Mathematics for linguists

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Equivalent derivations

Sample grammar

$$
\begin{aligned}
V_T &= \{\text{banana, the, green, buys, big, man}\} \\
V_N &= \{S, NP, VP, N, A, D, V\} \\
S & \\
R &= \left\{ \begin{array}{lcl}
S & \rightarrow & NP\ VP \\
NP & \rightarrow & D\ N \\
N & \rightarrow & A\ N \\
VP & \rightarrow & V\ NP \\
D & \rightarrow & \text{the} \\
N & \rightarrow & \text{banana} \\
N & \rightarrow & \text{man} \\
A & \rightarrow & \text{big} \\
A & \rightarrow & \text{green} \\
V & \rightarrow & \text{buys}
\end{array} \right\}
\end{aligned}
$$

Equivalent derivations

Sample derivation 1

	S	
$\Longrightarrow$	$NP\ \ VP$	($S \rightarrow NP\ VP$)
$\Longrightarrow$	$NP\ V\ \ NP$	($VP \rightarrow V\ \ NP$)
$\Longrightarrow$	$D\ \ N\ \ V\ \ NP$	($NP \rightarrow D\ N$)
$\Longrightarrow$	$D\ \ A\ \ N\ \ V\ \ NP$	($N \rightarrow A\ N$)
$\Longrightarrow$	$D\ \ A\ \ N\ \ V\ \ D\ \ N$	($NP \rightarrow D\ N$)
$\Longrightarrow$	$D\ \ A\ \ N\ \ V\ \ D\ \ A\ \ N$	($N \rightarrow A\ N$)
$\Longrightarrow$	the $A\ \ N\ \ V\ \ D\ \ A\ \ N$	($D \rightarrow$ the)
$\Longrightarrow$	the big $N\ \ V\ \ D\ \ A\ \ N$	($A \rightarrow$ big)
$\Longrightarrow$	the big man $V\ \ D\ \ A\ \ N$	($N \rightarrow$ man)
$\Longrightarrow$	the big man buys $D\ \ A\ \ N$	($V \rightarrow$ buys)
$\Longrightarrow$	the big man buys the $A\ \ N$	($D \rightarrow$ the)
$\Longrightarrow$	the big man buys the green N	($A \rightarrow$ green)
$\Longrightarrow$	the big man buys the green banana	($N \rightarrow$ banana)

