The evolution of convex categories

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Cognitive semantics

Gärdenfors (2000):

- meanings are arranged in **conceptual spaces**
- conceptual space has geometrical structure
- dimensions are founded in perception/cognition

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Convexity

A subset C of a conceptual space is said to be *convex* if, for all points x and y in C, all points between x and y are also in C.

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Criterion P

A *natural property* is a convex region of a domain in a conceptual space.

Examples

- spatial dimensions: *above, below, in front of, behind, left, right, over, under, between ...*
- temporal dimension: *early, late, now, in 2005, after, ...*
- sensual dimenstions: *loud, faint, salty, light, dark, ...*
- abstract dimensions: *cheap, expensive, important, ...*

Signaling game

- two players:
  - **S**ender
  - **R**eceiver
- infinite set of **M**eanings, arranged in a finite metrical space
  distance is measured by function $d : M^2 \mapsto R$
- finite set of **F**orms
- sequential game:
  1. nature picks out $m \in M$ according to some probability distribution p and reveals m to S
  2. S maps m to a form f and reveals f to R
  3. R maps f to a meaning m'

Signaling game

- **Goal:**
  - optimal communication
  - both want to minimize the distance between m and m'
- **Strategies:**
  - speaker: mapping S from M to F
  - hearer: mapping R from F to M
- **Average utility:** (identical for both players)

$$u(S, R) = \sum_m p_m \times sim(m, R(S(m)))$$

vulgo: average similarity between speaker's meaning and hearer's meaning

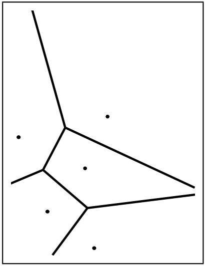
Similarity

Similarity function

- similarity is inversely related to distance
- requirements:

$$\begin{aligned}
\forall x : sim(x,x) &= 1 \\
\forall x,y : sim(x,y) &> 0 \\
\forall x,y,z : \|x-y\| > \|x-z\| &\rightarrow sim(x,y) < sim(x,z) \\
\forall x,y,z,w : \|x-y\| = \|z-w\| &\rightarrow sim(x,y) = sim(z,w)
\end{aligned}$$

- suppose R is given and known to the speaker: which speaker strategy would be the best response to it?
    - every form f has a "prototypical" interpretation: $R(f)$
    - for every meaning m: S's best choice is to choose the f that minimizes the distance between m and $R(f)$
    - optimal S thus induces a (quasi-)**partition** of the meaning space
    - Voronoi tesselation, induced by the range of R



Voronoi tesselation

Okabe et al. (1992) prove the following lemma (quoted from Gärdenfors 2000):

Lemma

The Voronoi tessellation based on a Euclidean metric always results in a partioning of the space into convex regions.

Evolutionary stability

Definition

A set E of symmetric Nash equilibria is an *evolutionarily stable set* (ESSet) if, for all $x^* \in E, u(x^*, y) > u(y, y)$ whenever $u(y, x^*) = u(x^*, x^*)$ and $y \notin E$. (Cressman 2003)

Evolutionary stability

Observation

If R is a pure receiver strategy, the inverse image of any $S \in BR(R)$ is consistent with the Voronoi tessellation of the meaning space that is induced by the image of R.

Evolutionary stability

Theorem

If a symmetric strategy is an element of some ESSet, the inverse image of its sender strategy is consistent with the Voronoi tessellation that is induced by the image of its receiver strategy.

sketch of proof:

- game in question is symmetrized asymmetric game
- ESSets of symmetrized games coincide with SESets of asymmetric game (Cressman, 2003)
- SESets are sets of NE
- SESets are finite unions of Cartesian producs of faces of the state space
- hence every component of an element of an SESet is a best reply to some pure strategy

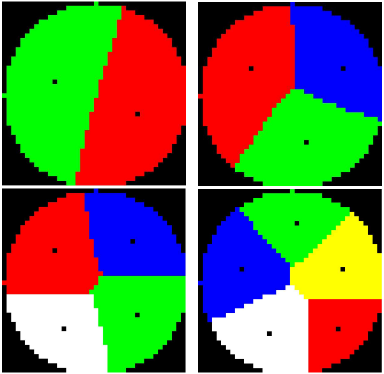
Static and dynamic stability

asymptotic stability

- in symmetrized games, a set E is an *asymptotically stable set of rest points* if and only if it is an ESSet
- in partnership games, at least one ESSet exists
- intuitive interpretation: under replicator dynamics + small effect of drift, system will eventually converge into some ESSet

