The different flavors of the Iterated Best Response Model of game theoretic pragmatics

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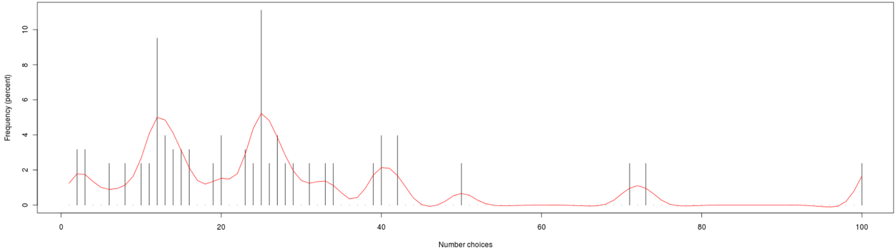
The Beauty Contest

- each participant has to write down a number between 0 and 100
- all numbers are collected
- the person whose guess is closest to $2/3$ of the arithmetic mean of all numbers submitted is the winner

The Beauty Contest



(data from Camerer 2003, *Behavioral Game Theory*)

Signaling games

- sequential game:
  1. **nature** chooses a world w
     - out of a pool of possible worlds W
     - according to a certain probability distribution p^*
  2. nature shows w to sender **S**
  3. S chooses a message m out of a set of possible signals M
  4. S transmits m to the receiver **R**
  5. R chooses an action a, based on the sent message.
- Both S and R have preferences regarding R's action, depending on w.
- S might also have preferences regarding the choice of m (to minimize signaling costs).

Tea or coffee?

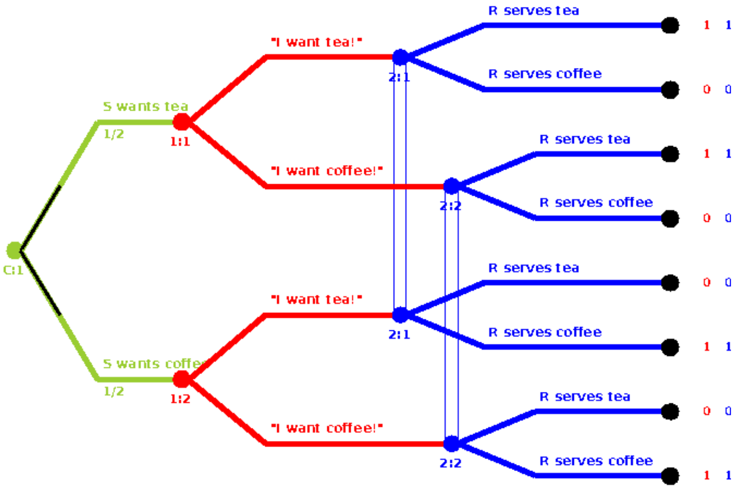
An example

- Sally either prefers tea (w_1) or coffee (w_2), with $p^*(w_1) = p^*(w_2) = 1/2$.
- Robin either serves tea (a_1) or coffee (a_2).
- Sally can send either of two messages:
  - m_1: *I prefer tea.*
  - m_2: *I prefer coffee.*
- Both messages are costless.

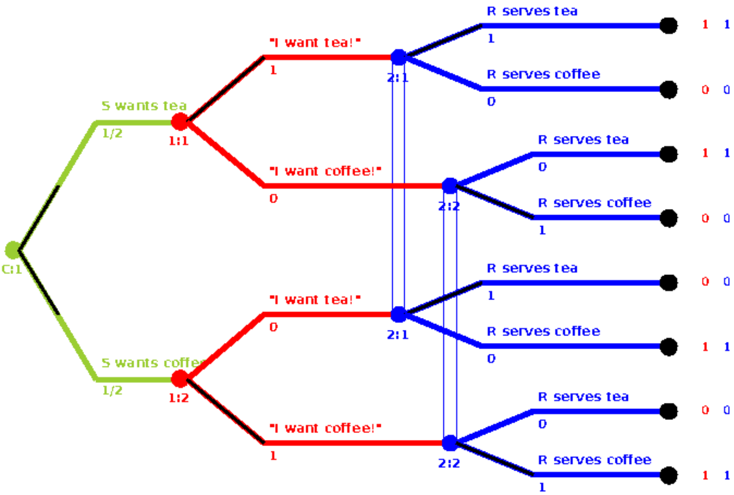
	a_1	a_2
w_1	$1, 1$	$0, 0$
w_2	$0, 0$	$1, 1$

