

# Only Updates

On the Dynamics of the Focus Particle *only*

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# 1 Introduction

The Montagovian paradigm of natural language semantics relies on the two crucial assumptions that (a) the meaning of a sentence can exhaustively be described by means of its truth conditions, and (b) this meaning can be built up compositionally from its parts. Unfortunately, empirical observations force us to the conclusion that the mentioned assumptions cannot be true at the same time, as soon we shift our attention to discourse phenomena. This is exemplified by the discourses in (1):

- (1) a. Peter came in. He wore a hat.  
b. John came in. He wore a hat.

The second sentence in (1a) is true just in case that Peter wore a hat, while the corresponding sentence in (1b) is equivalent to *John wore a hat*. These two sentences are syntactically identical, at least as far as their surface is concerned. Hence they have the same parts, and, *ceteris paribus*, they should have the same meaning. Nevertheless their truth conditions differ.

This and related problems led several authors to the conclusion that we have to give up the principle of compositionality in the strict Montagovian sense. Most influentially, Kamp 1981 and Heim 1982 propose that there is an additional level of representation relating syntactic structure and semantic interpretation. In their systems, syntactic structure has to undergo a process of “DRS-construction” (Kamp) or “LF-construal” (Heim), and it is the output of this process that is interpreted compositionally in terms of truth conditions.

Groenendijk and Stokhof 1991a and Groenendijk and Stokhof 1991b choose the other direction. They point out that the principle of compositionality can be maintained as soon as we do without the assumption that sentence meanings coincide with truth conditions. Instead they propose that sentences (and discourses) denote transition function over information states. These transition functions are connected to truth conditions, but in an indirect way. The advantage of this strategy is a methodological rather than an empirical one, since compositional and non-representational semantic theories are generally more restrictive in their predictive power.

It is the aim of this paper to extend the coverage of the dynamic paradigm to phenomena involving the focus sensitive operator *only*. Constructions involving this item show a dependency on linguistic context reminiscent to the behavior of anaphora. Existing approaches address this phenomenon by weakening the compositionality of interpretation in several respects. Instead we will try to outline a dynamic theory of the semantics of this constructions that preserves compositionality both on the sentence and on the text level.

## 2 The Problem

Consider the following contrast:

- (2) Who is wise? Only [<sub>+F</sub> SOCRATES] is wise.
- (3) Which Athenians are wise? Only [<sub>+F</sub> SOCRATES] is wise.

Although the answers in (2) and (3) are identical at the surface, they show up different truth conditions. The answer in (2) forms a blasphemy since it entails that Zeus is unwise, while the corresponding sentence in (3) is much weaker in the sense that only the wisdom of all Athenians except Socrates is denied. Nothing is said about other individuals like, say, gods. More generally, the respective answers are truthconditionally equivalent to the first order formulae in (4a) and (b) respectively.

- (4) a.  $\forall x(\text{wise}(x) \rightarrow x = s)$   
b.  $\forall x(\text{athenian}(x) \wedge \text{wise}(x) \rightarrow x = s)$

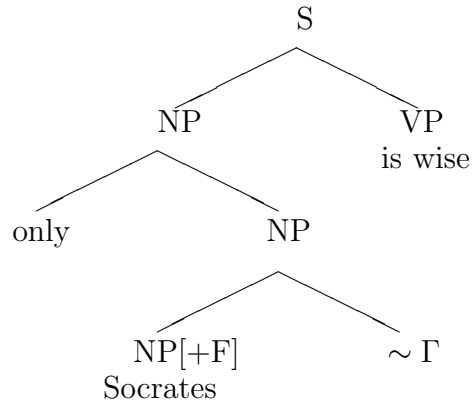
In both cases, the universal quantification is restricted by the non-focused part (the “background”) of the sentence (*is wise*). Besides this, there is an additional restrictor in (4b), corresponding to the argument of the *wh*-word in the question (*Athenian*). Hence it is quite obvious that the truth conditions of a sentence involving *only* somehow depend on the question the sentence is an answer to.

We will proceed as follows. In the subsequent section, the proposal made in Rooth 1992 – which can be seen as a kind of paradigmatic approach in a static setting – is briefly presented and discussed. In section 4 we develop a dynamic system that covers both interrogative and declarative sentences. Finally, in section 5 it is shown that this system is able to account for the kind of dependency between questions and answers illustrated above.

