Mathematical and computational models of language evolution

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DGfS Summer School

August 20, 2013

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Cognitive semantics

Gärdenfors (2000):

- meanings are arranged in **conceptual spaces**
- conceptual space has geometrical structure
- dimensions are founded in perception/cognition

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Criterion P

A *natural property* is a convex region of a domain in a conceptual space.

Examples

- spatial dimensions: *above, below, in front of, behind, left, right, over, under, between ...*
- temporal dimension: *early, late, now, in 2005, after, ...*
- sensual dimenstions: *loud, faint, salty, light, dark, ...*
- abstract dimensions: *cheap, expensive, important, ...*

Signaling game

- two players:
  - **S**ender
  - **R**eceiver
- infinite set of **M**eanings, arranged in a finite metrical space
  distance is measured by function $d : M^2 \mapsto R$
- finite set of **F**orms
- sequential game:
  1. nature picks out $m \in M$ according to some probability distribution p and reveals m to S
  2. S maps m to a form f and reveals f to R
  3. R maps f to a meaning m'

Signaling game

- **Goal:**
  - optimal communication
  - both want to minimize the distance between m and m'
- **Strategies:**
  - speaker: mapping S from M to F
  - hearer: mapping R from F to M
- **Average utility:** (identical for both players)

$$u(S, R) = \sum_m p_m \times \sim (m, R(S(m)))$$

vulgo: average similarity between speaker's meaning and hearer's meaning

Similarity

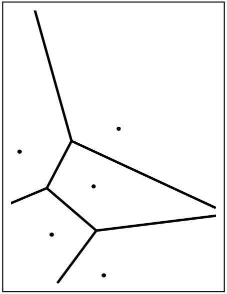
Similarity function

- similarity is inversely related to distance
- requirements:

$$\begin{aligned}
\forall x :\sim (x, x) &= 1 \\
\forall x, y :\sim (x, y) &> 0 \\
\forall x, y, z : \|x - y\| > \|x - z\| &\rightarrow \sim (x, y) <\sim (x, z) \\
\forall x, y, z, w : \|x - y\| = \|z - w\| &\rightarrow \sim (x, y) =\sim (z, w)
\end{aligned}$$

Voronoi tesselations

- suppose R is given and known to the speaker: which speaker strategy would be the best response to it?
  - every form f has a "prototypical" interpretation: $R(f)$
  - for every meaning m: S's best choice is to choose the f that minimizes the distance between m and $R(f)$
  - optimal S thus induces a (quasi-)**partition** of the meaning space
  - Voronoi tesselation, induced by the range of R



Voronoi tesselation

Okabe et al. (1992) prove the following lemma (quoted from Gärdenfors 2000):

Lemma

The Voronoi tessellation based on a Euclidean metric always results in a partioning of the space into convex regions.

Evolutionary stability

Definition

A set E of symmetric Nash equilibria is an *evolutionarily stable set* (ESSet) if, for all $x^* \in E, u(x^*, y) > u(y, y)$ whenever $u(y, x^*) = u(x^*, x^*)$ and $y \notin E$. (Cressman 2003)

Evolutionary stability

Observation

If R is a pure receiver strategy, the inverse image of any $S \in BR(R)$ is consistent with the Voronoi tessellation of the meaning space that is induced by the image of R.

Evolutionary stability

Theorem

If a symmetric strategy is an element of some ESSet, the inverse image of its sender strategy is consistent with the Voronoi tessellation that is induced by the image of its receiver strategy.

sketch of proof:

- game in question is symmetrized asymmetric game
- ESSets of symmetrized games coincide with SESets of asymmetric game (Cressman, 2003)
- SESets are sets of NE
- SESets are finite unions of Cartesian producs of faces of the state space
- hence every component of an element of an SESet is a best reply to some pure strategy

