# Evolutionary game theory and language evolution 

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## Conceptualization of language evolution

prerequisites for evolutionary dynamics

- replication
- variation
- selection


## Linguemes

- "any piece of structure that can be independently learned and therefore transmitted from one speaker to another" (Nettle 1999:5)
- Croft (2000) attributes the name lingueme to Haspelmath (Nettle calls them items)
- Examples:
- phonemes
- morphemes
- words
- constructions
- idioms
- collocations
- ...


## Linguemes

- Linguemes are replicators
- comparable to genes
- structured configuration of replicators
- Biology: genotype
- Linguistics: utterance


## Croft:

The utterance is the genome!

## Evolution

## Replication

(at least) two modes of lingueme replication:

- acquisition
- priming (Jäger and Rosenbach 2005; Croft and Nettle would perhaps not agree)


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- linguistic creativity
- reanalysis
- language contact
- ...


## Evolution

## Replication

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## Variation

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- reanalysis
- language contact
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## Selection

- social selection
- selection for learnability
- selection for primability


## Fitness

learnability/primability

- selection against complexity
- selection against ambiguity
- selection for frequency


## EGT and pragmatics

Horn strategies: prototypical meanings tend to go with simple expressions and less prototypical meanings with complex expressions.
(1) a. John went to church/jail. (prototypical interpretation)
b. John went to the church/jail. (literal interpretation)
(2) a. I am going to marry you. (no indirect speech act)
b. I will marry you. (indirect speech act)
(3) a. I need a new driller/cooker.
b. I need a new drill/cook.

## Horn strategies

- simple game:
- players: speaker and hearer
- two forms: $f_{0}$ (short) and $f_{1}$ (long)
- two meanings: $m_{0}$ (frequent) and $m_{1}$ (rare)
- speaker strategies: mappings from meanings to forms
- hearer strategies: mappings from forms to meanings


## Speaker strategies

- $S_{1}: m_{0} \mapsto f_{0}, m_{1} \mapsto f_{1}: \xrightarrow{\bullet \bullet}$ "Horn strategy"
- $S_{2}: m_{0} \mapsto f_{1}, m_{1} \mapsto f_{0}$ : $\therefore$
"anti-Horn strategy"
- $S_{3}: m_{0} \mapsto f_{0}, m_{1} \mapsto f_{0}:$

"Smolensky strategy"
"anti-Smolensky strategy"


## Hearer strategies

- $H_{1}: f_{0} \mapsto m_{0}, f_{1} \mapsto m_{1}: \bullet \bullet$ 'Horn strategy'
- $H_{2}: f_{0} \mapsto m_{1}, f_{1} \mapsto m_{0}:$

- $H_{3}: f_{0} \mapsto m_{0}, f_{1} \mapsto m_{0}:$ $\rightarrow-$
"Smolensky strategy"
"anti-Smolensky strategy"


## Utility of Horn games

- whether communication works depends both on speaker strategy $S$ and hearer strategy $H$
- two factors for functionality of communication
- communicative success ("hearer economy")

$$
\delta_{m}(S, H)= \begin{cases}1 & \text { iff } \quad H(S(m))=m \\ 0 & \text { else }\end{cases}
$$

- least effort ("speaker economy")

$$
\operatorname{cost}(f) \ldots \text { measure of complexity of expression }
$$

## Utility of Horn games

$$
u_{s / h}(S, H)=\sum_{m} p_{m} \times\left(\delta_{m}(S, H)-\operatorname{cost}(S(m))\right)
$$

$p \ldots$ probability distribution over meanings

## Utility of Horn game

Let's make up some numbers:

- $p\left(m_{0}\right)=.75$
- $p\left(m_{1}\right)=.25$
- $\operatorname{cost}\left(f_{0}\right)=.1$
- $\operatorname{cost}\left(f_{1}\right)=.2$


## Utility of Horn game

|  | $H_{1}$ | $H_{2}$ | $H_{3}$ | $H_{4}$ |
| ---: | ---: | ---: | ---: | ---: |
| $S_{1}$ | .875 | -.125 | .625 | .125 |
| $S_{2}$ | -.175 | .825 | .575 | .25 |
| $S_{3}$ | .65 | .15 | .65 | .15 |
| $S_{4}$ | .05 | .55 | .55 | .05 |



## Utility of Horn game

|  | $H_{1}$ | $H_{2}$ | $H_{3}$ | $H_{4}$ |
| ---: | ---: | ---: | ---: | ---: |
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| $S_{4}$ | .05 | .55 | .55 | .05 |



