Anaphora and Ellipsis in Type-Logical Grammar

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1 Introduction

The aim of the present paper is to outline a unified account of anaphoricity and ellipsis phenomena within the framework of Type Logical Categorial Grammar.[1] There is at least one conceptual and one empirical reason to pursue such a goal. Firstly, both phenomena are characterized by the fact that they re-use semantic resources that are also used elsewhere. Secondly, they show a striking similarity in displaying the characteristic ambiguity between strict and sloppy readings. This supports the assumption that in fact the same mechanisms are at work in both cases.

(1) a. John washed his car, and Bill did, too.
    b. John washed his car, and Bill waxed it.

In (1a), the second conjunct can mean that Bill washed Bill's car or that he washed John's car. Similarly, (1b) is ambiguous between a strict reading where Bill waxed John's and a sloppy/lazy one where he waxed his own car.

2 Structural Rules and Semantic Composition

One of the attractive features of Type-Logical Grammar is its commitment to a very strict correspondence between syntactic and semantic composition. Both are two sides of the same coin rather than two independent modules. Hence the decision for a particular syntax logic automatically restricts the possible ways of semantic combination. An instance of this tight connection is the fact that in grammars based on the associative Lambek calculus **L** (c.f. [7]) the meaning of a complex sign can be represented by a term of the typed λ-calculus where each meaning representation of a lexical item involved occurs once, and every λ-operator binds exactly one variable.
This seems to be too restrictive if we turn our attention to anaphora and ellipsis. Consider a simple elliptic sentence like

(2) a. John walked, and Bill did, too
    b. $and'(walk'b)(walk'j)$

In its semantic representation, the meaning of the VP in the first conjunct occurs twice. There are several ways to deal with this fact. The source of this meaning duplication could be located in the lexical entry of *did*. For pronouns, this has been proposed in [11]. However, this treatment only captures bound pronouns in the sense of [10]. Since we want to maintain the option for a unified treatment of intra- and inter-sentential ellipses, this seems to be too narrow. Hence we are left with the need to allow duplicating meanings during syntactic composition.[2] In a Lambek-style grammar, this amounts to enriching **L** with the structural rules of Contraction and—since source and target of the ellipsis are not adjacent—Permutation, which results in **LPC**.

(3) $$\frac{\Gamma[(\Delta, \Pi)] \Rightarrow t : A}{\Gamma[(\Pi, \Delta)] \Rightarrow t : A}[P] \qquad \frac{\Gamma[(x : A, y : A)] \Rightarrow t : B}{\Gamma[x : A] \Rightarrow t_{[y \leftarrow x]} : B}[C]$$

[1] As introductions to this theory of grammar, the interested reader is referred to [1, 8, 9]

[2] This has been proposed for pronouns already in [6] in the framework of Combinatory Categorial Grammar. Due to the intrinsic properties of this framework, Jacobson's proposal only captures bound pronouns, too.

For the time being, we treat *did* as a VP anaphor, and we follow [6] in considering the meaning of an anaphor to be the identity function. The essential steps of the derivation of (2) are (omitting redundant bracketing):

$$\cfrac{\cfrac{\cfrac{x:\mathbf{n}, y:\mathbf{n}\setminus\mathbf{s}, z:(\mathbf{s}\setminus\mathbf{s})/\mathbf{s}, u:\mathbf{n}, v:\mathbf{n}\setminus\mathbf{s} \Rightarrow z(vu)(yx):\mathbf{s}}{x:\mathbf{n}, y:\mathbf{n}\setminus\mathbf{s}, z:(\mathbf{s}\setminus\mathbf{s})/\mathbf{s}, v:\mathbf{n}\setminus\mathbf{s}, u:\mathbf{n} \Rightarrow z(vu)(yx):\mathbf{s}}\,[P]}{x:\mathbf{n}, y:\mathbf{n}\setminus\mathbf{s}, v:\mathbf{n}\setminus\mathbf{s}, z:(\mathbf{s}\setminus\mathbf{s})/\mathbf{s}, u:\mathbf{n} \Rightarrow z(vu)(yx):\mathbf{s}}\,[P]}{x:\mathbf{n}, y:\mathbf{n}\setminus\mathbf{s}, z:(\mathbf{s}\setminus\mathbf{s})/\mathbf{s}, u:\mathbf{n} \Rightarrow z(yu)(yx):\mathbf{s}}\,[C]$$