Table: utility matrix

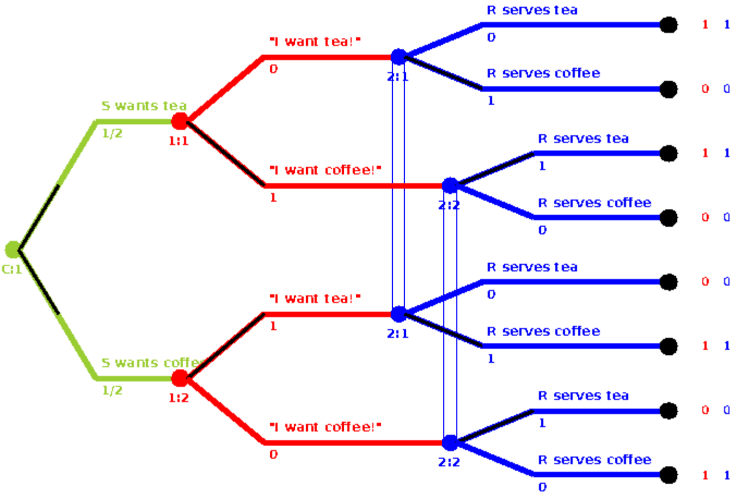
Extensive form



Extensive form



Extensive form



A coordination problem

- two strict Nash equilibria
  - S always says the truth and R always believes her.
  - S always says the opposite of the truth and R interprets everything ironically.
- Both equilibria are equally rational.
- Still, first equilibrium is more reasonable because it employs exogenous meanings of messages for equilibrium selection.
- Criterion for equilibrium selection:

> *As a default, S and R use/interpret signals according to their literal meaning. They only deviate from this if there self-interest dictates them to do so.*

- What exactly does this mean?

The Iterated Best Response sequence

sends any true message → S_0

interprets messages literally → R_0

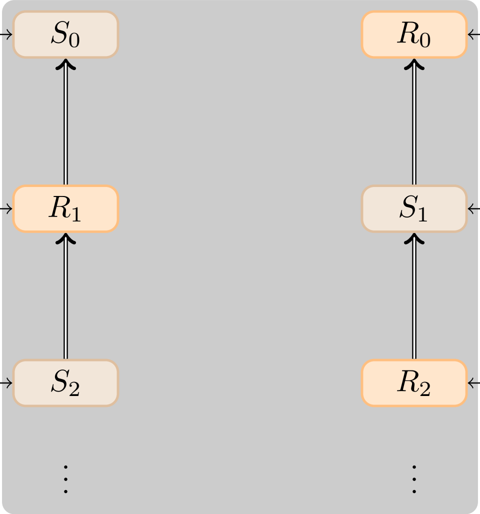
best response to S_0 → R_1

best response to R_0 → S_1

best response to R_1 → S_2

best response to S_1 → R_2

⋮



Interpretation games

- How does this relate to linguistic examples?
- There is a quasi-algorithmic procedure (due to Franke 2009) how to construct a game from an example sentence.

What is given?

- example sentence
- set of expression alternatives
- jointly form set of messages
- question under discussion QUD
- set of complete answers to QUD is the set of possible worlds

What do we need?

- interpretation function $\|\cdot\|$
- prior probability distribution p^*
- set of actions
- utility functions

Interpretation games

QUD

- often QUD is not given explicitly
- procedure to construct QUD from expression m and its alternatives $ALT(m)$:
  - Let ct be the context of utterances, i.e. the maximal set of statements that is common knowledge between Sally and Robin.
  - any subset w of $ALT(m) \cup \{\neg m' | m' \in ALT(m)\}$ is a possible world iff
    - w and ct are consistent, i.e. $w \cup ct \nvdash \bot$
    - for any set $X : w \subset X \subseteq ALT(m) \cup \{\neg m' | m' \in ALT(m)\}$, $ct \cup X$ is inconsistent

Interpretation games

Game construction

- interpretation function:

$$\|m'\| = \{w|w \vdash m\}$$

- p^* is uniform distribution over W
- justified by principle of insufficient reason
- set of actions is W
- intuitive idea: Robin's task is to figure out which world Sally is in
- utility functions:

$$u_{s/r}(w,a) = \begin{cases} 1 & \text{iff } w = a \\ 0 & \text{else} \end{cases}$$

- both players want Robin to succeed

Quantity implicatures

(1) a. Who came to the party?
b. SOME: Some boys came to the party.
c. ALL: All boys came to the party.

