Language change & cultural evolution
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Evolutionary game theory in linguistics
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A case study: the progressive cycle
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Language Change & Cultural Evolution – Game-Theoretic Approaches

Current Trends in Linguistics

Roland Mühlenbernd

EBERHARD KARLS
UNIVERSITÄT
TÜBINGEN

PHILOSOPHISCHE
FAKULTÄT
Seminar für Sprachwissenschaft

EVOLAEMP
LANGUAGE EVOLUTION: THE EMPIRICAL TURN

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Overview

The Tree of Life

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Natural Selection

- **Replication:** entities can produce copies

- **Mutation:** copies are not faithful

- **Variation:** plenty of variants exists (due to replication & mutation)

- **Differential Fitness:** number of offspring depends on contextual factors → selection

Mutation creates variation

Unfavorable mutations selected against

Reproduction and mutation occur

Favorable mutations more likely to survive

... and reproduce

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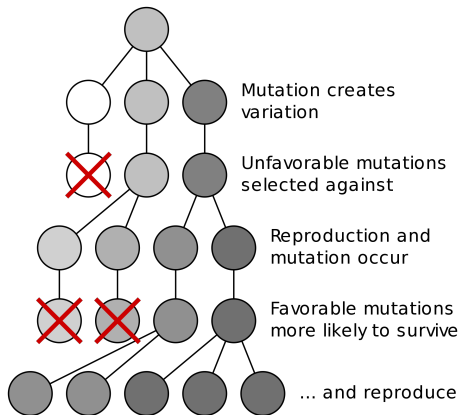
Evolution Theory & Language

The formation of different languages and of distinct species, and the proofs that both have been developed through a gradual process, are curiously parallel. ...Max Müller has well remarked: 'A struggle for life is constantly going on amongst the words and grammatical forms in each language. The better, the shorter, the easier forms are constantly gaining the upper hand, and they owe their success to their inherent virtue.' ...The survival or preservation of certain favoured words in the struggle for existence is natural selection.
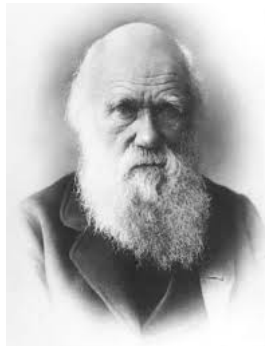
Charles Darwin, 1871[1]

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[1]The Descent of Man, and Selection in Relation to Sex, C2: *Mental Powers*

Evolution Theory & Language

Cultural transmission is analogous to genetic transmission in that, although basically conservative, it can give rise to a form of evolution. [...] Language seems to evolve by non-genetic means, and at a rate which is orders of magnitude faster than genetic evolution.

Richard Dawkins, 1976[2]

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[2]The Selfish Gene, Chapter 11: *Memes: the new replicators*

Mechanisms of Universal Darwinisms

1. *Variation*: continuing (abundance) of different elements

2. *Replication*: elements have the capacity to create copies or replicas of themselves

3. *Differential "Fitness"*: the number of copies of an element that are created in a given time varied, depending on interactions between

   ▶ features of that element
   ▶ features of the environment in which it persists

   ⇒ *Selection*
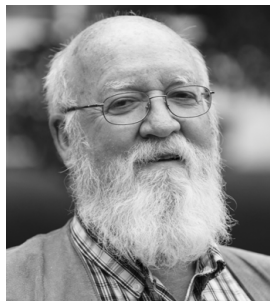
Daniel Dennett, 1995[3]

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[3]Darwin's Dangerous Ideas, Chapter 12: *The Cranes of Culture*

Rosenbach's Differentiation

- ▶ We have to distinguish i) the evolution of language faculty of Homo Sapiens from ii) the change of human language
- ▶ language change is part of cultural evolution

− language	+ language
————— language evolution —————→	————— language change —————→
↓	↓
biological evolution	cultural evolution
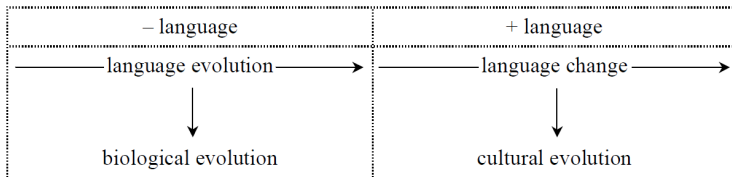
Quelle: *Language change as cultural evolution* (Rosenbach 2008)

Linguistic Replication

1. **What are linguistic replicators in the first place?**

Thus, the paradigm replicator in language is the lingueme, parallel to the gene as the basic replicator in biology; an utterance is made up of linguemes and linguemes possess structure.

