Vagueness, Signaling & Bounded Rationality

Michael Franke[1] and Gerhard Jäger[2]
joint work with Robert van Rooij

[2]University of Tübingen

[1]University of Amsterdam

Vagueness in Language, Reasoning, and Cognition
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EBERHARD KARLS UNIVERSITÄT TÜBINGEN

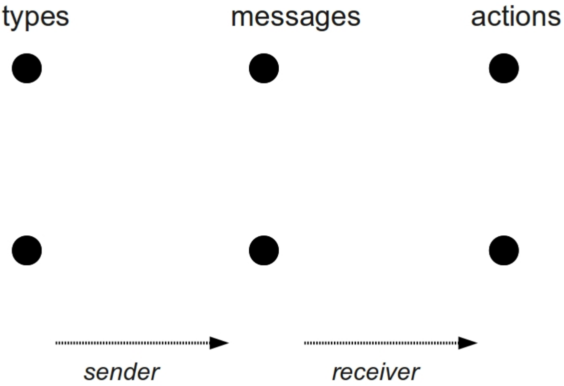
Overview

- Strategic communication
- Why vagueness is not rational
- Reinforcement learning with limited memory
- Quantal Best Response

Strategic communication: signaling games

- sequential game:
  1. **nature** chooses a type w (think of it as a possible world or an information state)
     - out of a pool of possible types W
     - according to a certain probability distribution P
  2. nature shows w to sender **S**
  3. S chooses a message m out of a set of possible messages M
  4. S transmits m to the receiver **R**
  5. R chooses an action a, based on the sent message.
- Both S and R have preferences regarding R's action, depending on w.
- S might also have preferences regarding the choice of m (to minimize signaling costs).

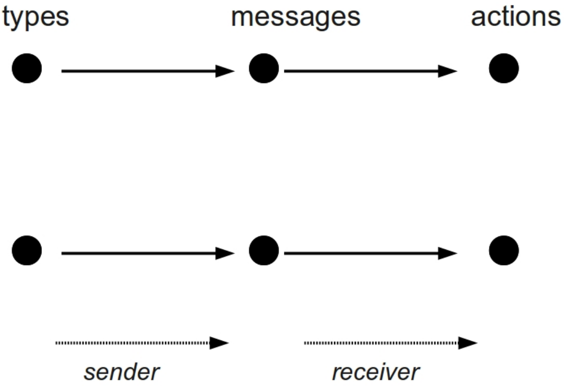
Basic example



utility matrix

	a_1	a_2
w_1	$1, 1$	$0, 0$
w_2	$0, 0$	$1, 1$

Basic example: Equilibrium 1



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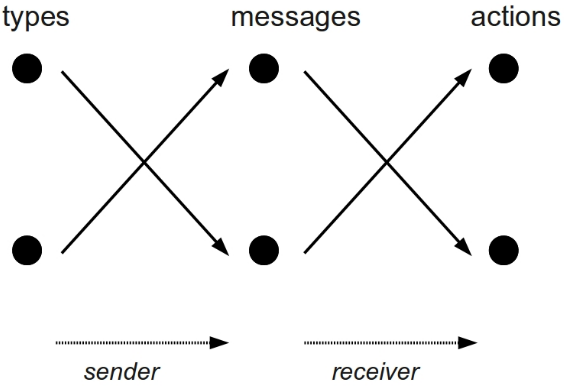
Basic example: Equilibrium 2