- both Horn and anti-Horn are evolutionarily stable
- EGT explains the aversion of natural languages against synonymy and ambiguity
- preference for Horn not directly explainable in standard EGT
- rationalistic considerations favor Horn over anti-Horn:
- Horn strategy is Pareto efficient (nobody can do better in absolute terms)
- Horn strategy risk dominates anti-Horn (if you know the population is in an equilibrium but you do not know in which one, going for Horn is less risky than anti-Horn)
- replicator dynamics favors Horn over anti-Horn:
- complete random state evolves to Horn/Horn
- basin of attraction of Horn is about 20 times as large as basin of attraction of anti-Horn (numerical approximation-does anybody know how to do this analytically?)


## Dynamics starting from random state



## The evolution of differential case marking

## Ways of argument identification

- transitivity may lead to ambiguity
die Frau, die Maria kennt
the woman that Maria knows
the woman that knows Maria
- three ways out
(1) word order
(2) agreement
(3) case
die Frau, die er kennt

the woman that he knows
die Frau, die inn kennt

the woman that knows him
- Suppose one argument is a pronoun and one is a noun (or a phrase)


## \{I, BOOK, KNOW

- both conversants have an interest in successful communication
- case marking (accusative or ergative) is usually more costly than zero-marking (nominative)
- speaker wants to avoid costs

| speaker strategies | hearer strategies |
| :--- | :--- |
| always case mark the object <br> (accusative) | ergative is agent <br> and accusative object |
| always case mark the agent <br> (ergative) | pronoun is agent |
| case mark the object |  |
| if it is a pronoun | pronoun is object |
| $\vdots$ |  |$\quad$| pronoun is agent |
| :--- |
| unless it is accusative |

## Statistical patterns of language use

four possible clause types

|  | $O / p$ | $O / n$ |
| :---: | :---: | :---: |
| $A / p$ | he knows it | he knows the book |
| $A / n$ | the man knows it | the man knows the book |

statistical distribution (from a corpus of spoken English)

|  | $O / p$ | $O / n$ |
| :---: | :---: | :---: |
| $A / p$ | $\mathrm{pp}=198$ | $\mathrm{pn}=716$ |
| $A / n$ | $\mathrm{np}=16$ | $\mathrm{nn}=75$ |

$$
\mathrm{pn} \gg \mathrm{np}
$$

- functionality of speaker strategies and hearer strategies depends on various factors:
- How often will the hearer get the message right?
- How many case markers does the speaker need per clause on average?


## speaker strategies that will be considered

| agent is pronoun | agent is noun | obiect is pronoun | object is noun |
| :---: | :---: | :---: | :---: |
| e(rgative) | e(rgative) | a(ccusative) | a(ccusative) |
| e | e | a | z(ero) |
| e | e | z | a |
| e | e | z | z |
| e | z | a | a |
| ... | ... | ... | ... |
| Z | e | z | z |
| z | z | a | a |
| z | z | a | z |
| z | z | z | a |
| z | z | z | z |

- hearer strategies:
- strict rule: ergative means "agent", and accusative means "object"
- elsewhere rules:
(1) $S O$ : "The first phrase is always the agent."
(2) $p A$ : "Pronouns are agents, and nouns are objects."
(3) $p O$ : "Pronouns are objects, and nouns are agents."
(9) $O S$ : "The first phrase is always the object."
- strategy space and utility function are known
- probability of meaning types can be estimated from corpus study
- hard to estimate how the complexity of a case morpheme compares to its benefit for disambiguation from the speaker perspective
- parameterized utility function

$$
u(S, H)=\sum_{m} p_{m} \times\left(\delta_{m}(S, H)-k \times \operatorname{cost}(S(m))\right)
$$

## Utility of case marking

- let us assume $k=.1$

| Speaker <br> strategies | Hearer strategies |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
|  | $S O$ | $p A$ | $p O$ | $O S$ |
| $e e z z$ | 0.90 | 0.90 | 0.90 | 0.90 |
| $z z a a$ | 0.90 | 0.90 | 0.90 | 0.90 |
| $e z a z$ | 0.85 | 0.85 | 0.85 | 0.85 |
| $z e z a$ | 0.81 | 0.81 | 0.81 | 0.81 |
| $z e a z$ | 0.61 | 0.97 | 0.26 | 0.61 |
| $e z z z$ | 0.86 | 0.86 | 0.87 | 0.86 |
| $z e z z$ | 0.54 | 0.89 | 0.54 | 0.54 |
| $z z a z$ | 0.59 | 0.94 | 0.59 | 0.59 |
| $z z z a$ | 0.81 | 0.81 | 0.82 | 0.81 |
| $z z z z$ | 0.50 | 0.85 | 0.15 | 0.50 |

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| $z z z z$ | 0.50 | 0.85 | 0.15 | 0.50 |

## Utility of case marking

- only one evolutionarily stable state: zeaz/pA (split ergative)
- very common among Australian aborigines languages


## Non-strict Nash equilibria

Why are non-strict Nash Equilibria unstable?