Game construction

- $ct = \emptyset$
- $W = \{w_{\exists\neg\forall}, w_{\forall}\}$
- $w_{\exists\neg\forall} = \{\text{SOME}\}, w_{\forall} = \{\text{SOME}, \text{ALL}\}$
- $p^* = (1/2, 1/2)$

- interpretation function:

$$\|\text{SOME}\| = \{w_{\exists\neg\forall}, w_{\forall}\}$$
$$\|\text{ALL}\| = \{w_{\forall}\}$$

- utilities:

	$a_{\exists\neg\forall}$	$a_{\forall}$
$w_{\exists\neg\forall}$	$1, 1$	$0, 0$
$w_{\forall}$	$0, 0$	$1, 1$

The recursion base

- IBR style reasing requires a default assumption that
  - reflects the literal meanings of the messages involved
  - is not the result of higher-order rational reasoning

Honest sender S_0

- in each type, Sally utters a true message
- does not involve a model of how Robin interprets messages

Credulous receiver R_0

- whatever messages Robin observes, he takes it to be true
- does not involve a model of how Sally uses messages

- both S_0 and R_0 are *sets* of (pure) strategies that need not be singleton (due to synonymy/homonymy)

Recursion bases for some-all game

- literal meanings:

	SOME	ALL
$w_{\exists\neg\forall}$	1	0
$w_{\forall}$	1	1

- default strategies

S_0	SOME	ALL
$w_{\exists\neg\forall}$	1	0
$w_{\forall}$	α	$1-\alpha$

R_0	$w_{\exists\neg\forall}$	$w_{\forall}$
SOME	β	$1-\beta$
ALL	0	1

$$\alpha, \beta \in \{0, 1\}$$

What does “best response” mean?

- suppose a player knows that their opponent some strategy out of a set A of strategies
- no *a priori* information about how the opponent selects elements of A
- two approaches to model rational behavior:
  - Franke (2009, 2011): a rational response to A is any strategy that maximizes utility against the **uniform** distribution over A
  - Jger (2013): a rational response to A is any strategy that maximizes utility against **some proper** distribution over A
- Terminology:
  - Franke-style rationality is called **best response** (as a technical term)
  - Jäger-style rationality (borrowed from Pearce 1984) is called **cautious response**

Some-all game again

S_0	SOME	ALL
$w_{\exists\neg\forall}$	1	0
$w_{\forall}$	α	$1-\alpha$

R_0	$w_{\exists\neg\forall}$	$w_{\forall}$
SOME	β	$1-\beta$
ALL	0	1

- **IBR** (Iterated Best Response *sensu strictu*; Franke-style rationality): $\alpha = \beta = 1/2$
- **ICR** (Iterated Cautious Response; Jäger-style rationality): $\alpha, \beta \in (0, 1)$
- in the running example, predictions coincide:

S_1	SOME	ALL
$w_{\exists\neg\forall}$	1	0
$w_{\forall}$	0	1

R_1	$w_{\exists\neg\forall}$	$w_{\forall}$
SOME	1	0
ALL	0	1

- all four theory variants (S-based vs. R-based, IBR vs. ICR) correctly predict quantity implicature

Biased some-all game

- Rothschild (2013): IBR makes wrong predictions when prior probabilities are non-uniform
- for instance: $p^*(w_{\exists\neg\forall}) = {}^1\!/_{10}, p^*(w_\forall) = {}^9\!/_{10}$

S_0	p^*	SOME	ALL
$w_{\exists\neg\forall}$	$1/10$	1	0
$w_\forall$	$9/10$	α	$1-\alpha$

R_0	$w_{\exists\neg\forall}$	$w_\forall$
SOME	β	$1-\beta$
ALL	0	1

Biased some-all game; IBR

S_0	p^*	SOME	ALL
$w_{\exists\neg\forall}$	$1/10$	1	0
$w_{\forall}$	$9/10$	$1/2$	$1/2$

R_0	$w_{\exists\neg\forall}$	$w_{\forall}$
SOME	$1/2$	$1/2$
ALL	0	1

R_1	$w_{\exists\neg\forall}$	$w_{\forall}$
SOME	0	1
ALL	0	1

S_1	p^*	SOME	ALL
$w_{\exists\neg\forall}$	$1/10$	1	0
$w_{\forall}$	$9/10$	0	1

S_2	p^*	SOME	ALL
$w_{\exists\neg\forall}$	$1/10$	1	0
$w_{\forall}$	$9/10$	$1/2$	$1/2$