utility matrix

	a_1	a_2
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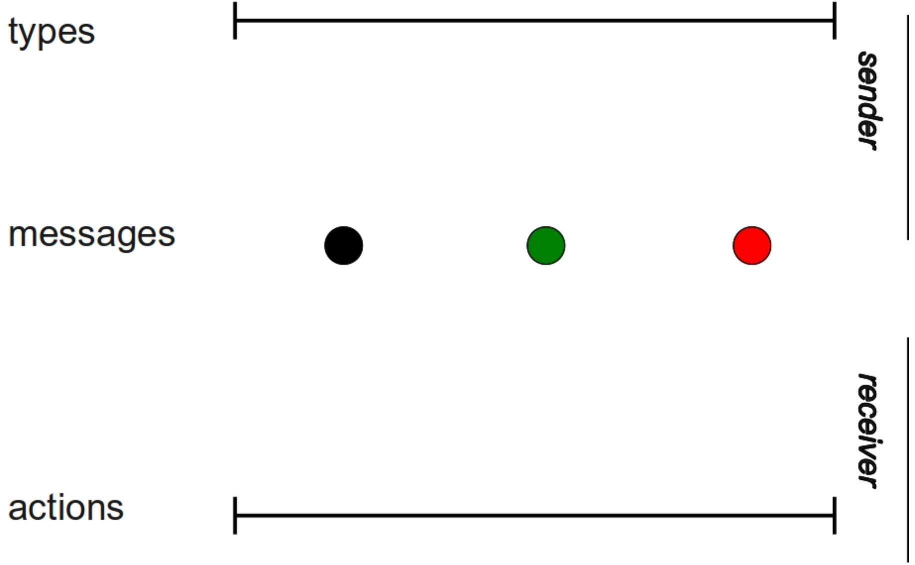
Equilibria

- two strict Nash equilibria
- these are the only ‘reasonable’ equilibria:
  - they are evolutionarily stable (self-reinforcing under iteration with positive feedback)
  - they are Pareto optimal (cannot be outperformed)

Euclidean meaning space

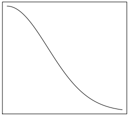


Utility function

General format

$$u_{s/r}(w, m, w') \quad = \quad \text{sim}(w, w')$$

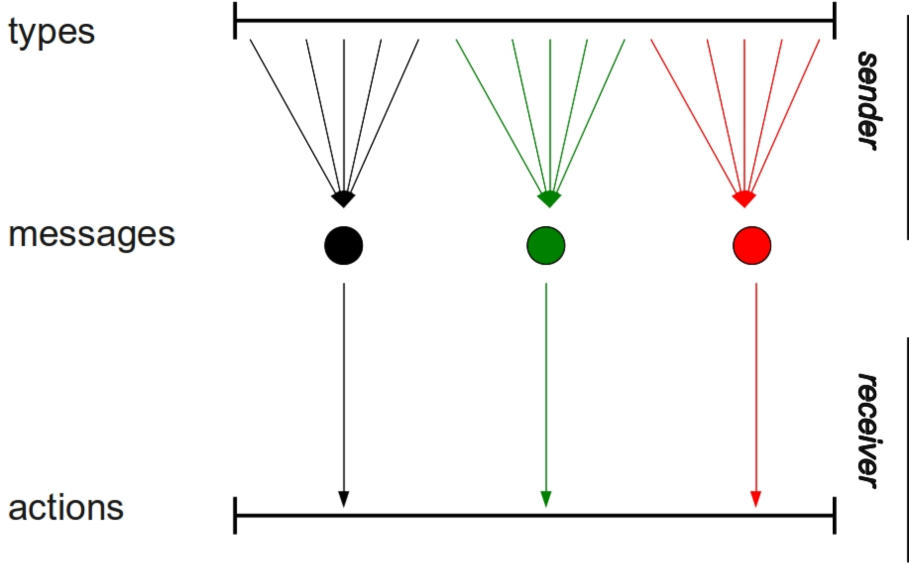
- $\text{sim}(x, y)$ is strictly monotonically decreasing in Euclidean distance $\|x - y\|$



In this talk, we assume a **Gaussian** similarity function

$$\text{sim}(x, y) \doteq \exp(-\frac{\|x - y\|^2}{2\sigma}).$$

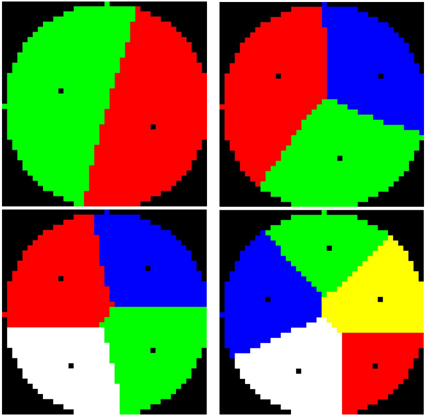
Euclidean meaning space: equilibrium



Simulations

- two-dimensional circular meaning space
- finitely many pixels (meanings)
- uniform distribution over meanings