## 3 Rooth 1992

In the sense of the discussion above, Rooth’s proposal can be seen on a par with Heim 1982. As the feature that matters most for our purposes, he assumes that it is not surface structure that serves as input for semantic interpretation, but that there is an intermediate level of “Logical Form”. The proposed LF for the sentence under discussion is roughly as in (5):

- (5)



This is not the proper place to discuss the details and merits of Rooth’s semantics of focus in general. It is only important that the LF contains – besides the overt material – an additional item “ $\Gamma$ ” that is adjoined to the sister constituent of *only* by means of the operator “ $\sim$ ”.  $\Gamma$  is interpreted as a restriction of the domain of the universal quantification induced by *only*, such that we end up with an interpretation as it is given in (6).

$$(6) \forall x(C(x) \wedge wise(x) \rightarrow x = s)$$

“ $\Gamma$ ” is considered to be a kind of anaphor that is interpreted as the free variable  $C$  above. The value of this variable is – according to Rooth 1992 – determined by a variety of factors that are external to the compositional interpretative machinery. Although he does not address comparable examples directly, it is very much in his spirit to assume that  $\Gamma$  should be coindexed with the *wh*-phrase of the preceding question in (2) and (3) by means of some pragmatic mechanism.

More generally, to achieve the appropriate truth conditions for constructions involving *only* in a static setup, we are forced to assume that (a) there is at least one level of representation distinct from surface structure that serves as input for the compositional interpretation, and (b) truth conditions, i.e. meaning is not completely determined by lexicon and syntax but relies on pragmatic processes. In view of this fact it strikes me a fruitful enterprise to develop a semantical analysis that avoids these stipulations, even if we have to give up the equation *meaning* = *truthconditions*. More in detail, our aim is

- to get rid of any kind of syntactic placeholders like the anaphor  $\Gamma$  in Rooth’s proposal
- to derive the meaning of constructions involving *only* fully compositionally, i.e. without making reference to pragmatics.

## 4 Update Logic for Questions and Answers

### 4.1 A Static Approach to the Semantics of Questions: Groenendijk and Stokhof 1989

It is quite obvious that the different domain restrictions in the answers in (2) and (3) depend on the preceding question. Since we aim at an approach that is purely semantical, we have to assume that it is the semantics of the respective questions that trigger this effect. As basis for further argumentation, I adopt the framework laid down in Groenendijk and Stokhof 1984 without further argumentation.<sup>1</sup> It relies on the assumption that each question determines a unique proposition that constitutes the exhaustive true answer to the question.<sup>2</sup> This is best illustrated by an example. Take some simple yes-no question like

(7) Is it raining?

If it is raining, the unique exhaustive true answer is the proposition *It is raining*, and in case it is not raining, this answer is constituted by *It is not raining*. This can easily be expressed in a two-sorted extensional type theory:

(8)  $\lambda w.(rain!(w) \leftrightarrow rain!(w_0))$

This expression denotes a proposition in every possible world, although it is not necessarily the same proposition in different worlds. To get a fixed semantic object, we have to  $\lambda$ -abstract over the world of evaluation  $w_0$ . According to Groenendijk and Stokhof 1984, the meaning of a question is just this new object – the “concept of the true answer”.

(9)  $Is\ it\ rainig? \rightsquigarrow \lambda v \lambda w (rain!(v) \leftrightarrow rain!(w))$

It is obvious that this denotes an equivalence relation on the set of possible worlds. Hence a question defines an exhaustive partition on this set into a set mutually exclusive proposition. In the example above, these are just the propositions *It is raining* and *It is not raining*. Generally, the elements of the partition are those propositions that constitute exhaustive – but not necessarily true – answers to the question.

### 4.2 Information States and Updates

The common strategy for dynamisizing a certain static semantics runs roughly as follows: If sentences statically denote objects from some domain  $\alpha$ , the corre-

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1. In Groenendijk and Stokhof 1989 it is shown that the propositional accounts to question semantics given in Hamblin 1976, Karttunen 1977, and Groenendijk and Stokhof 1984 are fundamentally equivalent, such that the choice does not matter too much.

2. Counterexamples to this claim like free-choice questions are ignored throughout this paper.

sponding dynamic formulae denote functions  $F : \alpha \rightarrow \alpha$ . Since we want to deal with question-answer sequences in a single dynamic system, we are faced with a serious problem. Declarative sentences statically denote propositions, i.e. sets of possible worlds, while interrogatives denote relations on possible worlds or – equivalently – sets of propositions. Hence there are two candidates for update-denotations: functions from propositions to propositions or functions from relations to relations. By “generalizing to the worst case”, we adopt the latter option. Formulae of the dynamic logic to be defined below denote functions over information states (or simply “states”), where states are equivalence relations on possible worlds.

Let a nonempty set  $W$  of possible be given. We define:

**Definition 1 (Information States)**

$\sigma$  is an information state iff  $\sigma \subseteq W \times W$  and  $\sigma$  is an equivalence relation.

This immediately gives us a partial order on the set of states, corresponding to the intuitive notion of informativity. The minimal and the maximal elements of this order are called **1** (state of ignorance) and **0** (absurd state) respectively.

