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COMMUNICATION, COORDINATION AND NASH EQUILIBRIUM

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I note a methodological problem in studying the role of pre-play communication in ensuring equilibrium. To deal with this problem, I define a solution concept for the extended game (in which talk is followed by play) that is intermediate between Nash equilibrium and rationalizability. In some games our solution concept implies a Nash outcome, while in others it does not.

1. Introduction

Recent work has done more to refine the concept of Nash equilibrium than to justify our working assumption that *some* equilibrium will occur. Yet equilibrium often demands considerable coordination: a problem seen most clearly in games of pure coordination, but present in most games.

One important reason why we might expect Nash outcomes despite this coordination problem involves informal pre-play communication. Suppose, following Aumann (1974), that players can talk before choosing their actions, but cannot bind themselves. Aumann suggests that they will reach some agreement on how to play, and that since no external enforcement is available, they can only consider self-enforcing, or Nash, outcomes. This justifies focusing on Nash outcomes, *if* the players reach an agreement; but they might not.

Cheap talk is notoriously hard to model: there are no obviously 'right' rules about who speaks when, what he may say, and when discussion ends. Rather than struggle with those problems, I will assume them away, in order to address a fundamental problem: *what solution concept do we use for the extended game* of communication followed by play?

If we solve the extended game using Nash equilibrium, we can get exactly any Nash equilibrium in the original game. This might seem just the conclusion we wanted; but we assumed (for the extended game) what we set out to prove (for the original game). This is unsatisfactory. If instead we use the weaker concept of rationalizability in the extended game, then *any* rationalizable outcome of the original game is rationalizable in the extended game. This is implausible, so this approach too is unsatisfactory.

To escape this dilemma, I propose an intermediate solution concept for the extended game, recognizing that the players share a common language and that they will believe a speaker if there is no reason for him to deceive them. I show that in some games all 'sensible' outcomes are Nash; but that in others that is not so. [1] Briefly, this is because although Nash suggestions would be followed, they may never be made.

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[1] Farrell (1987) and Farrell and Saloner (1988) have discussed the coordinating effects of pre-play communication, but they *assumed* a (mixed-strategy) Nash equilibrium in the extended game. See also Crawford and Haller (1987).

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2. Analysis

Consider a simultaneous-move game G in which player i chooses a (mixed) strategy s_i. Construct a two-stage *extended game* G^*: In the first stage, players talk about play in G. This is *cheap talk*: what they say does not directly affect payoffs. In the second stage, they play G. Payoffs in G^* depend only on what happens in G, not directly on first-stage talk. However, the talk may affect second-stage actions.

We apply various solution concepts to G^*, and ask for what solution concepts, and for what games G, every solution of G^* involves second-stage choices that constitute a Nash equilibrium of G. We begin with two observations whose proofs are simple [2].

First, every Nash equilibrium of G^* has as its second stage a Nash equilibrium of G, and every Nash equilibrium of G is the second stage of a subgame-perfect Nash equilibrium of G^*. Consequently, we would like to see whether a weaker solution concept in G^* implies Nash outcomes in G. But our second observation is that every (cautiously) rationalizable outcome of G is the second stage of a (cautiously) rationalizable outcome of G^*.

The problem is that we have no link between words and actions. As a result, since only actions count, talk does nothing. This is unrealistic: talk does matter. For instance, people do better in coordination games when they can talk first. We want a solution concept for G^* reflecting the fact that, while no player is bound to tell the truth or to follow suggestions, nonetheless if there is no incentive to cheat then honesty is focal. Consider a simple example.

Example 1.

	A_2	B_2
A_1	(2, 2)	(0, 0)
B_1	(0, 0)	(1, 1).

Intuitively, player 1 should say 'I will do A_1; you do A_2', and then the players should choose A_1 and A_2 respectively. Yet, all probability distributions of outcomes of G^* (and of G) are rationalizable, and all outcomes occur with positive probability in some perfect Nash equilibrium. For player 1 cannot be sure that 2 will believe and understand what he says, even though their interests completely coincide.

To overcome this problem, we now introduce an assumption that links talk (in the first stage of G^*) to actions (in the second stage). Our assumption restricts beliefs [3] after certain messages in the first stage, in a similar spirit to refinements of the Nash equilibrium concept by Myerson (1983) and Farrell (1985). Here, we use it to refine rationalizability.