- Dynamics without mutation



## Non-strict Nash equilibria

Why are non-strict Nash Equilibria unstable?

- Dynamics with mutation



## Utility of case marking

If speakers get lazier...

- $k=0.45$

| Speaker <br> strategies | Hearer strategies |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
|  | $S O$ | $p A$ | $p O$ | $O S$ |
| $e e z z$ | 0.550 | 0.550 | 0.550 | 0.550 |
| $z z a a$ | 0.550 | 0.550 | 0.550 | 0.550 |
| $e z a z$ | 0.458 | 0.458 | 0.458 | 0.458 |
| $z e z a$ | 0.507 | 0.507 | 0.507 | 0.507 |
| $z e a z$ | 0.507 | 0.863 | 0.151 | 0.507 |
| $e z z z$ | 0.545 | 0.538 | 0.553 | 0.545 |
| $z e z z$ | 0.505 | 0.861 | 0.148 | 0.505 |
| $z z a z$ | 0.510 | 0.867 | 0.154 | 0.510 |
| $z z z a$ | 0.539 | 0.531 | 0.547 | 0.539 |
| $z z z z$ | 0.500 | 0.849 | 0.152 | 0.500 |

## Utility of case marking

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| $e e z z$ | 0.550 | 0.550 | 0.550 | 0.550 |
| $z z a a$ | 0.550 | 0.550 | 0.550 | 0.550 |
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| $z z z a$ | 0.539 | 0.531 | 0.547 | 0.539 |
| $z z z z$ | 0.500 | 0.849 | 0.152 | 0.500 |

## Utility of case marking

... and lazier ...

- $k=0.53$

| Speaker <br> strategies | Hearer strategies |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
|  | $S O$ | $p A$ | $p O$ | $O S$ |
| $e e z z$ | 0.470 | 0.470 | 0.470 | 0.470 |
| $z z a a$ | 0.470 | 0.470 | 0.470 | 0.470 |
| $e z a z$ | 0.368 | 0.368 | 0.368 | 0.368 |
| $z e z a$ | 0.436 | 0.436 | 0.436 | 0.436 |
| $z e a z$ | 0.483 | 0.839 | 0.127 | 0.483 |
| $e z z z$ | 0.473 | 0.465 | 0.480 | 0.473 |
| $z e z z$ | 0.497 | 0.854 | 0.141 | 0.497 |
| $z z a z$ | 0.494 | 0.850 | 0.137 | 0.494 |
| $z z z a$ | 0.476 | 0.468 | 0.484 | 0.476 |
| $z z z z$ | 0.500 | 0.848 | 0.152 | 0.500 |

## Utility of case marking

... and lazier ...

- $k=0.53$

| Speaker <br> strategies | Hearer strategies |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
|  | $S O$ | $p A$ | $p O$ | $O S$ |
| $e e z z$ | 0.470 | 0.470 | 0.470 | 0.470 |
| $z z a a$ | 0.470 | 0.470 | 0.470 | 0.470 |
| $e z a z$ | 0.368 | 0.368 | 0.368 | 0.368 |
| $z e z a$ | 0.436 | 0.436 | 0.436 | 0.436 |
| $z e a z$ | 0.483 | 0.839 | 0.127 | 0.483 |
| $e z z z$ | 0.473 | 0.465 | 0.480 | 0.473 |
| $z e z z$ | 0.497 | 0.854 | 0.141 | 0.497 |
| $z z a z$ | 0.494 | 0.850 | 0.137 | 0.494 |
| $z z z a$ | 0.476 | 0.468 | 0.484 | 0.476 |
| $z z z z$ | 0.500 | 0.848 | 0.152 | 0.500 |

## Utility of case marking

... and lazier...