R_2	$w_{\exists\neg\forall}$	$w_{\forall}$
SOME	1	0
ALL	0	1

$R_3 = R_1$

$S_3 = S_1$

Biased some-all game; ICR

S_0	p^*	SOME	ALL
$w_{\exists\neg\forall}$	$1/10$	1	0
$w_\forall$	$9/10$	α	$1-\alpha$

R_0	$w_{\exists\neg\forall}$	$w_\forall$
SOME	γ	$1-\gamma$
ALL	0	1

R_1	$w_{\exists\neg\forall}$	$w_\forall$
SOME	β	$1-\beta$
ALL	0	1

S_1	p^*	SOME	ALL
$w_{\exists\neg\forall}$	$1/10$	1	0
$w_\forall$	$9/10$	0	1

S_2	p^*	SOME	ALL
$w_{\exists\neg\forall}$	$1/10$	1	0
$w_\forall$	$9/10$	0	1

R_2	$w_{\exists\neg\forall}$	$w_\forall$
SOME	1	0
ALL	0	1

R_3	$w_{\exists\neg\forall}$	$w_\forall$
SOME	1	0
ALL	0	1

$S_3 = S_1$

$S_4 = S_2$

Biased some-all game; summary

- prior preference for $w_\forall$ is a problem for S-based IBR
- the other three theory variants get the right result

Some but not all

- extension of the previous game with additional message

  (1) **d.** SBNA: Some but not all boys came to the party

- SBNA is only true in $w_{\exists\neg\forall}$ and incurs a nominal cost ($^1/_{100}$, for concreteness sake)

	p^*	SOME	ALL	SBNA
$w_{\exists\neg\forall}$	$^1/_2$	1	0	1
$w_{\forall}$	$^1/_2$	1	1	0
c		0	0	$^1/_{100}$

SBNA-game; IBR

S_0	SOME	ALL	SBNA
$w_{\exists\neg\forall}$	1	0	0
$w_{\forall}$	$1/2$	$1/2$	0

R_0	$w_{\exists\neg\forall}$	$w_{\forall}$	c
SOME	$1/2$	$1/2$	0
ALL	0	1	0
SBNA	1	0	$1/100$

R_1	$w_{\exists\neg\forall}$	$w_{\forall}$	c
SOME	1	0	0
ALL	0	1	0
SBNA	1	0	$1/100$

S_2	SOME	ALL	SBNA
$w_{\exists\neg\forall}$	0	0	1
$w_{\forall}$	0	1	0

S_3	SOME	ALL	SBNA
$w_{\exists\neg\forall}$	1	0	0
$w_{\forall}$	0	1	0

R_3	$w_{\exists\neg\forall}$	$w_{\forall}$	c
SOME	$1/2$	$1/2$	0
ALL	0	1	0
SBNA	1	0	$1/100$

SBNA-game; ICR

S_0	SOME	ALL	SBNA
$w_{\exists\neg\forall}$	1	0	0
$w_{\forall}$	α	$1-\alpha$	0

R_0	$w_{\exists\neg\forall}$	$w_{\forall}$	c
SOME	β_1	$1-\beta_1$	0
ALL	0	1	0
SBNA	1	0	$1/100$

R_1	$w_{\exists\neg\forall}$	$w_{\forall}$	c
SOME	1	0	0
ALL	0	1	0
SBNA	1	0	$1/100$

S_1	SOME	ALL	SBNA
$w_{\exists\neg\forall}$	β_2	0	$1-\beta_2$
$w_{\forall}$	0	1	0

S_2	SOME	ALL	SBNA
$w_{\exists\neg\forall}$	1	0	0
$w_{\forall}$	0	1	0

$R_2 = R_1$

SBNA-game; summary

- IBR/R: no scalar implicature for SOME
- IBR/S; ICR: same scalar implicature as in simple some-all game
- simple-minded responses to surprise messages
- in line with general spirit of IBR vs. ICR:
  - IBR: surprise message induces belief in uniform distribution over the set of worlds where the message is true
  - ICR: surprise message induces belief in some proper probability distribution over the set of worlds where the message is true
- Franke (2009, 2011) uses more sophisticated form of forward induction; correctly predicts scalar implicature for both IBR sequences