However, the unrestricted usage of Contraction would lead to a heavy over-generation (not to mention the effect of unrestricted Permutation). For instance, *John shows Mary* would be predicted to be a grammatical sentence with the meaning of *John shows Mary herself.* Therefore we have to impose constraints on the applicability of these rules to avoid such a collapse.

3 A Multi-Modal System

Research in recent years has shown that none of the pure categorial logics (like **NL**, **L**, **LP** or **LPC**) is well-suited for a comprehensive description of natural language, each of them by itself being either too restrictive or too permissive. That's why combinations of several systems have attracted much attention. In the simplest case, such a multi-modal logic has more than one n-place product connective together with the corresponding residuation connectives. Each family is characterized by the usual logical rules and a set of characteristic structural rules. In more elaborate systems, these different modes of composition are allowed to communicate via certain *interaction postulates.* This technique can be exploited to control the availability of Contraction and Permutation in the context of anaphora and ellipsis resolution.

Besides concatenation, we propose to use a second mode of combination "$\sim$" (with corresponding residuation operations $\hookleftarrow$ and $\hookrightarrow$). Formally, we augment **L** with the usual logical rules for $\hookleftarrow, \sim$, and $\hookrightarrow$. The bracketing of structured antecedent sequences corresponding to $\bullet$ and $\sim$ are denoted by $(\ldots)$ and $\{\ldots\}$ respectively henceforth.

In (4) the structural rules for the hybrid system **LA** (**L**ambek Calculus with **A**naphora) are given. "C" allows unrestricted Contraction for $\sim$. Permutation is distributed over three rules. Since $\sim$ can be thought of as combining an index with a constituent, the first two are dubbed "IM" for Index Movement and "IP" for Index Percolation. "P" assures that the collection of indices attached to a constituent form a multiset.

(4)
$$\frac{\Gamma[\{x:A, y:A\}] \Rightarrow t:B}{\Gamma[y:A] \Rightarrow t_{[x\leftarrow y]}:B}\,[C] \qquad \frac{\Gamma[(\Delta, \{\Pi, \Theta\})] \Rightarrow t:A}{\Gamma[(\{\Pi, \Delta\}, \Theta)] \Rightarrow t:A}\,[IM]$$

$$\frac{\Gamma[(\{\Pi, \Delta\}, \Theta)] \Rightarrow t:A}{\Gamma[\{\Pi, (\Delta, \Theta)\}] \Rightarrow t:A}\,[IP] \qquad \frac{\Gamma[\{\Pi, \{\Sigma, \Delta\}\}] \Rightarrow t:A}{\Gamma[\{\Sigma, \{\Pi, \Delta\}\}] \Rightarrow t:A}\,[P]$$

LA has two desirable logical properties, namely:

Fact 1

(i) ***LA*** *enjoys Cut Elimination.*

(ii) ***LA*** *is decidable.*

Sketch of Proof: An inspection of the inference rules of **LA** shows that the Cut Elimination algorithm from [7] in its extension by [4] carries over to **LA**. Due to Contraction, this does not guarantee decidability *per se*. But decidability does hold if we have an upper bound for the number of applications of Contraction in a proof. Such an upper bound can be obtained in the following way. There is an obvious translation from **LA** to **LPC** which preserves validity and proof term assignment (but not invalidity). Since proof search space for **LPC** is finite (c.f. [12]), there is only a finite number of potential proof terms for a given **LA**-sequent. Since proof terms code the number of applications of Contraction in a proof, this gives us the desired upper bound. ⊣

4 VP Ellipsis

To illustrate the system with a simple example, take the sentence

(5) John walks, and Bill does, too.