- $k=0.7$

| Speaker <br> strategies | Hearer strategies |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
|  | $S O$ | $p A$ | $p O$ | $O S$ |
| $e e z z$ | 0.300 | 0.300 | 0.300 | 0.300 |
| $z z a a$ | 0.300 | 0.300 | 0.300 | 0.300 |
| $e z a z$ | 0.177 | 0.177 | 0.177 | 0.177 |
| $z e z a$ | 0.287 | 0.287 | 0.287 | 0.287 |
| $z e a z$ | 0.431 | 0.788 | 0.075 | 0.431 |
| $e z z z$ | 0.318 | 0.310 | 0.326 | 0.318 |
| $z e z z$ | 0.482 | 0.838 | 0.126 | 0.482 |
| $z z a z$ | 0.457 | 0.814 | 0.101 | 0.457 |
| $z z z a$ | 0.343 | 0.335 | 0.350 | 0.343 |
| $z z z z$ | 0.500 | 0.848 | 0.152 | 0.500 |

## Utility of case marking

... and lazier...

- $k=0.7$

| Speaker <br> strategies | Hearer strategies |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
|  | $S O$ | $p A$ | $p O$ | $O S$ |
| $e e z z$ | 0.300 | 0.300 | 0.300 | 0.300 |
| $z z a a$ | 0.300 | 0.300 | 0.300 | 0.300 |
| $e z a z$ | 0.177 | 0.177 | 0.177 | 0.177 |
| $z e z a$ | 0.287 | 0.287 | 0.287 | 0.287 |
| $z e a z$ | 0.431 | 0.788 | 0.075 | 0.431 |
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| $z e z z$ | 0.482 | 0.838 | 0.126 | 0.482 |
| $z z a z$ | 0.457 | 0.814 | 0.101 | 0.457 |
| $z z z a$ | 0.343 | 0.335 | 0.350 | 0.343 |
| $z z z z$ | 0.500 | 0.848 | 0.152 | 0.500 |

## Utility of case marking

- $k=1$

| Speaker <br> strategies | Hearer strategies |  |  |  |
| :--- | ---: | ---: | ---: | ---: |
|  | $S O$ | $p A$ | $p O$ | $O S$ |
| $e e z z$ | 0.000 | 0.000 | 0.000 | 0.000 |
| $z z a a$ | 0.000 | 0.000 | 0.000 | 0.000 |
| $e z a z$ | -0.160 | -0.160 | -0.160 | -0.160 |
| $z e z a$ | 0.024 | 0.024 | 0.024 | 0.024 |
| $z e a z$ | 0.340 | 0.697 | -0.016 | 0.340 |
| $e z z z$ | 0.045 | 0.037 | 0.053 | 0.045 |
| $z e z z$ | 0.455 | 0.811 | 0.099 | 0.455 |
| $z z a z$ | 0.394 | 0.750 | 0.037 | 0.394 |
| $z z z a$ | 0.106 | 0.098 | 0.144 | 0.106 |
| $z z z z$ | 0.500 | 0.848 | 0.152 | 0.500 |

## Utility of case marking

- $k=1$

| Speaker <br> strategies | Hearer strategies |  |  |  |
| :--- | ---: | ---: | ---: | ---: |
|  | $S O$ | $p A$ | $p O$ | $O S$ |
| $e e z z$ | 0.000 | 0.000 | 0.000 | 0.000 |
| $z z a a$ | 0.000 | 0.000 | 0.000 | 0.000 |
| $e z a z$ | -0.160 | -0.160 | -0.160 | -0.160 |
| $z e z a$ | 0.024 | 0.024 | 0.024 | 0.024 |
| $z e a z$ | 0.340 | 0.697 | -0.016 | 0.340 |
| $e z z z$ | 0.045 | 0.037 | 0.053 | 0.045 |
| $z e z z$ | 0.455 | 0.811 | 0.099 | 0.455 |
| $z z a z$ | 0.394 | 0.750 | 0.037 | 0.394 |
| $z z z a$ | 0.106 | 0.098 | 0.144 | 0.106 |
| $z z z z$ | 0.500 | 0.848 | 0.152 | 0.500 |