**Definition 2 (Informativity)**

$$\begin{aligned} \sigma_1 \leq \sigma_2 &\Leftrightarrow_{def} \sigma_1 \supseteq \sigma_2 \\ \mathbf{1} &=_{def} W \times W \\ \mathbf{0} &=_{def} \emptyset \end{aligned}$$

Note that these relations may be partial, i.e. their domains may be proper subsets of  $W$ . Hence each state nontrivially determines a certain proposition, which can be thought of as the factual knowledge shared by the conversants.

**Definition 3 (Domain of a state)**

$$D(\sigma) =_{def} \{w | w\sigma w\}$$

Since each equivalence relation uniquely defines a partition on its domain, it is worth considering the structure of the space of partitions determined by the space of states.

**Definition 4 (Partitions)**

Let  $\sigma, \tau$  be information states.

$$P_\sigma =_{def} \{\{v | v\sigma w\} | w \in W\} \setminus \{\emptyset\}$$

$$\begin{aligned}
P_\sigma \sqsubseteq_d P_\tau &\Leftrightarrow_{def} P_\sigma \supseteq P_\tau \\
P_\sigma \sqsubseteq_i P_\tau &\Leftrightarrow_{def} \bigcup P_\sigma = \bigcup P_\tau \wedge \forall x \in P_\sigma \exists y \in P_\tau : x \supseteq y
\end{aligned}$$

Equivalence classes of possible worlds can be thought of as epistemic alternatives in a certain stage of conversation. Information growth can affect these alternatives in two ways. Either some of them are eliminated – this is covered by  $\sqsubseteq_d$  – or they are made more finegrained without changing the domain of the equivalence relation itself ( $\sqsubseteq_i$ ). The latter corresponds to the effect of asking a question, while the former is the purpose of declarative utterances.<sup>3</sup> The notion of informativity given above covers both ways of information growth.

**Fact 1**

For all states  $\sigma$  and  $\tau$ , it holds that:

$$\begin{aligned}
P_\sigma \sqsubseteq_d P_\tau &\rightarrow \sigma \leq \tau \\
P_\sigma \sqsubseteq_i P_\tau &\rightarrow \sigma \leq \tau
\end{aligned}$$

Furthermore, the notion of informativity is exhausted by  $\sqsubseteq_d$  and  $\sqsubseteq_i$ :

**Fact 2**

For all states  $\sigma$  and  $\tau$ , it holds that:

$$\sigma \leq \tau \leftrightarrow \exists v : P_\sigma \sqsubseteq_i P_v \wedge P_v \sqsubseteq_d P_\tau$$

The intended interpretations of sentences/formulae are updates, i.e. transition functions over states that increase information.

**Definition 5 (Updates)**

Let  $\Sigma$  be the set of information states.

$$UP =_{def} \Sigma^\Sigma \cap POW(\leq)$$

Updates may be classified according to the way how they increase information.

**Definition 6 (Interrogative and Declarative Updates)**

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3. This is reminiscent to Dekker’s EDPL (cf. Dekker 1993), where information growth either introduces new variables into the state or eliminates possible values of familiar variables.

- An update  $u$  is called declarative iff

$$\forall \sigma : P_\sigma \sqsubseteq_d P_{u(\sigma)}.$$

- An update  $u$  is called interrogative iff

$$\forall \sigma : P_\sigma \sqsubseteq_i P_{u(\sigma)}.$$

There are only two updates that are both declarative and interrogative, namely the identity function and the constant function that always gives  $\mathbf{0}$  as output. On the other hand, there are many updates that are neither declarative nor interrogative. Nevertheless, according to Fact 2, every update can be decomposed into an interrogative and a declarative one (in this order).

Since interrogative and declarative sentences denote different kinds of objects statically, there are two pairs of operators that switch between static and dynamic meanings.

**Definition 7 (Up-Arrow and Down-Arrow)**

Let  $\sigma$  be an information state,  $p$  a proposition,  $q$  a static question denotation, and  $u$  an update.

$$\begin{aligned} \downarrow_d u &=_{def} \{w | u(\{\langle w, w \rangle\}) = \{\langle w, w \rangle\}\} \\ (\uparrow_d p)(\sigma) &=_{def} \sigma \cap p \times p \\ \downarrow_i u &=_{def} u(\mathbf{1}) \\ (\uparrow_i q)(\sigma) &=_{def} \sigma \cap q \end{aligned}$$

**Fact 3** For all propositions  $p$  and static question denotations  $q$ , it holds that:

$$\begin{aligned} \downarrow_d \uparrow_d p &= p \\ \downarrow_i \uparrow_i q &= q \end{aligned}$$

Please note that the converse neither holds for  $\uparrow_d \downarrow_d$  nor for  $\uparrow_i \downarrow_i$ . The  $\uparrow_d$ -operator is of particular interest since it enables us to add factual knowledge to a state without destroying the structure of the partition (of course, this holds only to the extent that the alternatives are compatible with this factual knowledge).