Simulations

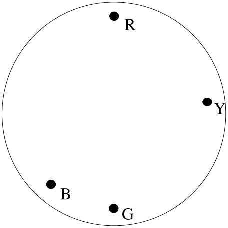
- two-dimensional circular meaning space
- discrete approximation
- uniform distribution over meanings
- initial stratgies are randomized
- update rule according to (discrete time version of) replicator dynamics



A toy example

- suppose
  - circular two-dimensional meaning space
  - four meanings are highly frequent
  - all other meanings are negligibly rare
- let's call the frequent meanings
  Red, Green, Blue and Yellow

  $p_i(\text{Red}) > p_i(\text{Green}) > p_i(\text{Blue}) > p_i(\text{Yellow})$



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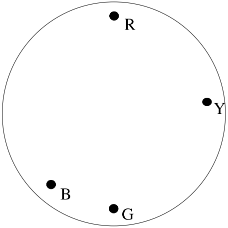
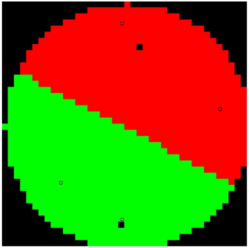


$$p_i(\text{Red}) > p_i(\text{Green}) > p_i(\text{Blue}) > p_i(\text{Yellow})$$

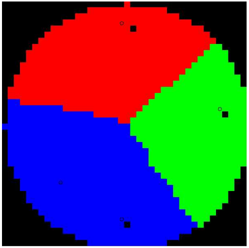
Yes, I made this up without empirical justification.

- suppose there are just two forms
- only one Strict Nash equilibrium (up to permuation of the forms)
- induces the partition {**Red, Blue**}/{**Yellow, Green**}



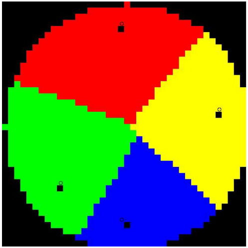
Three forms

- if there are three forms
- two Strict Nash equilibria (up to permuation of the forms)
- partitions {Red}/{Yellow}/{Green, Blue} and {Green}/{Blue}/{Red, Yellow}
- only the former is **stochastically stable** (resistent against random noise)



Four forms

- if there are four forms
- one Strict Nash equilibrium (up to permuation of the forms)
- partitions {Red}/{Yellow}/{Green}/{Blue}



Measure terms

Krifka's observations

- measure terms are vague
- some measure terms are ambiguous between different degrees of vagueness
- usually only simple expressions are ambiguous in this way
- complexifying an expression may reduce ambiguitiy

Measure terms

vagueness

95 m: between 94.5 and 95.5 m

ambiguity

- *The water has a temperature of 40°:* $38^\circ < T < 42^\circ$
- *His body temperature is 40°:* $39.95^\circ < T < 40.05^\circ$

simple and complex expression

His body temperature is 39°: cannot mean $37^\circ < T < 41^\circ$

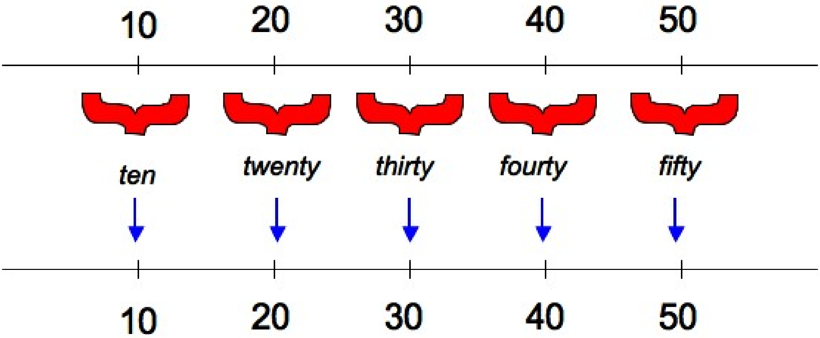
complexification

The water has a temperature of exactly 40°: $39.9^\circ < T < 40.1^\circ$

- Suppose the game setup is as before, with arithmetic difference as distance function

ESS

- Sender:
  - meaning space is partitioned into continuous intervals of equal length
  - each interval is correlated with one signal
- Receiver:
  - each signal is mapped to the center of the corresponding interval


10
20
30
40
50
ten
twenty
thirty
fourty
fifty
10
20
30
40
50

Costly signaling

- suppose signals incur a cost for both sender and receiver
- modified utility function

$$u(S, R) = \sum_m p_m \exp(-(m - R(S(m)))^2) - c(S(m))$$

- intuitive idea:

$$c(\text{thirty-nine}) > c(\text{forty})$$

etc.