(cf. Jäger & van Rooij, 2007)



Vagueness

- many evolutionarily stable/Pareto optimal equilibria
- all are strict (except for a null set at category boundaries)
- a *vague* language would be one where the sender plays a mixed strategy

Vagueness is not rational

Rational players will never prefer a vague language over a precise one in a signaling game. (Lipman 2009)

- similar claim can be made with regard to evolutionary stability (as corollary to a more general theorem by Reinhard Selten)

Vagueness is not evolutionarily stable

In a signaling game, a vague language can never be evolutionarily stable.

Vagueness and bounded rationality

- Lipman's result depends on assumption of perfect rationality
- we present two deviations from perfect rationality that support vagueness:
  - Learning: players have to make decisions on basis of limited experience
  - Stochastic decision: players are imperfect/non-deterministic decision makers

Learning and vagueness

Fictitious play

- model of learning in games
- indefinitely iterated game
- player memorize game history
- decision rule:
  - assume that other player plays a stationary strategy
  - make a maximum likelihood estimate of this strategy
  - play a best response to this strategy
- always converges against some Nash equilibrium

Limited memory

- more realistic assumption: players only memorize last k rounds (for fixed, finite k)
- consequence: usually no convergence
- long-term behavior depends on number of states — in relation to k

Formal definitions

$$\sigma(m|w) = \begin{cases} \frac{|\{k|\bar{s}(k)=\langle w,m\rangle\}|}{|\{k|\exists m':\bar{s}(k)=\langle w,m'\rangle\}|} & \text{if divisor } \neq 0 \\ \frac{1}{|M|} & \text{otherwise} \end{cases}$$

$$\rho(w|m) = \begin{cases} \frac{|\{k|\bar{r}(k)=\langle m,w\rangle\}|}{|\{k|\exists w':\bar{r}(k)=\langle m,w'\rangle\}|} & \text{if divisor } \neq 0 \\ \frac{1}{|W|} & \text{otherwise.} \end{cases}$$

A simulation

Game

- signaling game
- 500 possible worlds, evenly spaced in unit interval $[0, 1]$
- 3 distinct messages
- Gaussian utility function ($\sigma = 0.1$)

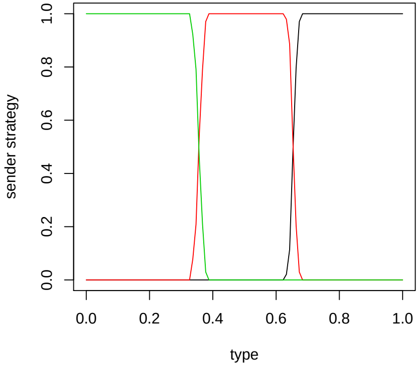
Fictitious play with limited memory

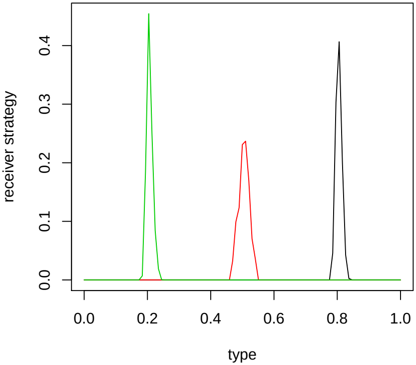
- $k = 200$
- simulation ran over 20,000 rounds

▸ start simulation ▸ stop simulation

A simulation

average over 10,000 rounds:





Intermediate summary

- Signaling games + fictitious play with limited memory:
  - predicts sharp category boundaries/unique prototypes for each agent at every point in time
  - strategies undergo minor changes over time tough
  - in multi-agent simulations, we also expect minor inter-speaker variation
  - vagueness emerges if we average over several interactions
- captures some aspect of vagueness (may provide solution for some instances of Sorites paradox)
- still: even at this very moment, I do not know the exact boundary between red and orange $\Rightarrow$ vagueness also applies to single agents