**Fact 4** For all states  $\sigma$  and propositions  $p$ , it holds that:

$$\begin{aligned} D(\uparrow_d p(\sigma)) &= D(\sigma) \cap p & (1) \\ \forall x(x \in P_\sigma \wedge x \cap p \neq \emptyset) &\rightarrow x \cap p \in P_{\uparrow_d p(\sigma)} & (2) \end{aligned}$$



### 4.3 The Semantics of ULQA

Having developed the necessary ontological background, we can start define a simple language that serves to reason about the described kind of updates. The syntax of ULQA is just the syntax of first order logic without functions symbols and with identity, with the single extension that there is a oneplace propositional operator “?” that makes formulae out of formulae. For convenience, we take  $\wedge$ ,  $\neg$ ,  $\rightarrow$ , and  $\forall$  as logical constants and the other connectives as abbreviations in the usual way.

A model for ULQA consists of an individual domain and a collection of classical interpretation functions (= possible worlds).

#### Definition 8 (Model)

A model  $M$  for ULQA is a triple  $\langle E, W, F \rangle$ , where  $E$  is a denumerable infinite set,  $W$  is non-empty, and  $F$  is a function that assigns a first-order interpretation function based on  $E$  to every member of  $W$ .

In the definition of the semantics of ULQA, we follow common practise in writing  $\sigma[\phi]_g$  instead of  $\|\phi\|_g(\sigma)$  in case  $\phi$  denotes an update. By  $g[e/x]$  we mean the assignment function  $g'$  that is exactly like  $g$  except that it maps  $x$  to  $e$ .

#### Definition 9 (The Semantics of ULQA)

$$\begin{aligned}
\|c\|_{w,g} &=_{def} F(w)(c) \text{ iff } c \text{ is an individual constant} \\
\|v\|_{w,g} &=_{def} g(v) \text{ iff } v \text{ is a variable} \\
\sigma[P(t_1, \dots, t_n)]_g &=_{def} \sigma \cap \{ \langle v, w \rangle \mid \forall w' (v\sigma w' \rightarrow \langle \|t_1\|_{w',g}, \dots, \|t_n\|_{w',g} \rangle \in F(w')(P)) \} \\
\sigma[t_1 = t_2]_g &=_{def} \sigma \cap \{ \langle v, w \rangle \mid \forall w' (v\sigma w' \rightarrow \|t_1\|_{w',g} = \|t_2\|_{w',g}) \} \\
\sigma[\phi \wedge \psi]_g &=_{def} \sigma[\phi]_g[\psi]_g \\
\sigma[\neg\phi]_g &=_{def} \sigma \setminus \sigma[\phi]_g \\
\sigma[?\phi]_g &=_{def} \sigma \cap \{ \langle v, w \rangle \mid \{ \langle v, v \rangle \}[\phi]_g = \emptyset \leftrightarrow \{ \langle w, w \rangle \}[\phi]_g = \emptyset \} \\
\sigma[\forall x.\phi]_g &=_{def} \bigcap_{e \in E} \sigma[\phi]_{g[e/x]} \\
\sigma[\phi \rightarrow \psi]_g &=_{def} \sigma \cap \{ \langle v, w \rangle \mid v\sigma[\phi]_g w \rightarrow v\sigma[\phi \wedge \psi]_g w \}
\end{aligned}$$

The interpretation of atomic formulae is based on the corresponding classical interpretation. Updating a certain state  $\sigma$  with a classical formula  $\phi$  amounts saying that only those alternatives in  $P_\sigma$  survive that are completely included in the set of possible worlds where  $\phi$  is true under its classical interpretation. If you consider  $\sigma$  as an accesibility relation in a Kripke model, the domain of the output is restricted to those worlds where  $\phi$  is necessarily true in its static interpretation.

The clauses for dynamic conjunction, dynamic negation, and dynamic implication are familiar from other dynamic systems like Veltman’s Update Se-

mantics (cf. Veltman 1990) and do not need much explanation. The semantics of universal quantification is a straightforward extrapolation from its classical counterpart.

The key feature of ULQA is the  $?$ -operator. To explain its impact on a rather intuitive level, each proposition in  $P_\sigma$  is splitted into those worlds where the formula in the scope of “?” is true and those where it is false. To put it another way, “ $? \phi$ ” defines an equivalence relation by its own, and the output of the update is the intersection of  $\sigma$  with this relation.

It does not come as a surprise that atomic formulae denote declarative updates and formulae prefixed with “?” interrogative ones. Both properties are preserved under conjunction and universal quantification. Being a declarative update is also preserved under negation. Negating an interrogative update, on the other hand, returns you in all non-trivial cases a relation as output that is reflexive and symmetric but not transitive. Hence the negation of an interrogative update isn’t an update at all in the general case.