Lifted SBNA-game with competence assumption

- Pavan (2013) notices a problem with Jäger's (2011) version of ICR
- main difference to Jäger (2013): improper probability distributions are admitted
- Pavan's example:
    - SBNA-game
    - types are *information states*, i.e. sets of possible worlds:

$$\begin{aligned} t_{\exists\neg\forall} &= \{w_{\exists\neg\forall}\} \\ t_{\forall} &= \{w_{\forall}\} \\ t_{\exists} &= \{w_{\exists\neg\forall}, w_{\forall}\} \end{aligned}$$

    - *competence assumption:* $p^*(t_\exists) = 0$

Lifted SBNA-game with competence assumption

	p^*	SOME	ALL	SBNA
$t_{\exists\neg\forall}$	$1/2$	1	0	1
$t_{\forall}$	$1/2$	1	1	0
$t_{\exists}$	0	1	0	0
c		0	0	$1/100$

Lifted SBNA-game with competence assumption; IBR

S_0	p^*	SOME	ALL	SBNA
$t_{\exists\neg\forall}$	$1/2$	1	0	0
$t_\forall$	$1/2$	$1/2$	$1/2$	0
$t_\exists$	0	1	0	0

R_0	$t_{\exists\neg\forall}$	$t_\forall$	$t_\exists$	c
SOME	$1/2$	$1/2$	0	0
ALL	0	1	0	0
SBNA	1	0	0	$1/100$

R_1	$t_{\exists\neg\forall}$	$t_\forall$	$t_\exists$	c
SOME	1	0	0	0
ALL	0	1	0	0
SBNA	1	0	0	$1/100$

S_1	p^*	SOME	ALL	SBNA
$t_{\exists\neg\forall}$	$1/2$	0	0	1
$t_\forall$	$1/2$	0	1	0
$t_\exists$	0	1	0	0

S_2	p^*	SOME	ALL	SBNA
$t_{\exists\neg\forall}$	$1/2$	1	0	0
$t_\forall$	$1/2$	0	1	0
$t_\exists$	0	1	0	0

R_2	$t_{\exists\neg\forall}$	$t_\forall$	$t_\exists$	c
SOME	$1/3$	$1/3$	$1/3$	0
ALL	0	1	0	0
SBNA	1	0	0	$1/100$

$R_3 = R_1$

$S_3 = S_1$

Lifted SBNA-game with competence assumption; ICR

S_0	p^*	SOME	ALL	SBNA
$t_{\exists\neg\forall}$	$1/2$	1	0	0
$t_{\forall}$	$1/2$	α_1	$1-\alpha_1$	0
$t_{\exists}$	0	1	0	0

R_0	$t_{\exists\neg\forall}$	$t_{\forall}$	$t_{\exists}$	c
SOME	β_1	$1-\beta_1$	0	0
ALL	0	1	0	0
SBNA	1	0	0	$1/100$

R_1	$t_{\exists\neg\forall}$	$t_{\forall}$	$t_{\exists}$	c
SOME	1	0	0	0
ALL	0	1	0	0
SBNA	1	0	0	$1/100$

S_1	p^*	SOME	ALL	SBNA
$t_{\exists\neg\forall}$	$1/2$	α_2	0	$1-\alpha_2$
$t_{\forall}$	$1/2$	0	1	0
$t_{\exists}$	0	1	0	0

S_2	p^*	SOME	ALL	SBNA
$t_{\exists\neg\forall}$	$1/2$	1	0	0
$t_{\forall}$	$1/2$	0	1	0
$t_{\exists}$	0	1	0	0

R_2	$t_{\exists\neg\forall}$	$t_{\forall}$	$t_{\exists}$	c
SOME	1	0	0	0
ALL	0	1	0	0
SBNA	1	0	0	$1/100$

$R_3 = R_1$

$S_3 = S_2$

Lifted SBNA-game with competence assumption; summary

- lifted SBNA-game + competence behaves as simple sbna-game
- scalar implicatures is prediced by IBR/S and ICR, but not by IBR/R
- latter problem can be fixed by means of sophisticated forward induction