Equivalent derivations

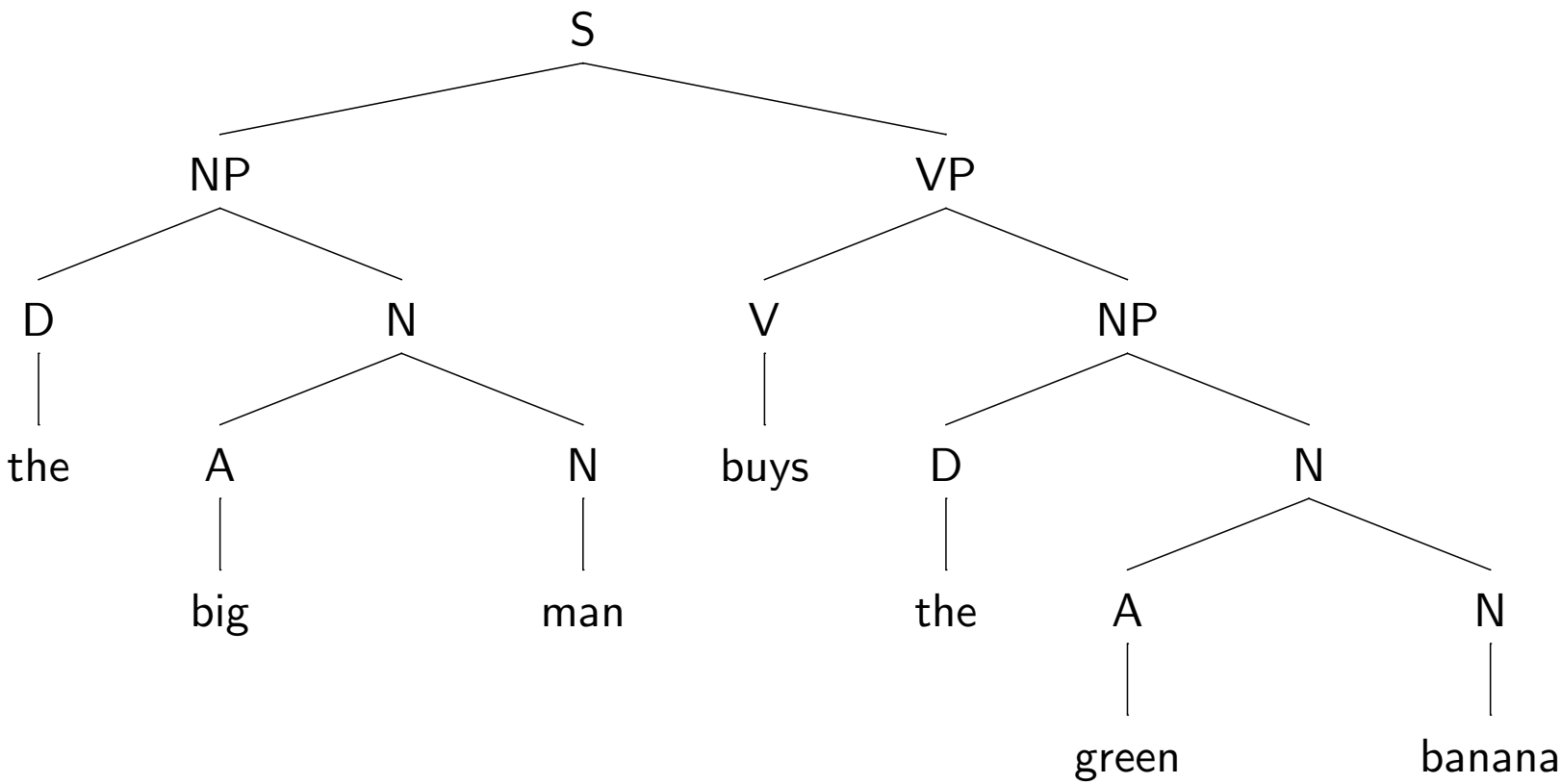
Sample derivation 2

	S	
$\Longrightarrow$	$NP\ \ VP$	$(\ S \rightarrow NP\ VP\ )$
$\Longrightarrow$	$D\ \ N\ \ VP$	$(\ NP \rightarrow D\ N\ )$
$\Longrightarrow$	the $\ N\ \ VP$	$(\ D \rightarrow$ the $)$
$\Longrightarrow$	the $A\ \ N\ \ VP$	$(\ N \rightarrow A\ N\ )$
$\Longrightarrow$	the big $\ N\ \ VP$	$(\ A \rightarrow$ big $)$
$\Longrightarrow$	the big man $\ VP$	$(\ N \rightarrow$ man $)$
$\Longrightarrow$	the big man $V\ \ NP$	$(\ VP \rightarrow V\ NP)$
$\Longrightarrow$	the big man buys $\ NP$	$(\ V \rightarrow$ buys $)$
$\Longrightarrow$	the big man buys $D\ \ N$	$(\ NP \rightarrow D\ N\ )$
$\Longrightarrow$	the big man buys the $\ N$	$(\ D \rightarrow$ the $)$
$\Longrightarrow$	the big man buys the $A\ \ N$	$(\ N \rightarrow A\ N\ )$
$\Longrightarrow$	the big man buys the green $\ N$	$(\ A \rightarrow$ green $)$
$\Longrightarrow$	the big man buys the green banana	$(\ N \rightarrow$ banana $)$

Derivation trees

- These two derivations are intuitively equivalent because the same rules are used, and furthermore the rules are applied to the same substirngs. Here we consider — informally — symbols and substrings in different stages of a derivation as equivalent if one is the result of simply copying the other.
- Notation of derivation can be simplified if
  - in each derivation step, only the substring that corresponds to the right hand side of the rule applied is written into the next line, and
  - all new new symbols are connected by a line to the corresponding left hand side symbol in the preceding line.

Derivation trees



Derivation trees

- Important: for some grammars, derivations cannot be represented as derivation trees.
- For linguistically interesting grammars this is always possible though. (To be more precise, this is always possible with so-called *context sensitive grammars* — we will return to this later in this course.)
- The derivation tree contains all linguistically relevant information; the relative order of the single derivaiton steps is, at least for the so-called *context free grammars*, irrelevant.