The definitions of truth and entailment in ULQA are fairly standard from related dynamic calculi. A formula is called *true in a state* if updating the state with the formula does not add information. By abstracting over particular contexts, we get the notion of *truth in a model*, and by abstracting over models, we can define *logical truth*.

### Definition 10 (Truth)

Let  $M$  be a model,  $\sigma$  be an information state and  $\phi$  be a formula.

$$\begin{aligned} M, \sigma \models \phi &\Leftrightarrow_{def} \forall g : \sigma[\phi]_{M,g} = \sigma \\ M \models \phi &\Leftrightarrow_{def} \forall \sigma : M, \sigma \models \phi \\ \models \phi &\Leftrightarrow_{def} \forall M : M \models \phi \end{aligned}$$

The definition of the consequence relation between formulae is straightforwardly derived from this truth definitions.  $\psi$  is said to be a consequence of  $\phi$  iff the output of  $\phi$  is always a state where  $\psi$  is true.

### Definition 11 (Entailment)

Let  $M$  be a model,  $\sigma$  an information state, and  $\phi, \psi$  formulae.

$$\begin{aligned} \phi \models_{M,\sigma} \psi &\Leftrightarrow_{def} \forall g : \sigma[\phi]_{M,g} = \sigma[\phi \wedge \psi]_{M,g} \\ \phi \models_M \psi &\Leftrightarrow_{def} \forall \sigma : \phi \models_{M,\sigma} \psi \\ \phi \models \psi &\Leftrightarrow_{def} \forall M, \phi \models_M \psi \end{aligned}$$

## 4.4 The Relation of ULQA to First-order Logic

Syntactically speaking, ULQA is a simple extension of first-order logic, and also semantically, there is a close connection between the classical fragment of ULQA

and PL1. Recall that the individual domain  $E$  of and ULQA-model  $M$  together with any possible world  $w$  from  $W$  forms a first-order model. A ULQA-formula is called *?-free* iff it does not contain any occurrence of “?”. Trivially, the ?-free formulae are just the formulae of PL1.

**Definition 12**

Let  $\phi$  be a ?-free ULQA formula.

- By  $\|\phi\|_M^{ULQA}$  we refer to the ULQA-interpretation of  $\phi$  in the ULQA-model  $M$ .
- By  $\|\phi\|_M^{PL1}$  we refer to the set of worlds  $w$  from  $W$  such that  $\|\phi\|$  is true in  $\langle E, F(w) \rangle$  under classical first-order interpretation.

**Fact 5**

Let  $\phi$  be a ?-free ULQA-formula. It holds in any model  $M$  that:

$$\downarrow_d \|\phi\|_M^{ULQA} = \|\phi\|_M^{PL1}$$

*Proof* : By induction on the complexity of  $\phi$ .

In a sense, the classical interpretation of some ?-free formula  $\phi$ , i.e.  $\downarrow_d \|\phi\|_M^{ULQA}$ , can be seen as the “context-free” truth-conditional or factual impact of that formula. Hence by updating a state  $\sigma$  with  $\uparrow_d \downarrow_d \|\phi\|$ , we add the factual content of  $\phi$  to  $\sigma$  without affecting the structure of  $P_\sigma$ .

**Fact 6**

Let  $\phi$  be ?-free.

$$\uparrow_d \downarrow_d \|\phi\|^{ULQA}(\sigma) = \sigma \cap \|\phi\|^{PL1} \times \|\phi\|^{PL1}$$

*Proof* : Immediately from the definition of  $\uparrow_d$ .

Following common practise, we call  $\uparrow_d \downarrow_d \|\phi\|$  the *static closure* of  $\phi$ . Fortunately, the operation of static closure can be expressed in the object language, as far as ?-free formulae are concerned.

**Fact 7**

Let  $\phi$  be ?-free.

$$\uparrow_d \downarrow_d \|\phi\| = \|\phi \wedge ?\phi\|$$

*Proof* : By **Fact 5** and induction on the complexity of  $\phi$ .

For convenience, we will henceforth abbreviate “ $?\phi \wedge \phi$ ” with “ $\uparrow\downarrow \phi$ ” and we will also refer to it as the “static closure of  $\phi$ ”. Context will make clear whether we use the term in its syntactic or its semantic sense.

Remember that the union of the propositions in  $P_\sigma$  (the epistemic alternatives) represents the knowledge that is shared by the conversants. Static closure enables us to make statements about this state of factual knowledge in the object language.