We assume the following lexical assignment:

(6)
- John–$j : \mathbf{n}$
- Bill–$b : \mathbf{n}$
- walks–$walk' : \mathbf{n} \setminus \mathbf{s}$
- and–$and' : (\mathbf{s} \setminus \mathbf{s})/\mathbf{s}$
- does–$\lambda x.x : (\mathbf{n} \setminus \mathbf{s}) \hookrightarrow (\mathbf{n} \setminus \mathbf{s})$

Starting with an **L**-derivable sequent, we obtain the following derivation (omitting redundant bracketings and intermediate steps):

(7)

$$\dfrac{\dfrac{\dfrac{x : \mathbf{n}, y : \mathbf{n} \setminus \mathbf{s}, z : (\mathbf{s} \setminus \mathbf{s})/\mathbf{s}, w : \mathbf{n}, r : (\mathbf{n} \setminus \mathbf{s}) \Rightarrow z(rw)(ux) : \mathbf{s}}{x : \mathbf{n}, y : \mathbf{n} \setminus \mathbf{s}, z : (\mathbf{s} \setminus \mathbf{s})/\mathbf{s}, w : \mathbf{n}, \{v : \mathbf{n} \setminus \mathbf{s}, u : (\mathbf{n} \setminus \mathbf{s}) \hookrightarrow (\mathbf{n} \setminus \mathbf{s})\} \Rightarrow z(uvw)(ux) : \mathbf{s}}[\hookrightarrow L]}{x : \mathbf{n}, \{v : \mathbf{n} \setminus \mathbf{s}, y : \mathbf{n} \setminus \mathbf{s}\}, z : (\mathbf{s} \setminus \mathbf{s})/\mathbf{s}, w : \mathbf{n}, u : (\mathbf{n} \setminus \mathbf{s}) \hookrightarrow (\mathbf{n} \setminus \mathbf{s}) \Rightarrow z(uvw)(ux) : \mathbf{s}}[IM]}{x : \mathbf{n}, y : \mathbf{n} \setminus \mathbf{s}, z : (\mathbf{s} \setminus \mathbf{s})/\mathbf{s}, w : \mathbf{n}, u : (\mathbf{n} \setminus \mathbf{s}) \hookrightarrow (\mathbf{n} \setminus \mathbf{s}) \Rightarrow z(uyw)(yx) : \mathbf{s}}[C]$$

After inserting the lexical meanings, we obtain the reading $and'((\lambda x.x)walk'\ b)(walk'\ j)$, which reduces to $and'(walk'\ b)(walk'\ j)$.
The mechanism works similar in the case of nominal anaphora. If we extend our lexicon with

(8)
- washed–$wash : (\mathbf{n} \setminus \mathbf{s})/\mathbf{n}$
- his–$\lambda xy.of'\ y\ x : \mathbf{n} \hookrightarrow (\mathbf{n}/\mathbf{cn})$
- car–$car' : \mathbf{cn}$

we derive the reading (9c) for (9a), corresponding to the provable sequent (9b).

(9)
a. John washed his car.
b. $x : \mathbf{n}, y : (\mathbf{n} \setminus \mathbf{s})/\mathbf{s}, z : \mathbf{n} \hookrightarrow (\mathbf{n}/\mathbf{cn}), w : \mathbf{cn} \Rightarrow y(zxw)x : \mathbf{s}$
c. $wash'((\lambda xy.of'\ y\ x)j\ car')j\ (= wash'(of'\ car'\ j)j)$

Before we proceed to the interaction of VP ellipsis and anaphoricity, observe that (9) shows a spurious ambiguity. After (9b) is derived, we can either stop or apply the rule "\L", which gives us the sequent

(10) $y : (\mathbf{n} \setminus \mathbf{s})/\mathbf{s}, z : \mathbf{n} \hookrightarrow (\mathbf{n}/\mathbf{cn}), w : \mathbf{cn} \Rightarrow \lambda x.y(zxw)x : \mathbf{n} \setminus \mathbf{s}$

This means that it is possible to resolve the anaphor *his* against the subject argument place of *washed*, assigning the meaning $\lambda x.wash'(of'\ car'\ x)x$ to the VP *washed his car*.[3] In (9) this ambiguity is spurious since after combining this VP with the subject *John*, we end up with the meaning (9c) again.