William Croft (2000): Explaining Language Change

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Linguistic Replication

2. **What are the units of linguistic replication?**

Well-known building blocks of linguistic structure:

▶ phonemes, morphemes, phrases, constructions, corresponding meaning (Croft 2000, Ritt 2004, Rosenbach 2008)

Linguistic Replication

3. **What is the replication mechanism?**

▶ first language acquisition (vertical transfer)
▶ repeated communication (horizontal transfer)
  ▶ imitation (Ritt 2004)
  ▶ accommodation (Croft 2000: important selection mechanism)
  ▶ priming (Rosenbach & Jäger 2008, Eckardt 2008)

Language change as cultural evolution

1. Replication: production/adoption through repeated communication, first language acquisition
2. Variation: number of variants to express something
3. Selection: learnability, speaker/hearer economy (communicative functionality), social factors (register, prestige, accommodation)

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Overview

1. Language change & cultural evolution
2. **Evolutionary game theory in linguistics**
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Explanatory Potential of EGT in Linguistics

> *Results and methods from game theory are apt to formalize the intuitive notion of 'linguistic universals' as emergent properties of communication. [...] Evolutionary game theory [...] provides a formal framework that models questions of language evolution that allows us to integrate the two approaches, innateness and social function of language.*

Gerhard Jäger, 2014[4]

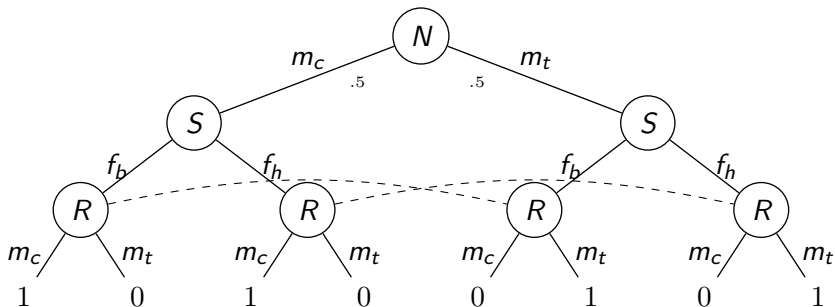[4]What is a universal? On the explanatory potential of evolutionary game theory in linguistics

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Coordination & Signaling Games

	m_c	m_t
m_c	1	0
m_t	0	1

$$U(m_i, m_j) = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{else} \end{cases}$$

signaling game: $\langle \{S, H\}, M, F, P, U \rangle$

▶ speaker S, hearer H

▶ meanings $M = \{m_c, m_t\}$

▶ forms $F = \{f_h, f_b\}$

▶ $P \in \Delta(M)$, $P(m_c) = P(m_t) = .5$

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Signaling Strategies

▶ Speaker strategy s: speaker behavior formalized as a function
  $s : M \to F$:

$$s_1: \begin{array}{c} m_c \longrightarrow f_b \\ m_t \longrightarrow f_h \end{array} \qquad s_2: \begin{array}{c} m_c f_b \\ \times \\ m_t f_h \end{array} \qquad s_3: \begin{array}{c} m_c f_b \\ \nearrow \\ m_t f_h \end{array} \qquad s_4: \begin{array}{c} m_c f_b \\ \searrow \\ m_t \longrightarrow f_h \end{array}$$

▶ Hearer strategy h: hearer behavior formalized as a function
  $h : F \to M$:

$$h_1: \begin{array}{c} f_b \longrightarrow m_c \\ f_h \longrightarrow m_t \end{array} \qquad h_2: \begin{array}{c} f_b m_c \\ \times \\ f_h m_t \end{array} \qquad h_3: \begin{array}{c} f_b m_c \\ \nearrow \\ f_h m_t \end{array} \qquad h_4: \begin{array}{c} f_b m_c \\ \searrow \\ f_h \longrightarrow m_t \end{array}$$

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Signaling Systems...