**Fact 8**

Let  $\phi$  be  $?$ -free.

$$\sigma \models \uparrow\downarrow \phi \Leftrightarrow D(\sigma) \subseteq \|\phi\|^{PL1} \tag{1}$$

$$\psi \models \uparrow\downarrow \phi \Leftrightarrow \forall \sigma : D(\sigma[\psi]) \subseteq \|\phi\|^{PL1} \tag{2}$$

## 5 Restricted Quantification in ULQA

### 5.1 English $\implies$ ULQA

Since ULQA is a first-order language, there cannot be an immediate compositional translation function from English into ULQA. But it is obvious from the definition of the semantics of ULQA that every semantic object that is the interpretation of some ULQA-formula is at the same time the interpretation of some  $Ty_2$ -formula. Hence ULQA can be seen as a convenient notation of a fragment of  $Ty_2$ . On the other hand, if the translation of a couple of English sentences into  $Ty_2$  are given, it is a technical exercise to develop a Montague-style compositional translation from this fragment of English into  $Ty_2$ . This in mind, I will content myself with stipulating the ULQA-translations of the English sentences under debate since the described procedure would take a lot of space without illuminating anything of particular interest.

The translation of simple clauses with names in the argument positions are straightforward and do not need much explanation.

$$(10) \text{ Socrates is wise } \rightsquigarrow \textit{wise}(s)$$

*Prima facie*, the  $?$ -operator only enables us to form yes-no questions. Nevertheless it is possible to deal with constituent questions appropriately. We start with the observation that a *which*-question is equivalent to a yes-no-question in the scope of a restricted universal quantifier.

- (11) a. Which Athenians are wise?
- b. For all Athenians: Is he or she wise?

In contrast to other approaches to the semantics of questions, the interpretations of interrogative and declarative sentences belong to the same logical type, namely updates. Hence there is no problem in quantifying into a question. Hence I assume:

$$(12) \text{ Which Athenians are wise? } \rightsquigarrow \forall x(\text{athenian}(x) \rightarrow ?\text{wise}(x))$$

*Who*-questions can be handled in a similar manner. The only difference lies in the absence of a restriction to the quantifier.

$$(13) \text{ Who is wise } \rightsquigarrow \forall x.?\text{wise}(x)$$

One of the crucial features of ULQA is the fact that universal quantification is – so to speak – automatically contextually restricted. This fact will be illustrated in some length in the subsequent paragraphs. Therefore the translation of *only*-constructions can be kept pretty simple.

$$(14) \text{ Only Socrates is wise } \rightsquigarrow \forall x(\text{wise}(x) \rightarrow x = s)$$

This strategy is of course an oversimplification in some respects, but Krifka 1992 shows convincingly that it is possible to derive corresponding translations of more complex construction involving VP-focus and multiple focus fully compositionally.

## 5.2 Some Properties of ULQA

Let us start with a couple of negative results. First of all, the consequence relation defined above is not reflexive.

### Fact 9 (Non-Identity)

There are formulae  $\phi$  such that

$$\phi \not\models \phi$$

As it will turn out, this is not an accident but even a quite desirable feature. An example will be given and discussed below. It is worth noting that identity does hold as far as ?-free formulae are concerned.

### Fact 10

Let  $\phi$  be ?-free. Then it holds that

$$\phi \models \phi$$

*Sketch of proof:* The semantics of ULQA can equivalently be redefined in such a way that formulae denote updates over partitions. Under this perspective, ?-free formulae denote updates that are both eliminative and distributive (cf. Groenendijk and Stokhof 1990). Hence there is a static interpretation to this fragment such that updating is just intersecting the state with the static meaning. The fact follows then from the idempotence of set intersection.  $\dashv$

For the present discussion, it is more important that we are not enabled to infer from a certain update to its static closure, even if identity holds for this update.

**Fact 11**

There are formulae  $\phi$  such that

$$\phi \models \phi \tag{1}$$

$$\phi \not\models \uparrow\downarrow \phi \tag{2}$$

*Proof :* Suppose  $\phi = \neg\psi$ , let  $\psi$  be a ?-free closed atomic formula such that  $\|\psi\|^{PL1} \neq \emptyset$ ,  $\|\psi\|^{PL1} \subset W$ . (1) follows immediately from Fact 10. For (2), observe that  $\mathbf{1}[\neg\psi] = \mathbf{1}$ ,  $\mathbf{1}[\uparrow\downarrow \neg\psi] = \|\neg\psi\|^{PL1}$ . Hence  $\mathbf{1}[\neg\psi] \neq \mathbf{1}[\neg\psi][\uparrow\downarrow \neg\psi] \dashv$

This implies that the context-free meaning of a certain formula may be logically stronger than its context-dependent version. A quite realistic example is

- (15) a. Only Socrates is wise.  
 b.  $\forall x.(wise! \rightarrow x = s!) \not\models \uparrow\downarrow \forall x.(wise! \rightarrow x = s!)$

The static closure of *Only Socrates is wise* means that Socrates is literally the only wise individual, and this of course cannot be inferred from the utterance of the sentence in a particular context, as (3) shows.