Stochastic choice

- real people are not perfect utility maximizers
- they make mistakes $\rightsquigarrow$ sub-optimal choices
- still, high utility choices are more likely than low-utility ones

Rational choice: best response

$$P(a_i) = \begin{cases} \frac{1}{|\arg_j \max u_i|} & \text{if } u_i = \max_j u_j \\ 0 & \text{else} \end{cases}$$

Stochastic choice: (logit) quantal response

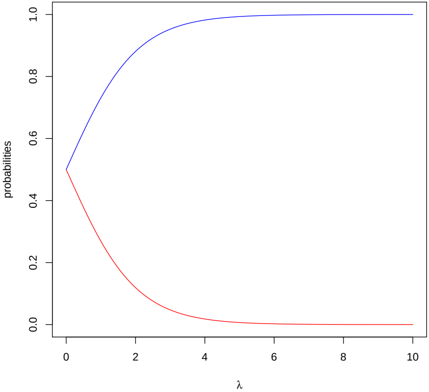
$$P(a_i) = \frac{\exp(\lambda u_i)}{\sum_j (\lambda \exp u_j)}$$

Quantal response

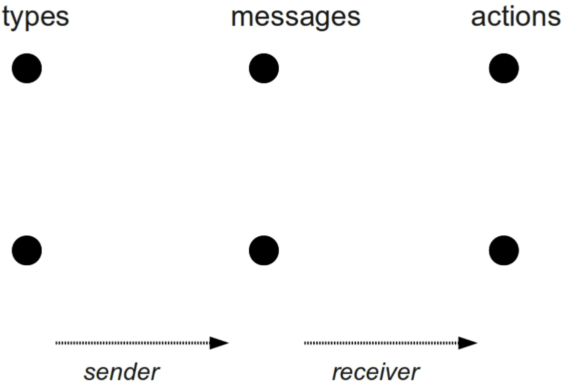
- λ measures degree of rationality
- $\lambda = 0$:
  - completely irrational behavior
  - all actions are equally likely, regardless of expected utility
- $\lambda \to \infty$
  - convergence towards behavior of rational choice
  - probability mass of sub-optimal actions converges to 0
- if everybody plays a quantal response (for fixed λ), play is in **quantal response equilibrium** (QRE)
- asl $\lambda \to \infty$, QREs converge towards Nash equilibria

Quantal response

- Suppose there are two choices, a_1 and a_2, with the utilities
  - $u_1 = 1$
  - $u_2 = 2$
- probabilities of a_1 and a_2:



Basic example



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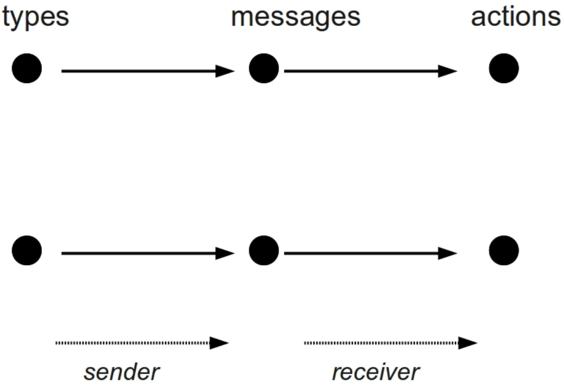
Basic example: Equilibrium 1



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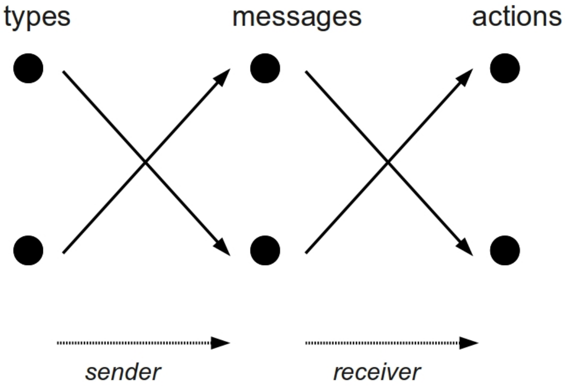
Basic example: Equilibrium 2