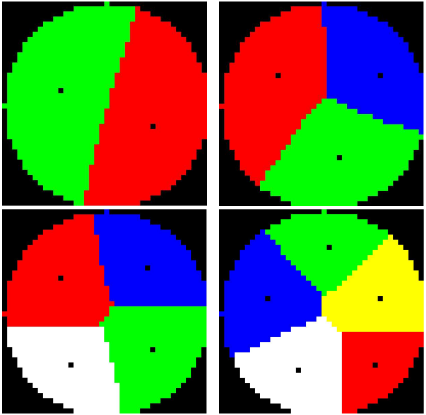
Static and dynamic stability

asymptotic stability

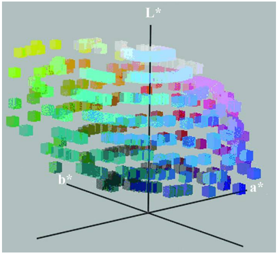
- in symmetrized games, a set E is an *asymptotically stable set of rest points* if and only if it is an ESSet
- in partnership games, at least one ESSet exists
- intuitive interpretation: under replicator dynamics + small effect of drift, system will eventually converge into some ESSet

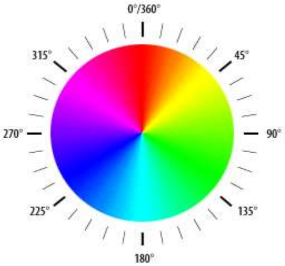
Simulations

- two-dimensional circular meaning space
- discrete approximation
- uniform distribution over meanings
- initial stratgies are randomized
- update rule according to (discrete time version of) replicator dynamics



The color space





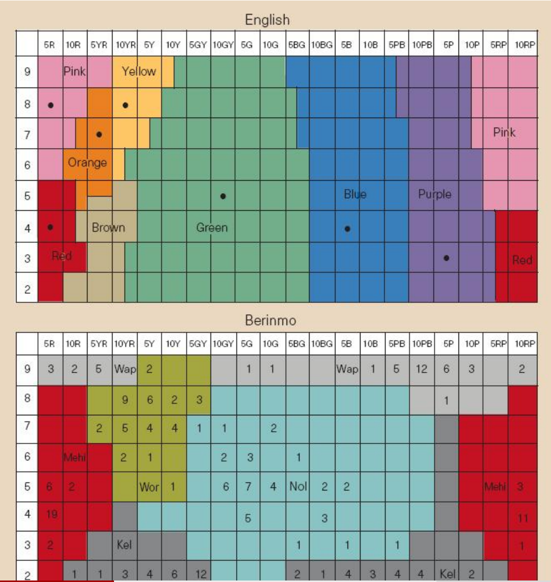
- physical color space is of infinite dimensionality
- psychological color space has only three dimensions:
  1. brightness
  2. hue
  3. saturation

Color words

- languages differ wildly in how they carve up the color space into categories
- experimental investigation of color categorization using Munsell array

- for instance: English vs. Berinmo (Papua New Guinea)