Neither can we infer from a universally quantified formula to the static closure of some instance.<sup>4</sup>

**Fact 12**

There are formulae  $\phi$  and individual constants  $a$  such that

$$\forall x.\phi \models \phi(a)$$

$$\forall x.\phi \not\models \uparrow\downarrow \phi(a)$$

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4. By  $\phi(a)$  we refer to  $\phi[a/x]$ , provided that there are no variables besides  $x$  free in  $\phi$ .

*Proof* : Let  $\phi$  be as in the proof of Fact 11. Since  $\phi$  is closed, the fact follows immediately from Fact 11.  $\dashv$

This is again a desired result, since we don't want to draw the conclusion *Zeus is not wise* ( $\Leftarrow$  *If Zeus is wise, then he is Socrates*) from *Only Socrates is wise* under all circumstances. Nevertheless there is a restricted version of the mentioned kind of universal instantiation.

### Fact 13

For all ?-free formulae  $\phi, \psi$  and individual constants  $a$ , it holds that:

$$\begin{aligned} \text{If } \psi &\models \uparrow\downarrow \psi \\ \text{then } ?\phi(a) \wedge \forall x(\phi \rightarrow \psi) &\models \uparrow\downarrow (\phi(a) \rightarrow \psi(a)) \end{aligned}$$

*Sketch of proof:*  $P_{\sigma[?\phi(a)]}$  contains only propositions that either entail  $\|\phi(a)\|^{PL1}$  or  $\|\neg\phi(a)\|^{PL1}$ . Among those that make  $\phi(a)$  classically true, only those can survive in  $P_{\sigma[?\phi(a) \wedge \forall x(\phi \rightarrow \psi)]}$  that survive under updating with  $\psi(a)$ . The premise ensures that these propositions are contained in  $\|\psi(a)\|^{PL1}$ . Hence if a proposition in  $P_{\sigma[?\phi(a) \wedge \forall x(\phi \rightarrow \psi)]}$  classically entails  $\phi(a)$ , it also entails  $\psi(a)$ . The conclusion follows by Fact 8 (2). $\dashv$ .

Applied to the example, it follows that we may infer from the utterance of *Only Socrates is wise* to, say, *Plato is unwise* provided that Plato's wisdom is **under debate** in the present state of conversation.

$$(16) \quad ?wise(p) \wedge \forall x(wise(x) \rightarrow x = s) \models \uparrow\downarrow (wise(p) \rightarrow p = s)$$

To put it another way round, besides the syntactically present restrictor to a universal quantifier, we have the implicit restriction that an individuals being an instance of the restrictor has to be under debate at the current state of conversation. Let me briefly explain why ULQA behaves in this way. A partition corresponding to a state is a collection of sets of worlds, i.e. total first-order interpretation functions. A set of interpretation functions can be identified with a partial function. If all total functions in the set agree about the value of a certain item  $x$ , the derived functions assigns this value to  $x$ , too. If  $x$  receives different values under different functions in the set, its value under the corresponding partial function is undefined. Hence an information state can be seen as a set of situations, i.e. partial first-order models. Asking a question in a certain state extends the domain of the situations in the state. The interpretation of  $\phi$  is defined in any situation in  $\sigma[?\phi]$ , no matter whether it was defined in  $\sigma$ . Hence  $\sigma \models ?\phi$  just if  $\phi$  is completely defined in  $\sigma$ .

This in mind, it becomes clear why identity should not hold in ULQA. Suppose that an atomic formula  $\phi$  is undefined in any situation in a state  $\sigma$ . Then  $\sigma[\phi] = \mathbf{0}$  and  $\sigma[\neg\phi] = \sigma$ . But updating  $\sigma$  with  $?\phi$  brings us in a state where  $\phi$  is defined. Now suppose that  $\phi$  expresses a proposition contingent in  $\sigma$ . In this situation,  $\sigma[?\phi \wedge \neg\phi] \neq \sigma[?\phi]$ . Hence  $\neg\phi \wedge ?\phi \not\models \neg\phi \wedge ?\phi$ .

A certain individual only falls in the domain of a restricted universal quantifier if it unequivocally either is an instance of the restrictor or it is not. Hence we have an implicit restriction of a quantifier domain given by definedness.

Now let us return to the example. *Only Socrates is wise* makes a statement about those objects whose wisdom is under debate. Asking the question *Which Athenians are wise?* brings you in a state where the wisdom of all individuals that are known to be Athenians is under debate. Let us make this slightly more precise. Firstly, we have a restricted version of Modus Ponens together with universal instantiation in ULQA. To give the restrictions precisely, we firstly need an auxiliary definition.

**Definition 13 (Persistence)**

A formula  $\phi$  is said to be *persistent* iff for all states  $\sigma, \tau$ :

$$\sigma \models \phi, \sigma \leq \tau \Rightarrow \tau \models \phi$$

Note that by the definitions, any formula prefixed with “?” is persistent.