Horn-game

	p^*	f_1	f_2
w_1	$^2/_3$	1	1
w_2	$^1/_3$	1	1
c		0	$^1/_{100}$

Horn-game: IBR

S_0	p^*	f_1	f_2
w_1	$2/3$	1	0
w_2	$1/3$	1	0

R_0	w_1	w_2	c
f_1	1	0	0
f_2	1	0	$1/100$

R_1	w_1	w_2	c
f_1	1	0	0
f_2	$1/2$	$1/2$	$1/100$

S_1	p^*	f_1	f_2
w_1	$2/3$	1	0
w_2	$1/3$	1	0

S_2	p^*	f_1	f_2
w_1	$2/3$	1	0
w_2	$1/3$	0	1

R_2	w_1	w_2	c
f_1	1	0	0
f_2	$1/2$	$1/2$	$1/100$

R_3	w_1	w_2	c
f_1	1	0	0
f_2	0	1	$1/100$

S_3	p^*	f_1	f_2
w_1	$2/3$	1	0
w_2	$1/3$	0	1

Horn-game: ICR

S_0	p^*	f_1	f_2
w_1	$2/3$	1	0
w_2	$1/3$	1	0

R_0	w_1	w_2	c
f_1	1	0	0
f_2	1	0	$1/100$

R_1	w_1	w_2	c
f_1	1	0	0
f_2	α	$1-\alpha$	$1/100$

S_1	p^*	f_1	f_2
w_1	$2/3$	1	0
w_2	$1/3$	1	0

S_2	p^*	f_1	f_2
w_1	$2/3$	1	0
w_2	$1/3$	0	1

R_2	w_1	w_2	c
f_1	1	0	0
f_2	β	$1-\beta$	$1/100$

R_3	w_1	w_2	c
f_1	1	0	0
f_2	0	1	$1/100$

S_3	p^*	f_1	f_2
w_1	$2/3$	1	0
w_2	$1/3$	0	1

Horn game: summary

- Horn's division of pragmatic labor (M-implicature) is predicted both by IBR and ICR

Generalized Horn game

	p^*	f_1	f_2	f_3
w_1	$^3/_6$	1	1	1
w_2	$^2/_6$	1	1	1
w_2	$^1/_6$	1	1	1
c		0	$^1/_{100}$	$^2/_{100}$

Generalized Horn: IBR

S_0	p^*	f_1	f_2	f_3
w_1	$3/6$	1	0	0
w_2	$2/6$	1	0	0
w_2	$1/6$	1	0	0

R_0	w_1	w_2	w_3	c
f_1	1	0	0	0
f_2	1	0	0	$1/100$
f_2	1	0	0	$2/100$

R_1	w_1	w_2	w_3	c
f_1	1	0	0	0
f_2	$1/3$	$1/3$	$1/3$	$1/100$
f_2	$1/3$	$1/3$	$1/3$	$2/100$

S_1	p^*	f_1	f_2	f_3
w_1	$3/6$	1	0	0
w_2	$2/6$	1	0	0
w_2	$1/6$	1	0	0

S_2	p^*	f_1	f_2	f_3
w_1	$3/6$	1	0	0
w_2	$2/6$	0	1	0
w_2	$1/6$	0	1	0

R_2	w_1	w_2	w_3	c
f_1	1	0	0	0
f_2	$1/3$	$1/3$	$1/3$	$1/100$
f_2	$1/3$	$1/3$	$1/3$	$2/100$

Generalized Horn: IBR (cont.)

S_2	p^*	f_1	f_2	f_3
w_1	$3/6$	1	0	0
w_2	$2/6$	0	1	0
w_2	$1/6$	0	1	0

R_2	w_1	w_2	w_3	c
f_1	1	0	0	0
f_2	$1/3$	$1/3$	$1/3$	$1/100$
f_2	$1/3$	$1/3$	$1/3$	$2/100$

R_3	w_1	w_2	w_3	c
f_1	1	0	0	0
f_2	0	1	0	$1/100$
f_2	$1/3$	$1/3$	$1/3$	$2/100$

S_3	p^*	f_1	f_2	f_3
w_1	$3/6$	1	0	0
w_2	$2/6$	0	1	0
w_2	$1/6$	0	1	0

S_4	p^*	f_1	f_2	f_3
w_1	$3/6$	1	0	0
w_2	$2/6$	0	1	0
w_2	$1/6$	0	0	1

$R_4 = R_3$

Generalized Horn: ICR

S_0	p^*	f_1	f_2	f_3
w_1	$3/6$	1	0	0
w_2	$2/6$	1	0	0
w_2	$1/6$	1	0	0

R_0	w_1	w_2	w_3	c
f_1	1	0	0	0
f_2	1	0	0	$1/100$
f_2	1	0	0	$2/100$

R_1	w_1	w_2	w_3	c
f_1	1	0	0	0
f_2	x_1	y_1	$1 - x_1 - y_1$	$1/100$
f_2	x_2	y_2	$1 - x_2 - y_2$	$2/100$

$S_1 = S_0$

S_2	p^*	f_1	f_2	f_3
w_1	$3/6$	1	0	0
w_2	$2/6$	0	x	$1 - x$
w_2	$1/6$	0	y	$1 - y$

$R_2 = R_1$

Generalized Horn: ICR (cont.)