- ► are combinations of pure strategies.
  The coffeehouse game has two: $L_1 = \langle s_1, h_1 \rangle$ and $L_2 = \langle s_2, h_2 \rangle$

$$L_1: \quad \begin{array}{ccccc} m_c & \longrightarrow & f_b & \longrightarrow & m_c \\ m_t & \longrightarrow & f_h & \longrightarrow & m_t \end{array} \qquad L_2: \quad \begin{array}{ccc} m_c & f_b & m_c \\ & \times \times & \\ m_t & f_h & m_t \end{array}$$
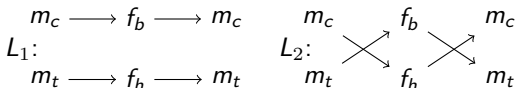
- ► are one-to-one mappings between form and meaning
- ► are strict *Nash equilibria* of the *EU-table*:

	h_1	h_2	h_3	h_4
s_1	1	0	.5	.5
s_2	0	1	.5	.5
s_3	.5	.5	.5	.5
s_4	.5	.5	.5	.5

  - ► whereby $EU(s, h) = \sum_{m \in M} P(m) \times U(m, h(s(m)))$

- ► and are therefore evolutionary stable under evolutionary dynamics

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Signaling Games and Cultural Evolution

- *Replicators:* form-meaning mappings in terms of strategy pairs
- *Variation:* is given for any signaling game with $|M| > 1$ and/or $|F| > 1$
- *Replication and Selection:* defined as 'update mechanism/rule' for 'repeated plays'

Overview: Update Mechanisms

	replicator dynamics	imitation	learning
perspective	population-based	agent-based	agent-based
memory size	0	1	(whole) history
game type	static	static	static/dynamic
strategy	pure	pure	pure/probabilistic

- ▶ replicator dynamics is standard model in EGT
- ▶ imitation is the basic mechanism for cultural evolution
- ▶ "it can be shown that exactly the same strategies are evolutionary stable under the replicator dynamics and under the imitation dynamics" (Jäger 2007)
- ▶ learning rules i) can be combined with actual 'communicative acts' and ii) add the aspect of language acquisition/ontogeny

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Applications of Signaling Games I

Functional explanations for typological universals:

- *morphology*: evolutionary stability of case marking systems for semantic core roles via (i) replicator dynamics, and (ii) stability analysis (Jäger 2007)
- *phonology*: stability of vowel systems (separating acoustic space) via (i) exemplar learning, and (ii) simulation experiments (Jäger 2008)
- *pragmatics*: evolutionary stability of the 'Horn strategy' via (i) reinforcement learning, fictitious play, and (ii) simulation experiments (Mühlenbernd 2011)

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Applications of Signaling Games II
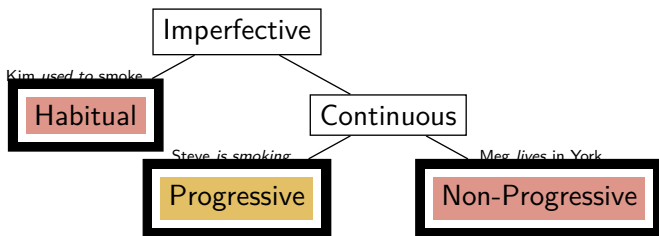
Functional explanations for 'diachronic' universals:

- *negation*: evolutionary trajectories of the 'Jespersen cycle' via (i) replicator dynamics, and (ii) stability/trajectory analysis (Ahern & Clark 2014, Ahern 2016)
- *morphosemantics*: evolutionary trajectories of the 'progressive cycle' via (i) replicator dynamics, and (ii) stability/trajectory analysis (Deo 2015, Yanovich 2016)
- *morphosemantics*: evolutionary trajectories of the 'progressive cycle' via (i) reinforcement learning, and (ii) simulation experiments (Enke, Mühlenbernd, Yanovich 2016)

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The Imperfective Domain

- ▶ the imperfective form makes explicit reference to the internal temporal structure of a situation
- ▶ the imperfective range (Comrie, 1976):

Imperfective

Kim *used to* smoke

Habitual

Continuous

Steve *is smoking*

Meg *lives* in York

Progressive

Non-Progressive

- ▶ fundamental classification of the types of knowledge we posses (Goldsmith & Woisetschlaeger, 1982):
- ▶ *structural:* stable facts, generalizations that characterize the world
- ▶ *phenomenal:* facts that pertain to specific times and the events that occupy such times

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Marking Strategies of the Imperfective Domain

	system	strategy	sample language
ZP	zero progressive	X_{imp}	*Russian*, Old English

(1) Olga   **pisa-la**   pis'ma   kogda
     Olga.NOM.SG write.IMPF-PST.F letter.ACC.PL when
     pojavilsja   Vadim
     appear.PERF.PST.M Vadim.NOM.SG
     'Olga *was writing* letters when Vadim appeared.'