$$
\begin{aligned}
& z e a z / p A \\
& \text { split ergative } \\
& \begin{array}{ll}
z z a z / p A & e z z z / p O \\
\text { differential object marking } & \text { inverse DOM }
\end{array} \\
& z e z z / p A \\
& \text { differential subject marking inverse DSM } \\
& z z z a / p O
\end{aligned}
$$

```
zeaz/pA
split ergative
Australian languages
\(z z a z / p A \quad e z z z / p O\)
differential object marking inverse DOM
\(z e z z / p A\)
differential subject marking inverse DSM
\(z z z z / p A\)
\(z z z a / p O\)
no case marking
\(z z z z / p A\)
```

```
zeaz/pA
split ergative
Australian languages
\(z z a z / p A \quad e z z z / p O\)
differential object marking inverse DOM
English, Dutch, ...
\(z e z z / p A \quad z z z a / p O\)
differential subject marking inverse DSM
\(z z z z / p A\)
\(z z z a / p O\)
no case marking
\(z z z z / p A\)
```

```
zeaz/pA
split ergative
Australian languages
\(z z a z / p A \quad e z z z / p O\)
differential object marking inverse DOM
English, Dutch, ...
\(z e z z / p A\)
\(z z z a / p O\)
differential subject marking
several caucasian languages
\(z z z z / p A\)
\(z z z a / p O\)
no case marking
\(z z z z / p A\)
```

```
zeaz/pA
split ergative
Australian languages
```

$z z a z / p A$
differential object marking English, Dutch, ...
$z e z z / p A$
differential subject marking several caucasian languages
$z z z z / p A$
no case marking Chinese, Thai
$z z z z / p A$
$e z z z / p O$
inverse DOM
-
$z z z a / p O$
inverse DSM
$z z z a / p O$

```
\(z e a z / p A\)
split ergative
Australian languages
\(z z a z / p A \quad e z z z / p O\)
differential object marking inverse DOM
English, Dutch, ...
\(z e z z / p A\)
\(z z z a / p O\)
differential subject marking
several caucasian languages
\(z z z z / p A\)
\(z z z a / p O\)
no case marking
Chinese, Thai
\(z z z z / p A\)
```


## Taking stock

- only very few languages are not evolutionary stable in this sense
zzaa: Hungarian, ezza: Parachi, Yazguljami (Iranian languages), eeaa: Wangkumara
- curious asymmetry: if there are two competing stable states, one is common and the other one rare
- similar pattern as with Horn vs. anti-Horn

Alle equilibria are stable, but some equilibria are more stable than others.

## Stochastic EGT

## Random mutation and stability

- idealizations of standard Evolutionary Game Theory
- populations are (practically) infinite
- mutations rate is constant and low
- better model (Young 1993; Kandori, Mailath and Rob 1993)
- finite population
- mutation is noisy


## Consequences of finite population model

- every mutation barrier will occasionally be taken
- no absolute stability
- if multiple Strict Nash Equilibria coexist, system will oscillate between them
- some equilibria are more stable than others
- system will spend most of the time in most robustly stable state
- stochastically stable states


## A particular model

- discrete time/finite population version of replicator dynamics
- mutations occur rarely (most generations have no mutants at all)
- if mutation occurs, each individual in this generation has same probability to be a mutant
- mutation frequency and mutation rate equal for both populations
- each strategy is equally likely for a mutant (within its population)

The formulas

$$
\begin{aligned}
& \frac{\Delta x_{i}}{\Delta t}=x_{i}\left(\tilde{u}_{i}-\tilde{u}^{A}\right)+\sum_{j} \frac{Z_{j i}-Z_{i j}}{n} \\
& \frac{\Delta y_{i}}{\Delta t}=y_{i}\left(\tilde{u}_{i}-\tilde{u}^{B}\right)+\sum_{j} \frac{Z_{j i}-Z_{i j}}{n}
\end{aligned}
$$

The formulas

$$
\begin{aligned}
& \frac{\Delta x_{i}}{\Delta t}=x_{i}\left(\tilde{u}_{i}-\tilde{u}^{A}\right)+\sum_{j} \frac{Z_{j i}-Z_{i j}}{n} \\
& \frac{\Delta y_{i}}{\Delta t}=y_{i}\left(\tilde{u}_{i}-\tilde{u}^{B}\right)+\sum_{j} \frac{Z_{j i}-Z_{i j}}{n}
\end{aligned}
$$

- $x_{i}$ : frequency of speaker strategy $i$

The formulas

$$
\begin{aligned}
& \frac{\Delta x_{i}}{\Delta t}=x_{i}\left(\tilde{u}_{i}-\tilde{u}^{A}\right)+\sum_{j} \frac{Z_{j i}-Z_{i j}}{n} \\
& \frac{\Delta y_{i}}{\Delta t}=y_{i}\left(\tilde{u}_{i}-\tilde{u}^{B}\right)+\sum_{j} \frac{Z_{j i}-Z_{i j}}{n}
\end{aligned}
$$

- $x_{i}$ : frequency of speaker strategy $i$
- $y_{i}$ : frequency of hearer strategy $i$