ESSets

- general pattern as before
- additional constraint: in an ESS (S, R), we have

$$\forall m : S(m) = \arg_f \max[\exp(-(m - R(f))^2) - c(f)]$$

- simultaneous
  - minimizing distance between m and $R(S(m))$
  - minimizing costs $c(S(m))$
- in equilibrium (ESSet), distance between m and $R(S(m))$ need not be minimal

Variable standard of precision

Assessment

- this setup
  - predicts the possibility of vague interpretation: **good**
  - fails to predict the ambiguity between precise and vague interpretations (or different degrees of vagueness): **bad**

Variable standard of precision

Proposal

- required degree of precision depends on context
- modeling as Bayesian game with different utility function
- both players still have same utility function and know that function

$$u(S,R) = \sum_{m,\sigma} p_{m,\sigma} \exp(-(m - R(S(m)))^2/\sigma^2) - c(S(m))$$

- high value of σ: precision doesnt matter very much
- low value of σ: precision is more important than economy of expression

An example

- Suppose:
  - just two meanings: 39, 40
  - just two forms: *thirty-nine*, *forty*

$$c(\textit{thirty-nine}) - c(\textit{forty}) = \mathbf{c} > 0$$

- two standards of precision, σ_1 and σ_2

$$\begin{aligned}
\sigma_1 &< \sigma_2 \\
\exp(-(1^2/\sigma_1^2)) &= d_1 \\
\exp(-(1^2/\sigma_2^2)) &= d_2 \\
1 - d_1 &> \mathbf{c} \\
1 - d_2 &< \mathbf{c} \\
\forall m, \sigma : p_{m,\sigma} &= .25
\end{aligned}$$

An example

Intuitive characterization

- two standards of precision
- utility loss under vague interpretation is $1 - d_i$
- utility loss due to usage of more complex expression is $\mathbf{c}$
- under σ_1 precision is more important
- under σ_2 economy of expression is more important
- uniform probability distribution over states

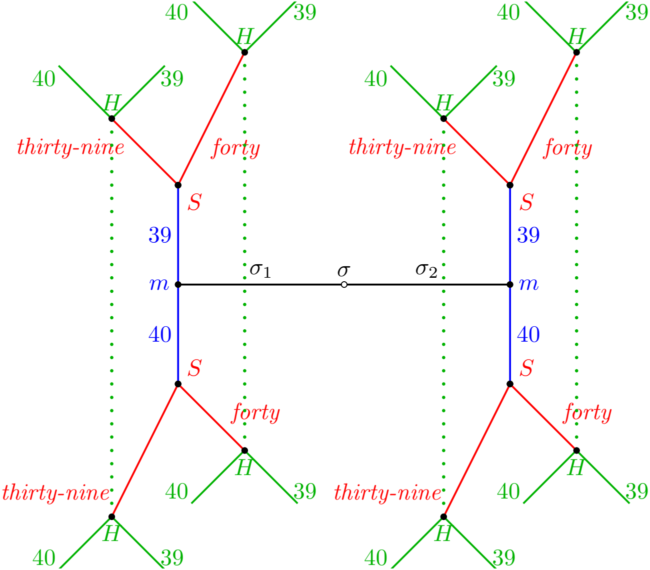
meanings/signals

S		R
39	*thirty-nine*	39
40	*forty*	40

strategies

- S_1/R_1 : 
- S_2/R_2 : 
- S_3/R_3 : 
- S_4/R_4 : 

Extensive form



Utility matrices

σ_1

				
	$1 - \frac{c}{2}$	$d_1 - \frac{c}{2}$	$\frac{1+d_1-c}{2}$	$\frac{1+d_1-c}{2}$
	$d_1 - \frac{c}{2}$	$1 - \frac{c}{2}$	$\frac{1+d_1-c}{2}$	$\frac{1+d_1-c}{2}$
	$\frac{1+d_1}{2}$	$\frac{1+d_1}{2}$	$\frac{1+d_1}{2}$	$\frac{1+d_1}{2}$
	$\frac{1+d_1}{2} - c$	$\frac{1+d_1}{2} - c$	$\frac{1+d_1}{2} - c$	$\frac{1+d_1}{2} - c$

σ_2

				
	$1 - \frac{c}{2}$	$d_2 - \frac{c}{2}$	$\frac{1+d_2-c}{2}$	$\frac{1+d_2-c}{2}$
	$d_2 - \frac{c}{2}$	$1 - \frac{c}{2}$	$\frac{1+d_2-c}{2}$	$\frac{1+d_2-c}{2}$
	$\frac{1+d_2}{2}$	$\frac{1+d_2}{2}$	$\frac{1+d_2}{2}$	$\frac{1+d_2}{2}$
	$\frac{1+d_2}{2} - c$	$\frac{1+d_2}{2} - c$	$\frac{1+d_2}{2} - c$	$\frac{1+d_2}{2} - c$