**Fact 14**

For all formulae  $\phi, \psi$  and individual constants  $a$ , it holds that:

$$\begin{aligned} &\text{If } \phi \models \phi, \psi \models \psi \text{ and } \psi(a) \text{ is persistent, then} \\ &\phi(a) \wedge \forall x(\phi \rightarrow \psi) \models \psi(a) \end{aligned}$$

*Sketch of proof:* Suppose that the premises hold for  $\phi$  and  $\psi$ . By the semantics of  $\forall$ , we have  $\sigma[\phi(a) \wedge (\phi(a) \rightarrow \psi(a))] \leq \sigma[\phi(a) \wedge \forall x(\phi \rightarrow \psi)]$ . From  $\phi \models \phi$ , we have  $\phi(a) \models \phi(a)$ , and together with the definition of dynamic implication, we then have  $\sigma[\phi(a) \wedge (\phi(a) \rightarrow \psi(a))] = \sigma[\phi(a) \wedge \psi(a)]$ . Hence we have  $\sigma[\phi(a) \wedge \psi(a)] \leq \sigma[\phi(a) \wedge \forall x(\phi \rightarrow \psi)]$ . From  $\psi \models \psi$ , we infer  $\psi(a) \models \psi(a)$ . Hence we have  $\sigma[\phi(a) \wedge \psi(a)] \models \psi(a)$ . By the persistence of  $\psi(a)$ , we have  $\sigma[\phi(a) \wedge \forall x(\phi \rightarrow \psi)] \models \psi(a)$ .

If we know that a person, say, Plato is an Athenian, after asking the question *Which Athenians are wise?* we are in a state where Plato’s wisdom is under debate.



$$(17) \text{ athenian}(p) \wedge \forall x.(\text{athenian}(x) \rightarrow ?\text{wise}(x)) \models ?\text{wise}(p)$$

Now from (17) and (16) we may conclude:

- (18) a. Which Athenians are wise? Only Socrates is wise.  
 b.  $\text{athenian}(p) \wedge \forall x(\text{athenian}(x) \rightarrow ?\text{wise}(x)) \wedge \forall x(\text{wise}(x) \rightarrow x = s)$

$$\models \uparrow\downarrow (\text{wise}(p) \rightarrow p = s)$$

Now suppose that a person, for instance Zeus, is neither known to be an Athenian nor to be wise in any of the epistemic alternatives in a certain state  $\sigma$  (formally:  $\sigma \models \neg\text{athenian}(z)$ ,  $\sigma \models \neg\text{wise}(z)$ ) In this case, there is nothing that can be inferred about Zeus' wisdom from the question-answer pair above. Hence the domain of the universal quantification introduced by *only* is in fact restricted to Athenians in this context.

To analyse *Who is wise?*, please observe that  $\forall x.?\text{wise}(x)$  is synonymous to  $\forall x(x = x \rightarrow ?\text{wise}(x))$ . Hence after asking that question, anybody's wisdom is under debate, and therefore we do not have any domain restriction at all in the subsequent statement.

$$(19) \forall x.?\text{wise}(x) \models ?\text{wise}(a) \text{ (where } a \text{ is an arbitrary individual constant)}$$

From (19) and (16), we may conclude:

- (20) a. Who is wise? Only Socrates is wise.  
 b.  $\forall x.?\text{wise}(x) \wedge \forall x(\text{wise}(x) \rightarrow x = s) \models \uparrow\downarrow (\text{wise}(z) \rightarrow z = s)$

## 6 Summary

The implicit universal quantification introduced by *only*-constructions is restricted by contextual information. This fact becomes most obvious in question-answer pairs. According to Rooth 1992, these constructions nevertheless involve classical universal quantification which is restricted by a syntactically present anaphor  $\Gamma$ . The interpretation of  $\Gamma$  is governed by some pragmatic mechanism. It was the aim of this paper to achieve the same result in a purely semantic manner. Firstly, to deal with context dependency semantically, we are forced to use a dynamic setup. Secondly, we changed the semantics of universal quantification in such a way that it is restricted implicitly by the context. Suppose a universal quantifier is syntactically restricted by some predicate  $P$ . In this case, the actual domain of quantification is not the entire universe but the set of individuals whose being  $P$

is under debate in the current state of conversation, and it is the purpose of questions to bring issues into the debate. Hence the dependency between questions and focus is a rather indirect one under the present approach.

There are plenty of questions left that require further investigations, concerning both the logic developed here and its linguistic application. As I mentioned in the preceding section, the semantics of ULQA makes use of a kind of simulated partiality. It strikes me an interesting issue to see what happens if we replace sets of propositions by sets of situations. Besides this, it is worth investigating whether it is possible to incorporate ULQA in a static logic with program modalities in a similar manner as presented in van Eijck and de Fries 1995 and van Eijck 1993 for Update Logic and Dynamic Predicate Logic. Linguistically, the coverage should be extended to other phenomena like the semantics of indirect questions of *wh*-pronouns.

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