[3] This can be seen as a reconstruction of Reinhart's ([10]) distinction between coreferential and bound pronouns.

(11) John washed his car, and Bill did, too.

In (11), on the other hand, this ambiguity, though spurious in the first conjunct, makes a difference for the interpretation of the second one. If we plug in (10) into the conclusion of (7) via the Cut rule, we immediately derive the sloppy reading of (11). This amounts to first resolving *his* against the subject argument place of *washed* and afterward resolving *did* against the VP derived in this way. If, on the other hand, *his* is resolved against *John* prior to resolution of *did*, the strict reading results.

5 Associativity?

[3] present an example of a cascaded ellipsis that allows to distinguish different ellipsis theories on a very fine-grained level.

(12) John revised his_1 paper before the teacher did [resolve his_2 paper], and Bill did [resolve his_3 paper before the teacher did resolve his_4 paper] too.

Among the six readings that are considered in [3], the present system generates four (*JJJJ, JJBB, JTJT, JTBT* as referents of the four occurrences of *he* respectively). This seems to be too few since in an appropriate contextual setting, JJBJ is a possible reading as well. The problem becomes even more obvious if we turn our attention to examples like the following (from [2]):

(13) John realizes that he is a fool, but Bill does not, even though his wife does.

An easy way to relax the constraints of the theory is to allow a lexical assignment like

(14) does–$\lambda x.x : (\mathbf{n} \hookrightarrow (\mathbf{n} \setminus \mathbf{s})) \hookrightarrow (\mathbf{n} \hookrightarrow (\mathbf{n} \setminus \mathbf{s}))$

for the first occurrence of *does*. This would enable us to resolve *it* against *realizes that he is a fool* **before** *he* is resolved. In this way, the silent *he* can be resolved independently from the overt one, yielding (among others) the desired reading.
While it seems to be *ad hoc* to assume such a lexical ambiguity for *does*, this type assignment can be derived if we add the a version of the Geach Rule to our calculus:

$$x : A \hookrightarrow B \;\Rightarrow\; \lambda yz.x(yz) : (C \hookrightarrow A) \hookrightarrow (C \hookrightarrow B)$$

Inserting the identity function (as the lexical meaning of *does*) for x gives us the semantic term $\lambda yz.yz$ for the derived category, which is equivalent to $\lambda y.y$. In terms of sequent rules, this amounts to extending **LA** to a new system, call it **LAA**, which includes the structural rule of Associativity for both modes of combination:

(15) $$\frac{\Gamma[\{\Delta, \{\Pi, \Sigma\}\}] \Rightarrow t : A}{\Gamma[\{\{\Delta, \Pi\}, \Sigma\}] \Rightarrow t : A}[\mathrm{A}\sim]$$

The decision between **LA** and **LAA** as appropriate calculus for anaphoricity and ellipsis is an empirical issue that has to be decided for each class of phenomena separately.
As far as English VP ellipsis is concerned, **LAA** predicts a very high degree of freedom. Besides the six readings for (12), it also admits readings like $JTTT$ etc. Two comments are in order here. First, something similar to $JTTT$ seems to be marginally possible indeed (judgments range from "impossible" to "perfect"):

(16) [Every bum on the streets of New York]$_j$ is more concerned about his_j safety than this crowd loving president Clinton_i is.

a. Fortunately for him_i, his_i bodyguard is too.

b. Fortunately for him_i, his_i bodyguard is more concerned about his_i safety than he_i is concerned about his_i safety.

