	 7	 7	 3		



Evaluation

- more than half of the typologically dominant patterns correspond to (experimentally determined) ESSs (150 out of 264 in the database)
- five out of seven ESSs correspond to empirically attested vowel systems
- even the two outliers look natural (symmetric systems with peripheral prototypes)

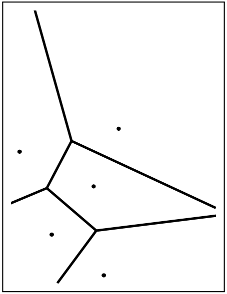
Theoretical considerations

ESS under replicator dynamics: strict Nash equilibria

- sender strategy: mapping from vowel categories to points in the vowel space
- receiver strategy: categorization of points

Voronoi tesselations

- suppose receiver strategy R is given and known to the sender: which sender strategy would be the best response to it?
  - every signal p has a "prototypical" interpretation: $R(p)$
  - for every vowel category v: S's best choice is to choose the p that minimizes the distance between p and $R(p)$
  - optimal S thus induces a **partition** of the meaning space
  - Voronoi tesselation, induced by the range of R



Open question

- numeric calculation of the ESSs for the human vowel space
- Exemplar Dynamics is similar but not identical to replicator dynamics
- conjecture: as the variance of the random variables goes to 0, the attractor states of the exemplar dynamics converges towards SNEs

de Boer, B. (2001). *The Origin of Vowel Systems*. Oxford University Press, Oxford.

Liljencrants, J. and B. Lindblom (1972). Numerical simulations of vowel quality systems: The role of perceptual contrast. *Language*, **48**:839–862.

Schwartz, J.-L., L.-J. Boe, N. Vallé, and C. Abry (1997). The dispersion-focalization theory of vowel systems. *Journal of Phonetics*, **25**:255–286.