(2) Olga   **pisa-la**   pis'mo   materi
     Olga.NOM.SG write.IMPF-PST.F letter.ACC.SG mother.DAT.SG
     po voskresenjam
     on Sunday.DAT.PL
     'Olga *used to write* a letter to her mother on Sundays.'

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Marking Strategies of the Imperfective Domain

	system	strategy	sample language
ZP	zero progressive	X_{imp}	*Russian*, Old English
OP	optional progressive	$X_{imp}, (Y_{prog})$	*Italian*, Early Modern English

(3)   Che fai?   Ridi?
      what do.PRS.1SG laugh.PRS.1SG
      'What are you doing? Are you laughing?'

(4)   Che **stai**   facendo? **Stai**   ridendo?
      what stay.PRS.1SG doing   stay.PRS.1SG laughing
      'What are you doing? Are you laughing?'

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Marking Strategies of the Imperfective Domain

	system	strategy	sample language
ZP	zero progressive	X_{imp}	*Russian*, Old English
OP	optional progressive	$X_{imp}, (Y_{prog})$	*Italian*, Early Modern English
CP	categorical progressive	X_{imp}, Y_{prog}	*PM Turkish*, English, Irish

(5) Genekkikle iki saat   çališ-**ir**-di-m
     Usually   for two hours work-IMPF-PST.COP-1.SG
     'I *usually work* for two hours.'

(6) Saat ikide   çališ-**iyor**-du-m
     At   two o'clock work-PROG-PST.COP-1.S
     'A two o'clock I *was working*.'

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Marking Strategies of the Imperfective Domain

|	system	strategy	sample language
ZP	zero progressive	X_{imp}	*Russian*, Old English
OP	optional progressive	$X_{imp}, (Y_{prog})$	*Italian*, Early Modern English
CP	categorical progressive	X_{imp}, Y_{prog}	*PM Turkish*, English, Irish
ZP^*	zero progressive	Y_{imp}	*Turkish*, Welsh

(7) Sen Ömer'i benden daha iyi   tan-**iyor**-du-n
    you Ömer me   better than know-PROG-PST.COP.2.SG
    'You *knew (were knowing)* Ömer better than me.'

(8) O zamanlarda Mehmet çok sigara
    At the time   Mehmet lot cigarette
    iç-**iyor**-du
    smoke-IMPF-PST.COP.3.SG
   'At that time, Mehmet *used to smoke* a lot.'

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The Functional Progressive Cycle

Old English,
Russian

Turkish,
Welsh

Keys:
P: phenomenal
S: structural

ZP

Generalization
Invasion

Innovation
Mutation

German,
Dutch

English?

CP

OP

English, Irish,
Old Turkish

Categorization
Adaptation
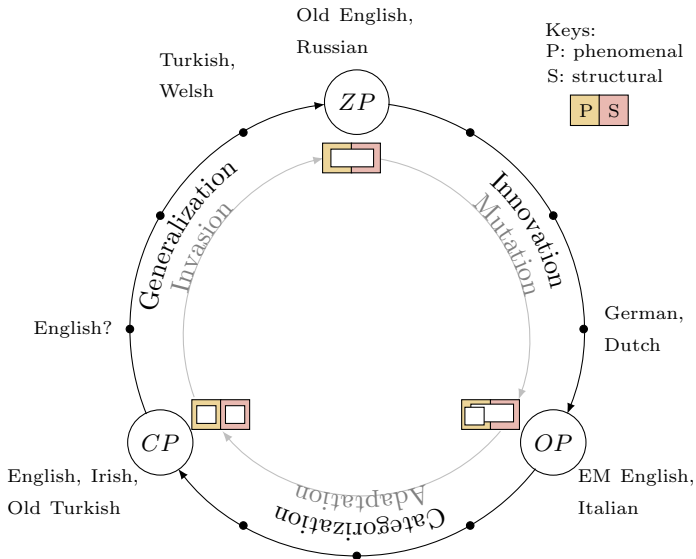
EM English,
Italian