To model G^* explicitly, we drastically simplify the first (talk) period by assuming that only player 1 may speak in the first stage. With this assumption, we simplify the problem [as in Myerson (1983)] and also rule out the problems of conflict over *which* efficient equilibrium to choose, as in the Battle of the Sexes. [4]

Player 1 can make a *suggestion* about what players should do in the second stage. A suggestion is a list specifying, for each player i, a non-empty subset T_i of i's (mixed) strategy space S_i. We can interpret this as a 'speech' proposing (precisely or vaguely) how everyone should behave in G. If this

[2] Proofs available on request from the author.

[3] Ben-Porath and Dekel (1987) discuss the coordinating and equilibrium-selection effects of *costly* talk, using dominance arguments.

[4] See Farrell and Saloner (1988).

speech is credible, we want to assume that everyone believes it: this is the link between words and actions.

Informally, we call a suggestion *consistent* if every move suggested to each player is a best response to some beliefs (about others' moves) that assume that others follow the suggestion. Formally, a suggestion T is consistent if, for each i, each s_i in T_i, and each j not equal to i, there is a probability distribution $B(i, j)$ on T_j, such that s_i is optimal for i given beliefs $B(i, .)$ about others' moves. In particular, every suggested strategy must be rationalizable. 'Suggesting' that every player just play 'some rationalizable move' is the vaguest possible consistent suggestion.

If each T_i is a singleton, then T is consistent if and only if it is Nash. Thus the idea of a consistent suggestion reduces to Nash equilibrium if all the T_i are singletons, but it also allows for vaguer suggestions. We cannot assume that player 1 would choose to make a precise suggestion, and indeed we show below that he will not always do so.

Player 1 will choose the suggestion T that he believes will be best for him. We want to suppose that he expects others to follow consistent suggestions, but that beyond that his expectations are cautious. This is intended to capture the idea that coordination, and confidence about what others will do, is generated by a combination of common knowledge of rationality and by credible communication; not by exogenous beliefs.

To formalize this, define a *prediction* as a function p taking suggestions T into probability distributions $p(T)$ on rationalizable outcomes of G (satisfying the condition that the distributions of i's and of j's choice must be independent). A prediction p is *respectable* if, (i) for each consistent suggestion T, the support of $p(T)$ is precisely equal to T, and (ii) for each consistent suggestion T, the support of $p(T)$ is the set of rationalizable outcomes of G. A distribution F of outcomes of G is *sensible* if there exists a respectable prediction p and a suggestion T, such that $p(T) = F$ and such that player 1 weakly prefers F to $p(T')$ for all other suggestions T'.

We now prove two results about sensible outcomes (or distributions of outcomes) in a game G. The first is a 'positive' result, confirming a natural intuition by showing that, in a class of games, the only sensible outcomes are certain Nash outcomes. The second is a 'negative' result, showing that sensible outcomes need to be Nash in general.

Proposition 1. If there is a Nash equilibrium $\mathbf{e}$ *in which player 1 gets his greatest rationalizable payoff* u_1^* *in G, then he must get* u_1^* *in every sensible outcome of G.*

Proof. First, $\{\mathbf{e}\}$ is a consistent suggestion. Therefore any respectable prediction p has $p(\{\mathbf{e}\})$ concentrated on $\mathbf{e}$. By assumption, then, player 1 weakly prefers $p(\{\mathbf{e}\})$ to any $p(T')$, with indifference only if $p(T')$ also has support entirely on outcomes that yield him u_1^*. Hence, he will make a suggestion that guarantees him u_1^*. QED.

Corollary 1A. If u_1^* *is attainable only in Nash equilibrium, then every sensible outcome is Nash.*

Corollary 1B. In a game of pure coordination (such as Example 1), in which players' interests completely coincide, the only sensible distributions of outcomes are the Pareto efficient distributions, i.e., the 'good' pure-strategy equilibria.

Notice that Corollary 1B plausibly does not depend on the assumed simple structure of the first phase of G^*: in such a game, any player benefits by making the appropriate suggestion.

We now prove our negative result: that in games in which the strong assumption of Proposition 1 does not hold, talk does not ensure equilibrium.

Proposition 2. If s is the rationalizable outcome of G that gives player 1 his best payoff (among rationalizable outcomes), then s is sensible.