Results

Evolutionary stability

- first subgame (σ_1; precision is important): two ESS
  - S_1/R_1
  - S_2/R_2
- in either case, both expressions have a precise meaning and are interpreted exactly as intended
- second subgame (σ_2; economy of expression is important): one ESSet
  - consists of S_3 and all mixed strategies of R
- Bayesian game:
  - two ESSets
  - any combination of ESSets of the two sub-games

Asymmetric information

Assessment

- this setup
- predicts that
  - all number words receive a precise interpretation if precision is important
  - only short number words are used and receive a vague interpretation if economy is important
- **good**
- with larger dictionary prediction that there is no correlation between the interpretation of words between the different subgames
- for instance:
  - *forty* could mean 40 for σ_1 and $\{28...32\}$ for σ_2
- **bad**

Asymmetric information

Modified information sets

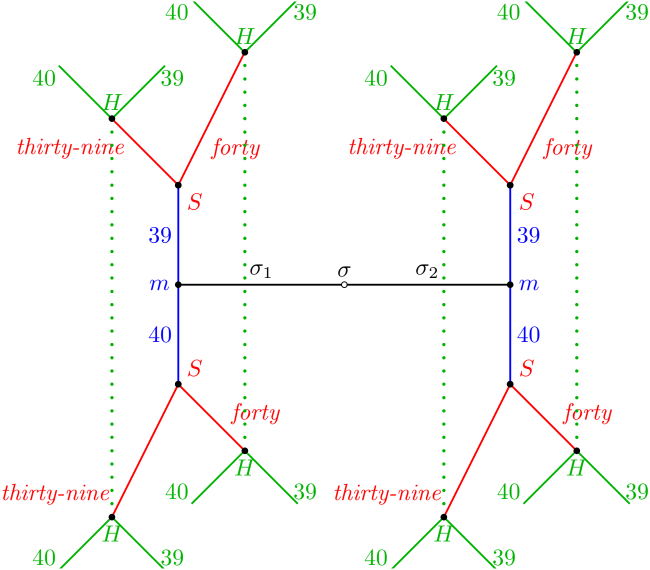
- idea
  - S knows σ, but
  - R doesn't
- then R's interpretation of a word cannot depend on σ

Strategy space

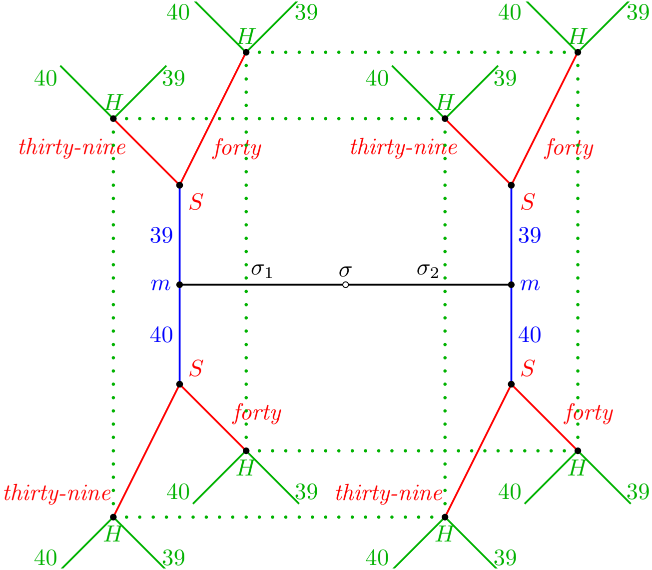
- Sender strategies:
  - functions from pairs (m, σ) to signals
  - in the example: $4 \times 4 = 16$ strategies, as before
- Receiver's strategies
  - functions from signals to meanings
  - in the example: only four such functions (as in the first version of the example)

Extensive form

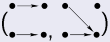
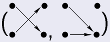
old game:



new game:


40
39
H
40
39
H
40
39
H
40
39
H
thirty-nine
forty
thirty-nine
forty
S
S
39
39
σ_1
σ
σ_2
m
m
40
40
S
S
forty
forty
H
H
thirty-nine
40
39
thirty-nine
40
39
H
H
40
39
40
39

ESS

- resulting game has only two ESSs
  - ESS 1:
    - S: 
    - R: 
  - ESS 2:
    - S: 
    - R: 
- in either case
  - R always assumes precise interpretation
  - S always chooses correct word if σ is low
  - S always chooses short word if σ is high

Loose ends

Open questions

- notion of ESS/ESSet only make sense for finite strategy space
- can results be maintained if meaning space is really continuous?
- S's signal gives information about value of σ
- perhaps R's guess about value of σ should enter the utility function
- would explain why
  - it can be rational for S to use excessively complex phrases like *exactly fourty* and short phrases like *fourty* synonymously
  - *exactly fourty* can only be interpreted precisely, while *fourty* is ambiguous