The Formal Progressive Path

	system	strategy	sample language
ZP	zero progressive	X_{imp}	*Russian*, Old English
OP	optional progressive	$X_{imp}, (Y_{prog})$	*Italian*, Early Modern English
CP	categorical progressive	X_{imp}, Y_{prog}	*PM Turkish*, English, Irish
ZP*	zero progressive	Y_{imp}	*Turkish*, Welsh
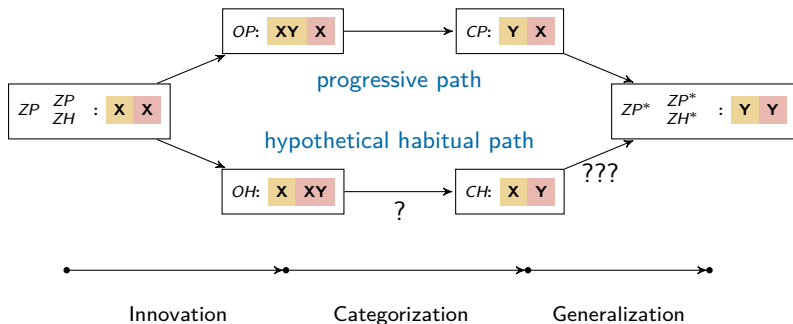
Research Questions

▶ What kind of conjectures can explain the existence of the following crosslinguistically attested 'universal' patterns inside the imperfective domain?

  ▶ the cyclic pattern of the progressive's functional evolution
  ▶ the non-cyclic (dead-end) trajectory of the habitual's functional evolution
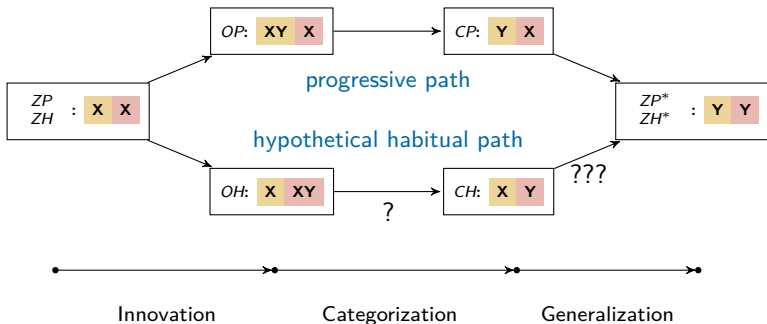
Research Project

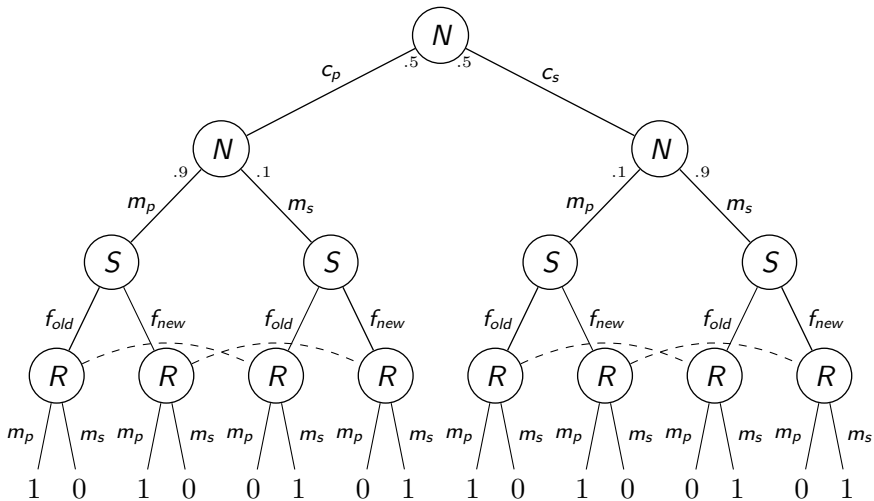**Given**: typologically supported theory of HOW a diachronic process (of morphosemantic change) happens

Wanted: potential conjectures responsible for WHY there is no alternative development (diachronic universal)

Approach: game theoretic model $\rightarrow$ evolutionary population model $\rightarrow$ synthetic analysis: simulation experiments for different conjectures

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Signaling game

- ▶ game theoretic model that represents a communicative situation between a speaker S and a hearer H

- ▶ Imperfective game: $IG = \langle (S, H, C, M, F, P, U \rangle$

- ▶ forms: $F = \{f_{old}, f_{new}\}$

- ▶ meanings: $M = \{ m_s , m_p \}$

- ▶ contexts: $C = \{ c_s , c_p \}$

- ▶ probabilities: $P(m_s|c_s) = P(m_p|c_p) = .9$

- ▶ utility function: $U(m, s, h) = \left\{ \begin{array}{ll} 1 & \text{if } h(s(m)) = m \\ 0 & \text{else} \end{array} \right.$

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Extensive Form of the Imperfective Game