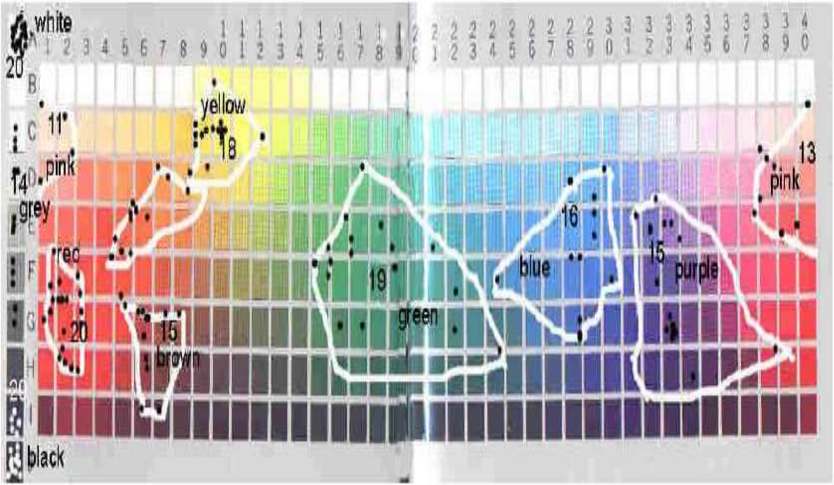
Color words


English
5R 10R 5YR 10YR 5Y 10Y 5GY 10GY 5G 10G 5BG 10BG 5B 10B 5PB 10PB 5P 10P 5RP 10RP
9 8 7 6 5 4 3 2
Pink
Yellow
Orange
Brown
Green
Blue
Purple
Pink
Red
Red
Berinmo
5R 10R 5YR 10YR 5Y 10Y 5GY 10GY 5G 10G 5BG 10BG 5B 10B 5PB 10PB 5P 10P 5RP 10RP
9 3 2 5 Wap 2 1 1 Wap 1 5 12 6 3 2
8 9 6 2 3 1
7 2 5 4 4 1 1 2
6 Mehi 2 1 2 3 1
5 6 2 Wor 1 6 7 4 Nol 2 2 Mehi 3
4 19 5 3 11
3 2 Kel 1 1 1 1
2 1 1 3 4 6 12 2 1 4 3 4 4 Kel 2

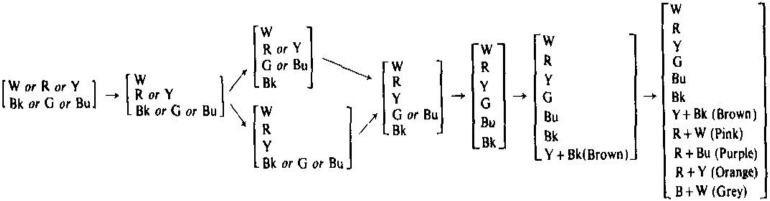
Color words

- Berlin and Kay (1969): study of the typology of color words
- subjects with typologically distant native languages
- subjects were asked about prototype and extension of the basic color words of their native language
- English: 11 basic colors

Berlin and Kay's study

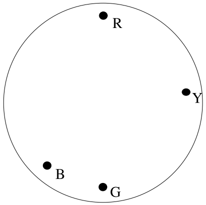


Implicational hierarchies



A toy example

- suppose
  - circular two-dimensional meaning space
  - four meanings are highly frequent
  - all other meanings are negligibly rare
- let's call the frequent meanings
  Red, Green, Blue and Yellow



$$p_i(\text{Red}) > p_i(\text{Green}) > p_i(\text{Blue}) > p_i(\text{Yellow})$$

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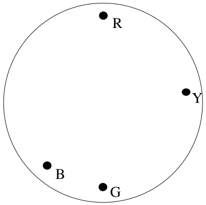
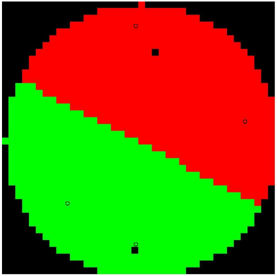


$$p_i(\text{Red}) > p_i(\text{Green}) > p_i(\text{Blue}) > p_i(\text{Yellow})$$

Yes, I made this up without empirical justification.

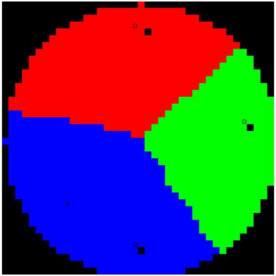
Two forms

- suppose there are just two forms
- only one Strict Nash equilibrium (up to permuation of the forms)
- induces the partition **{Red, Blue}/{Yellow, Green}**



Three forms

- if there are three forms
- two Strict Nash equilibria (up to permuation of the forms)
- partitions {Red}/{Yellow}/{Green, Blue} and {Green}/{Blue}/{Red, Yellow}
- only the former is **stochastically stable** (resistent against random noise)



Four forms

- if there are four forms
- one Strict Nash equilibrium (up to permuation of the forms)
- partitions {Red}/{Yellow}/{Green}/{Blue}