S_2	p^*	f_1	f_2	f_3
w_1	$3/6$	1	0	0
w_2	$2/6$	0	x	$1-x$
w_2	$1/6$	0	y	$1-y$

$R_2 = R_1$

R_3	w_1	w_2	w_3	c
f_1	1	0	0	0
f_2	0	x	$1-x$	$1/100$
f_2	0	y	$1-y$	$2/100$

$S_3 = S_2$

$S_4 = S_3$

$R_4 = R_3$

Generalized Horn: summary

- IBR predicts isomorphic map between types and signals
- ICR prdicts association between maximum-likelihood type and cheapest message; any mapping between lower-likelihood types and higher-cost signals are admitted in fixed point
- IBR result is aesthetically more pleasing, but it is unclear whether there are empirical arguments either way

Free-choice inference

	$m_{\diamond A}$	$m_{\diamond B}$	$m_{\diamond(A \vee B)}$	$m_{\diamond(A \wedge B)}$
$t_{\diamond A}$	1	0	1	0
$t_{\diamond B}$	0	1	1	0
$t_{\diamond AB}$	1	1	1	1
$t_{\diamond A\|B}$	1	1	1	0
c	0	0	$^1/_{100}$	$^1/_{100}$

Free-choice game: IBR

S_0	$m_{\diamond A}$	$m_{\diamond B}$	$m_{\diamond(A\vee B)}$	$m_{\diamond(A\wedge B)}$
$t_{\diamond A}$	1	0	0	0
$t_{\diamond B}$	0	1	0	0
$t_{\diamond AB}$	$1/2$	$1/2$	0	0
$t_{\diamond A\mid B}$	$1/2$	$1/2$	0	0

R_0	$t_{\diamond A}$	$t_{\diamond B}$	$t_{\diamond AB}$	$t_{\diamond A\mid B}$	c
$m_{\diamond A}$	$1/3$	0	$1/3$	$1/3$	0
$m_{\diamond B}$	0	$1/3$	$1/3$	$1/3$	0
$m_{\diamond A\vee B}$	$1/4$	$1/4$	$1/4$	$1/4$	$1/100$
$m_{\diamond A\wedge B}$	0	0	1	0	$1/100$

R_1	$t_{\diamond A}$	$t_{\diamond B}$	$t_{\diamond AB}$	$t_{\diamond A\mid B}$	c
$m_{\diamond A}$	1	0	0	0	0
$m_{\diamond B}$	0	1	0	0	0
$m_{\diamond A\vee B}$	$1/4$	$1/4$	$1/4$	$1/4$	$1/100$
$m_{\diamond A\wedge B}$	0	0	1	0	$1/100$

S_1	$m_{\diamond A}$	$m_{\diamond B}$	$m_{\diamond(A\vee B)}$	$m_{\diamond(A\wedge B)}$
$t_{\diamond A}$	1	0	0	0
$t_{\diamond B}$	0	1	0	0
$t_{\diamond AB}$	0	0	0	1
$t_{\diamond A\mid B}$	$1/2$	$1/2$	0	0

S_2	$m_{\diamond A}$	$m_{\diamond B}$	$m_{\diamond(A\vee B)}$	$m_{\diamond(A\wedge B)}$
$t_{\diamond A}$	1	0	0	0
$t_{\diamond B}$	0	1	0	0
$t_{\diamond AB}$	0	0	0	1
$t_{\diamond A\mid B}$	0	0	1	0

$R_2 = R_1$

Free-choice game: ICR

S_0	$m_{\diamond A}$	$m_{\diamond B}$	$m_{\diamond(A\vee B)}$	$m_{\diamond(A\wedge B)}$
$t_{\diamond A}$	1	0	0	0
$t_{\diamond B}$	0	1	0	0
$t_{\diamond AB}$	x	$1-x$	0	0
$t_{\diamond A\vert B}$	y	$1-y$	0	0