Speaker strategies
$s \in \mathbf{S} : M \times C \to F$

	c_p m_p	c_p m_s	c_s m_p	c_s m_s	
ZH s_0	f_{old}	f_{old}	f_{old}	f_{old}	ZP
s_1	f_{old}	f_{old}	f_{old}	f_{new}	
s_2	f_{old}	f_{old}	f_{new}	f_{old}	OP
s_3	f_{old}	f_{old}	f_{new}	f_{new}	
OH s_4	f_{old}	f_{new}	f_{old}	f_{old}	
CH s_5	f_{old}	f_{new}	f_{old}	f_{new}	
s_6	f_{old}	f_{new}	f_{new}	f_{old}	
s_7	f_{old}	f_{new}	f_{new}	f_{new}	
s_8	f_{new}	f_{old}	f_{old}	f_{old}	
s_9	f_{new}	f_{old}	f_{old}	f_{new}	
s_{10}	f_{new}	f_{old}	f_{new}	f_{old}	CP
s_{11}	f_{new}	f_{old}	f_{new}	f_{new}	
s_{12}	f_{new}	f_{new}	f_{old}	f_{old}	
s_{13}	f_{new}	f_{new}	f_{old}	f_{new}	
s_{14}	f_{new}	f_{new}	f_{new}	f_{old}	
ZH* s_{15}	f_{new}	f_{new}	f_{new}	f_{new}	ZP*

Hearer strategies
$h \in \mathbf{H} : F \times C \to M$

	c_p f_{new}	c_p f_{old}	c_s f_{new}	c_s f_{old}	
h_0	m_p	m_p	m_p	m_p	
h_1	m_p	m_p	m_p	m_s	OP
h_2	m_p	m_p	m_s	m_p	
h_3	m_p	m_p	m_s	m_s	ZP$^{(*)}$
h_4	m_p	m_s	m_p	m_p	
h_5	m_p	m_s	m_p	m_s	CP
h_6	m_p	m_s	m_s	m_p	
h_7	m_p	m_s	m_s	m_s	
h_8	m_s	m_p	m_p	m_p	
h_9	m_s	m_p	m_p	m_s	
h_{10}	m_s	m_p	m_s	m_p	
h_{11}	m_s	m_p	m_s	m_s	
h_{12}	m_s	m_s	m_p	m_p	
h_{13}	m_s	m_s	m_p	m_s	
h_{14}	m_s	m_s	m_s	m_p	
h_{15}	m_s	m_s	m_s	m_s	

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Evolutionary Population Model

- ▶ *population size*: of 200 agents
- ▶ *simulation*: agents play the imperfective game pairwise and repeatedly ($\rightarrow$ horizontal transfer)
- ▶ *learning*: agents 'learn' communication strategies by learning rule *reinforcement learning* (Roth & Erev 1995, Skyrms 2010)
- ▶ *replacement*: agents get older and old agents (here: 5000 interactions) get replaced by 'newbies' ($\rightarrow$ vertical transfer)
- ▶ *initial setting*: at the beginning agents can only use form f_{old}, after a while (here: 1000 simulation steps) new form f_{new} will be introduced

Experiments & Results

Exp. 1: basic model

Result: 'odd' context strategy $\langle s_6, h_9 \rangle$ (f_{new} marks 'context-unusual')

Exp. 2: reduced context cues (-10%)

Result: both categorical systems emerge with same probability; optional systems are short intermezzos

Exp. 3: increasing costs factor α

Result: both zero systems emerge with same probability via both paths

Exp. 4: asymmetry for input of newbies

Result: reconstruction of expected paths
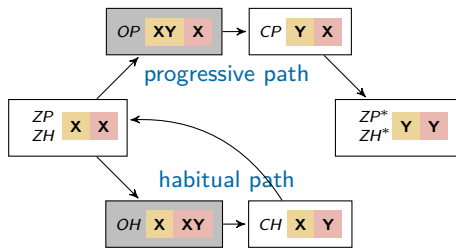
$$IR(m_s|n) = \begin{cases} \frac{0.5n}{n_{child}} & \text{if } n \leq n_{child} \\ 0.5 & \text{else} \end{cases}$$

$$IR(m_p|n) = 1.0 - IR(m_s|n)$$

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Summary

- given typological data, the progressive cycle can be seen as a diachronic language universal
- the cycle can be divided in 3 different stages
- the presented model can reproduce these stages in their assumed diachronic order, under the following assumptions:
  - reduced access to context information
  - increasing costs for complex systems (in favor of speaker economy)
  - asymmetry for young agents' input in favor of phenomenal situations

Language Change is Cultural Evolution

Vielen Dank!

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