Second, restrictions on anaphora resolution in constructions without ellipsis do not substantially differ from those with ellipsis. (17) shows exactly the same range of readings like (12).

(17) John revised his paper before the teacher revised his paper, and Bill revised his paper before the teacher revised his paper, too.

If *too* is understood as establishing a parallelism between *John* and *Bill*, we have just the same four or five readings we have in (12). This fact is well-known (see for instance [5]). One way to account for this is to assume that the deaccenting of the VPs in (17) that correspond to the elided material in (12) is the primary cause for this similarity. Ellipsis and deaccenting could be analyzed as largely two instances of the same phenomenon. Nevertheless another perspective is possible as well. The restrictions on anaphoric relationships that show up could be analyzed as consequences of the semantics/pragmatics of *too*, which simultaneously requires deaccenting of the second conjunct. This would make the differences between *and ... too*, *but*, *even though* etc. less mysterious. If such a line of research proves to be successful, this would allow a highly unrestrictive theory of ellipsis resolution like the one implied by **LAA**.

An **LAA** based account seems definitely to be preferable in the case of nominal anaphora, since this automatically captures *paycheck* pronouns.

(18) a. Bill spent his money, and John saved it.

b.
- spent–$spend'$: $(\mathbf{n} \setminus \mathbf{s})/\mathbf{n}$
- saved–$save'$: $(\mathbf{n} \setminus \mathbf{s})/\mathbf{n}$
- money–$money'$: $\mathbf{cn}$

c. $and'(save'(of'money'j)j)(spend'(of'money'b)b)$

Most importantly, *it* can get the derived category $(\mathbf{n} \hookrightarrow \mathbf{n}) \hookrightarrow (\mathbf{n} \hookrightarrow \mathbf{n})$, again with the interpretation as identity functions (over Skolem functions). Hence *his money* with the pronoun still unresolved (which denotes the Skolem function from individuals to their cars) can serve as antecedent for *it*.

In the case of stripping, **LA** seems to be the appropriate logic, although judgments are somewhat fuzzy here. In (19a) all contextual factors favor a mixed sloppy/strict reading (as indicated in (19b)), which is nevertheless only very marginally possible.

(19) a. Every candidate believes that he can win, even Smith, but not his wife.

b. Every candidate believes that he can win, even Smith believes that he can win, but his wife does not believe that Smith can win.

6 Conclusion

In this paper, I have outlined a theory of anaphoricity and ellipsis which shows some desirable properties from a conceptual point of view:

- The semantics is fully compositional. As a consequence, there is no need for a level of Logical Form where ellipsis resolution takes place. Since ellipsis phenomena are usually considered to be a strong indication for the presence of LF, this might have consequences for grammar architecture as a whole. Neither does the theory presented here crucially depend on the typed λ-calculus as a semantic representation language. That it has been used throughout the paper is merely a matter of convenience; everything could be reformulated in terms of set theory or Combinatory Logic without loss of generality.

- The theory is variable free. This removes a great deal of arbitrariness from semantic derivations. In traditional theories, anaphora and ellipses are translated as variables (i.e. they denote functions from assignment functions to objects of the appropriate type). Since there are infinitely many variables, one and the same pronoun is predicted to be infinitely ambiguous. Though this is compatible with the letter of the Principle of Compositionality, it is clearly against its spirit, since identical expressions with identical syntactic structure should have identical denotations. Here, resolution ambiguities are treated as structural ambiguities, corresponding to essentially different proofs of the same sequent.

- The theory is modular. Anaphora and ellipsis resolution take place at the syntax-semantics interface, without reference to pragmatics. Since both phenomena are subject to syntactic and semantic constraints, the syntax-semantics interface seems to be the natural place to deal with them. This does not exclude that some of the readings generated there are filtered out later on pragmatic grounds. This is not surprising since the same holds for other sources of structural ambiguities like quantifier scope or focus projection.

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