Proof. There exists a respectable prediction p that assigns to **s** almost all the weight after any suggestion that includes **s**. (That there are such suggestions follows from the rationalizability of **s**.) With such a prediction, player 1 will strictly prefer such a suggestion to any that does not include **s**, and so **s** is sensible. Q.E.D.

Corollary. When the condition of Proposition 1 fails (as it does for most games), then not all sensible outcomes are Nash.

We next discuss an example that illustrates and extends Proposition 2.

Example 2. In this symmetric two-player game there is a unique Nash equilibrium $\mathbf{e} = (A_1, A_2)$, while all outcomes are rationalizable. But *whatever* player 1 thinks player 2 will do if **e** is not suggested, he can only expect to lose by suggesting **e**. Thus, he will not suggest **e**, and so communication will not lead the players to equilibrium.

	A_2	B_2	C_2
A_1	$(-2, -2)$	$(1, -3)$	$(1, -3)$
B_1	$(-3, 1)$	$(2, -2)$	$(-2, 2)$
C_1	$(3, 1)$	$(-2, 2)$	$(2, -2)$.

Notice that the result of Example 2 does *not* depend on our simplifying assumption about the talk phase of G^*. Whatever the rules of that phase, neither player wants to suggest **e**, so they will not. They may talk, but only in the hope of fooling one another into playing the wrong move; not in order to coordinate.

3. Conclusion

We have analyzed a simple formulation of the incentives for players to engage in a form of pre-play cheap talk that can lead them to Nash equilibrium. After proposing a solution concept ('sensible outcomes') for the extended game G^*, we showed that in some games only Nash outcomes (indeed, only certain ones) are sensible, while in other (indeed, most) games there are sensible outcomes that are not Nash. Intuitively, in some games (including coordination games) one wants one's opponent to play his best response to one's own move. In others (such as two-person zero-sum games), one wants one's opponent to be as ignorant as possible; hence credible communication will not occur, and Nash outcomes cannot be expected in general without some other reason to expect coordinated beliefs.

We used a definition of 'consistent' suggestions intended to capture as closely as possible the idea of Nash equilibrium as a self-enforcing agreement, and we focused on the question whether such an agreement would be made. It might be argued that a more compelling definition, which we might call a *strictly consistent suggestion*, would require not only that every suggested move be rationalizable when others are expected to follow the suggestion, but also that no other move be so rationalizable. In the case of singleton (Nash) suggestions, strict consistency requires *strict* best responses, and thus rules out mixed-strategy equilibria and equilibria involving weakly dominated strategies: both of which some theorists find imperfectly convincing. Of course, strict consistency also rules out some vaguer consistent suggestions.

A possibly serious objection to our assumption that consistent suggestions are followed is due to Aumann (personal communication) and is based on the following example.

Example 3 (Aumann). There are two players, and payoffs are as follows:

	A_2	B_2
A_1	(7, 7)	(8, 1)
B_1	(1, 8)	(9, 9).

There are two pure-strategy Nash equilibria: (A_1, A_2) and (B_1, B_2). Each involves strict best-responses, so that the suggestion (B_1, B_2) is strictly consistent. But, Aumann has argued, that suggestion is not convincing, since *even if player 1 intended to play A_1, he would still want player 2 to play B_2.*

On the other hand (we would argue), if player 1 were to suggest the B-equilibrium, he would have to recognize that player 2 would then be at least quite likely to play B_2, and so perhaps he should follow his own (consistent) suggestion. The difference between our intuition and Aumann's is a matter of whether one thinks of player 1 deciding on his move at stage 2 'after' he chooses his stage-1 message, or deciding on his move first and then on his message. If the latter, then Aumanns' criticism is compelling; if the former, then matters are rather unclear.

Finally, we raise the question of inferences from the absence of suggestions, or from vague suggestions. While there may be some reason to make such inferences, and certainly one could tell an equilibrium story in which inferences would be appropriate, we find it implausible that coordination can be achieved by the absence of cheap talk. With costly messages, however, this may be quite possible, as Ben-Porath and Dekel (1987) have shown.

Pre-play communication may be important in achieving Nash equilibrium, but it need not guarantee an equilibrium outcome. Such talk may coordinate on some equilibrium, may do nothing, may introduce new equilibra [Farrell and Gibbons (1986), Matthews and Postlewaite (1987)], or may shrink the set of equilibria in complex ways [Myerson (1983), Farrell and Maskin (1987), and Bernheim and Ray (1987)]. The role of talk in games is still little understood.

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