The formulas

$$
\begin{aligned}
& \frac{\Delta x_{i}}{\Delta t}=x_{i}\left(\tilde{u}_{i}-\tilde{u}^{A}\right)+\sum_{j} \frac{Z_{j i}-Z_{i j}}{n} \\
& \frac{\Delta y_{i}}{\Delta t}=y_{i}\left(\tilde{u}_{i}-\tilde{u}^{B}\right)+\sum_{j} \frac{Z_{j i}-Z_{i j}}{n}
\end{aligned}
$$

- $x_{i}$ : frequency of speaker strategy $i$
- $y_{i}$ : frequency of hearer strategy $i$
- $\tilde{u}_{i}$ : expected utility of strategy $i$

$$
\begin{aligned}
& \frac{\Delta x_{i}}{\Delta t}=x_{i}\left(\tilde{u}_{i}-\tilde{u}^{A}\right)+\sum_{j} \frac{Z_{j i}-Z_{i j}}{n} \\
& \frac{\Delta y_{i}}{\Delta t}=y_{i}\left(\tilde{u}_{i}-\tilde{u}^{B}\right)+\sum_{j} \frac{Z_{j i}-Z_{i j}}{n}
\end{aligned}
$$

- $x_{i}$ : frequency of speaker strategy $i$
- $y_{i}$ : frequency of hearer strategy $i$
- $\tilde{u}_{i}$ : expected utility of strategy $i$
- $\tilde{u}^{R}$ : average utility of entire $R$-population

$$
\begin{aligned}
& \frac{\Delta x_{i}}{\Delta t}=x_{i}\left(\tilde{u}_{i}-\tilde{u}^{A}\right)+\sum_{j} \frac{Z_{j i}-Z_{i j}}{n} \\
& \frac{\Delta y_{i}}{\Delta t}=y_{i}\left(\tilde{u}_{i}-\tilde{u}^{B}\right)+\sum_{j} \frac{Z_{j i}-Z_{i j}}{n}
\end{aligned}
$$

- $x_{i}$ : frequency of speaker strategy $i$
- $y_{i}$ : frequency of hearer strategy $i$
- $\tilde{u}_{i}$ : expected utility of strategy $i$
- $\tilde{u}^{R}$ : average utility of entire $R$-population
- $Z_{i j}$ : random variable; distributed according to the binomial distribution $b\left(p_{i j},\left\lfloor x_{i} n\right\rfloor\right)$
- $p_{i j}$ : probability that an $i$-individual mutates to strategy $j$

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- $n$ : population size


## A simulation



## Stochastic stability

- punctuated equilibria
- long periods of dynamic stability alternate with short transition periods
- in the long run, more time in Horn state ( $67 \%$ vs. $26 \%$ in anti-Horn)
- simulation suggests that Horn is stable while anti-Horn is not
- can this be proved?


## Analytic considerations

- Simple recipes for finding stochastically stable state in $2 \times 2$ games
- not easily extrapolated to larger games
- basic idea:
- calculate the height of the invasion barrier of each ESS
- the ESSs with maximal invasion barrier is stochastically stable


## Analytic considerations

- invasion barrier $=$ amount of mutations necessary to push the system into the basin of attraction of another ESS
- Horn $\Rightarrow$ anti-Horn: 50\%
- anti-Horn $\Rightarrow$ Horn: 47.5\%
- Hence:


## Horn strategy is the only stochastically stable state

## Stochastic evolution of case marking

- $k=0.45$
- competition between $z z a z / p A$ and $e z z z / p O$
- evolution of speaker population:



## Stochastic evolution of case marking

- $k=0.45$
- competition between $z z a z / p A$ and $e z z z / p O$
- evolution of hearer population:



## Analysis

- invasion barriers:
- differential object marking: 45.2\%
- inverse differential subject marking: 2.06\%

Differential object marking is stochastically stable; inverse differential subject marking is not.

- likewise, differential subject marking is stochastically stable while inverse differential object marking is not.


## Stochastically stable states

zeaz/pA
split ergative
Australian languages
$z z a z / p A$
differential object marking
English, Dutch, ...
$z e z z / p A$
differential subject marking
several caucasian languages
$z z z z / p A$
no case marking
Chinese, Thai

## Conclusion

- out of $4 \times 16=64$ possible case marking patterns only four are stochastically stable
- vast majority of all languages that fit into this categorization are stochastically stable
- precise numbers are hard to come by though
- linguistic universals can be result of evolutionary pressure in the sense of cultural evolution