R_0	$t_{\diamond A}$	$t_{\diamond B}$	$t_{\diamond AB}$	$t_{\diamond A\vert B}$	c
$m_{\diamond A}$	x_1	0	y_1	$1-x_1-y_1$	0
$m_{\diamond B}$	0	x_2	y_2	$1-x_2-y_2$	0
$m_{\diamond A\vee B}$	x_3	y_3	z_3	$1-x_3-y_3-z_3$	$1/100$
$m_{\diamond A\wedge B}$	0	0	1	0	$1/100$

R_1	$t_{\diamond A}$	$t_{\diamond B}$	$t_{\diamond AB}$	$t_{\diamond A\vert B}$	c
$m_{\diamond A}$	1	0	0	0	0
$m_{\diamond B}$	0	1	0	0	0
$m_{\diamond A\vee B}$	x	y	z	$1-x-y-z$	$1/100$
$m_{\diamond A\wedge B}$	0	0	1	0	$1/100$

S_1	$m_{\diamond A}$	$m_{\diamond B}$	$m_{\diamond(A\vee B)}$	$m_{\diamond(A\wedge B)}$
$t_{\diamond A}$	x	0	$1-x$	0
$t_{\diamond B}$	0	y	$1-y$	0
$t_{\diamond AB}$	0	0	0	1
$t_{\diamond A\vert B}$	x_2	y_2	$1-x_2-y_2$	0

S_2	$m_{\diamond A}$	$m_{\diamond B}$	$m_{\diamond(A\vee B)}$	$m_{\diamond(A\wedge B)}$
$t_{\diamond A}$	1	0	0	0
$t_{\diamond B}$	0	1	0	0
$t_{\diamond AB}$	0	0	0	1
$t_{\diamond A\vert B}$	0	0	1	0

R_2	$t_{\diamond A}$	$t_{\diamond B}$	$t_{\diamond AB}$	$t_{\diamond A\vert B}$	c
$m_{\diamond A}$	x	0	0	$1-x$	0
$m_{\diamond B}$	0	y	0	$1-y$	0
$m_{\diamond A\vee B}$	x_3	y_3	0	$1-x_3-y_3$	$1/100$
$m_{\diamond A\wedge B}$	0	0	1	0	$1/100$

Free-choice game: ICR (cont.)

S_2	$m_{\diamond A}$	$m_{\diamond B}$	$m_{\diamond(A\vee B)}$	$m_{\diamond(A\wedge B)}$
$t_{\diamond A}$	1	0	0	0
$t_{\diamond B}$	0	1	0	0
$t_{\diamond AB}$	0	0	0	1
$t_{\diamond A\vert B}$	0	0	1	0

R_2	$t_{\diamond A}$	$t_{\diamond B}$	$t_{\diamond AB}$	$t_{\diamond A\vert B}$	c
$m_{\diamond A}$	x	0	0	$1-x$	0
$m_{\diamond B}$	0	y	0	$1-y$	0
$m_{\diamond A\vee B}$	x_3	y_3	0	$1-x_3-y_3$	$1/100$
$m_{\diamond A\wedge B}$	0	0	1	0	$1/100$

R_3	$t_{\diamond A}$	$t_{\diamond B}$	$t_{\diamond AB}$	$t_{\diamond A\vert B}$	c
$m_{\diamond A}$	1	0	0	0	0
$m_{\diamond B}$	0	1	0	0	0
$m_{\diamond A\vee B}$	0	0	0	1	$1/100$
$m_{\diamond A\wedge B}$	0	0	1	0	$1/100$

$S_3 = S_1$

$S_4 = S_2$

Free-choice game: summary

- IBR and ICR/S predict free-choice inference
- ICR/R fails

Summary IBR vs. ICR

	IBR/S	IBR/R	ICR/S	ICR/R
biased some-all	✗	✓	✓	✓
sbna	✓	✗	✓	✓
lifted sbna+competence	✓	✗	✓	✓
Horn	✓	✓	✓	✓
generalized Horn	✓	✓	✗	✗
free choice	✓	✓	✓	✗

Summary IBR vs. ICR

- problems for vanilla IBR/R are solved in sophisticated version of Franke (2009, 2011)
- problems for ICR not unsurmountable (more sophisticated forward induction protocol; preference ordering over types in R_0
- IBR: computationally simple and straightforward
- ICR: difficult (often virtually impossible) to compute solution manually for larger games
- automated solution requires Linear Programming
- while LP is generally fast, number of LP constraints grow exponentially with number of types/message
- ICR is computationally unfeasible already for moderately-sized games — at least in the worst case (as IA and equilibrium-based approaches)
- like IA, ICR is less sensitive to priors than IBR