utility matrix

	a_1	a_2
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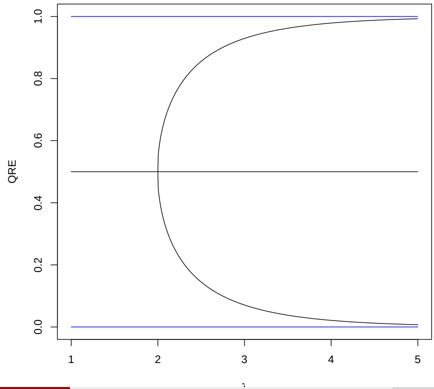
w_2	$0, 0$	$1, 1$

Quantal Response Equilibrium of 2×2 signaling game

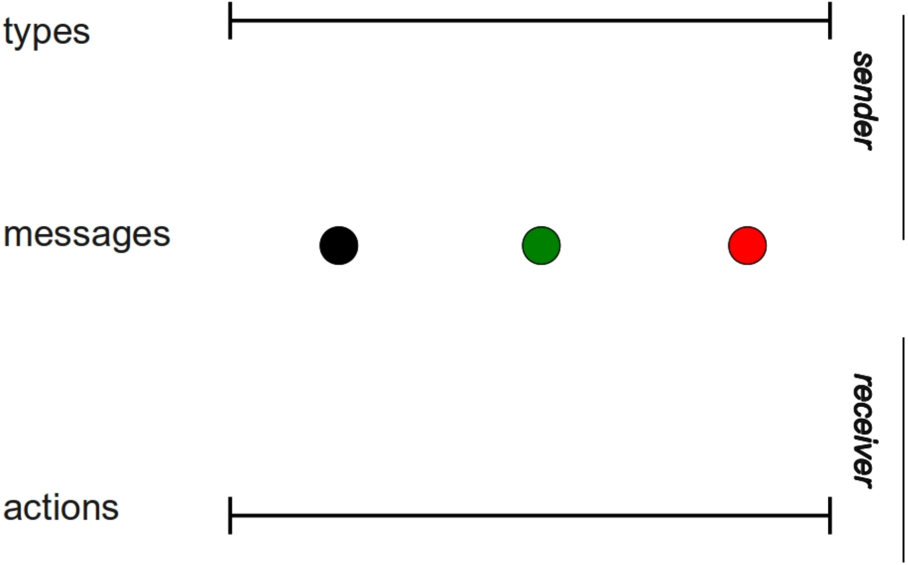
- for $\lambda \leq 2$: only babbling equilibrium
- for $\lambda > 2$: three (quantal response) equilibria:
  - babbling
  - two informative equilibria



QRE and vagueness

- similarity game
- 500 possible worlds, evenly spaced in unit interval $[0, 1]$
- 3 distinct messages
- Gaussian utility function ($\sigma = 0.2$)

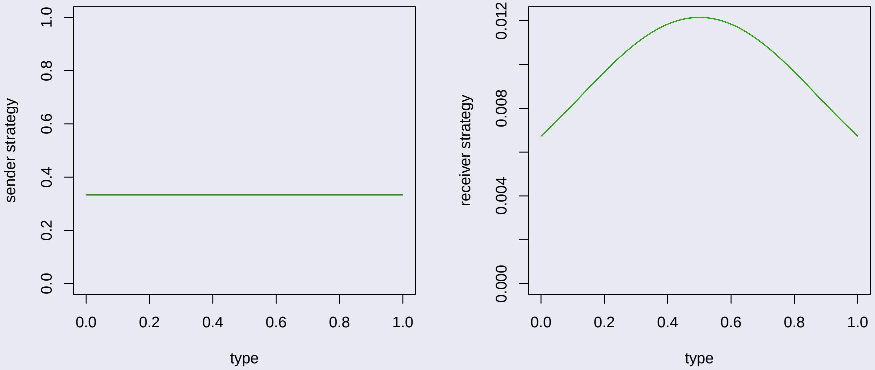
Euclidean meaning space



QRE and vagueness

$\lambda \leq 4$

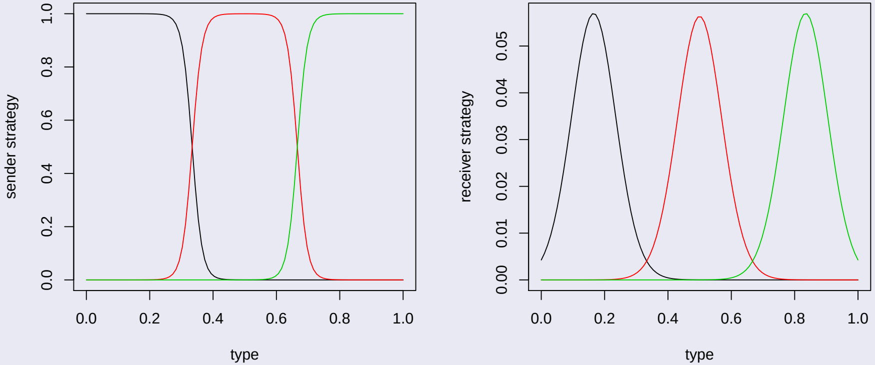
- only babbling equilibrium



QRE and vagueness

$\lambda > 4$

- separating equilibria
- smooth category boundaries
- prototype locations follow bell-shaped distribution



Imperfect Memory Fictitious Play (imf)

Motivation

- LMF still produced crisp meanings for single agent
- QRE gave vague individual languages but source of vagueness left implicit

⇒ synthesis: best responses to *imperfect* memory

Idea

- agents play best responses to finite set of past observations
- memory may be imperfect:
  - memory retrieval is noisy
  - noise is anti-proportional to recency of observation

Imperfect Memory Fictitious Play (imf)

Implementation

- $\bar{r} = \langle \langle m_1, w_1 \rangle, \ldots, \langle m_k, w_k \rangle \rangle$ — **S**'s observations of **R**'s behavior
  $\bar{s} = \langle \langle w_1, m_1 \rangle, \ldots, \langle w_k, m_k \rangle \rangle$ — **R**'s observations of **S**'s behavior
- each time a best response is computed, access memory as:
  $\bar{r}^* = \langle \langle m_1, w_1^* \rangle, \ldots, \langle m_k, w_k^* \rangle \rangle$
  $\bar{s}^* = \langle \langle w_1^*, m_1 \rangle, \ldots, \langle w_k^*, m_k \rangle \rangle$
- w_i^* is sampled from a normal distribution with mean w_i and standard deviation sd_i
- $sd_i = sd_{\max} \times \frac{i}{k}$
  $\Rightarrow$ $sd_{\max}$ and k are the relevant parameters of the game

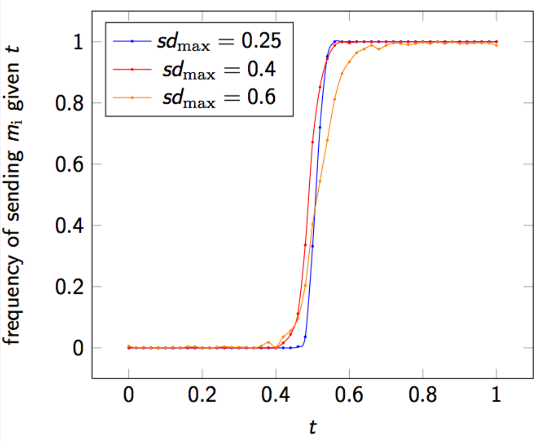
Imperfect Memory Fictitious Play (imf)

Experiment

- agents played IMF with $k = 50$ & $sd_{\max} \in \{0.25, 0.4, 0.6\}$
  $|W| = 51$, $M| = 2$, linear utilities
- freeze after 400 rounds
- compute best responses to current memories at all choice points 500 times

Imperfect Memory Fictitious Play (imf)

Results (Sender)



Imperfect Memory Fictitious Play (imf)

Results (Receiver)