Gradient constraints in finite state OT: The unidirectional and the bidirectional case

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Outline of talk

- Unidirectional OT with finite state methods
  - Binary constraints: Frank and Satta 1998
  - Gradient constraints: Gerdemann and van Noord 2000
  - Generalizations
- Bidirectional OT
  - Basic concepts
  - Binary constraints
  - Gradient constraints

Optimality Theory: The basic picture

- Three components:

  1. **GEN**: (very general) relation between input and output
  2. **CON**: set of ranked violable constraints on input-output pairs
  3. **EVAL**: Choice function that identifies optimal input-output pairs among a set of candidates (depending on **CON**)

Definition 1 (OT-System)

1. *An OT-system is a pair* $\mathcal{O} = \langle \textbf{GEN}, \textbf{CON} \rangle$*, where* **GEN** *is a relation, and* $\textbf{CON} = \langle c_1, \ldots, c_p \rangle, p \in \mathbb{N}$ *is a linearly ordered sequence of functions from* **GEN** *to* $\mathbb{N}$*.*

2. *Let* $a, b \in \textbf{GEN}$*.* $a <_{\mathcal{O}} b$ *iff there is an* i *with* $1 \leq i \leq p$ *such that* $c_i(a) < c_i(b)$ *and for all* $j < i : c_j(a) = c_j(b)$*.*

Definition 2 (Unidirectional optimality) *Let* $\mathcal{O} = \langle \textbf{GEN}, \textbf{CON} \rangle$ *be an OT-system. Then* $\langle i, o \rangle$ *is unidirectionally optimal with respect to* $\mathcal{O}$ *iff*

1. $\langle i, o \rangle \in$ **GEN**

2. *there is no* o' *such that* $\langle i, o' \rangle \in$ **GEN** *and* $\langle i, o' \rangle <_{\mathcal{O}} \langle i, o \rangle$

- Two types of constraints:
  1. Markedness constraints $\Rightarrow$ refer to output only
     - "syllables have onsets", "vowels are oral" ...
  2. Faithfulness constraints $\Rightarrow$ refer to i/o pairing
     - "don't delete material", "don't add material" ...

OT and finite state techniques: Frank and Satta 1998

- Naive algorithms only work with finite candidate sets
- Bad news: Set of optimal candidates might be undecidable if candidate set is infinite
- Good news: Large subclass of OT systems can even be implemented by finite state techniques

Finite state implementation

- Frank and Satta: If
  - there are no faithfulness constraints,
  - all constraints are binary (i.e. they don't count violations),
  - **GEN** is a rational relation, and
  - all constraints can be represented by a regular language,

  then the set of optimal input-output pairs is a rational relation

Some closure properties of regular languages and rational relations

- Every finite language is regular.
- If L_1 and L_2 are regular languages, then $L_1 \cap L_2, L_1 \cup L_2, L_1 - L_2, L_1 L_2$, and L_1^* are also regular languages.
- If R_1 and R_2 are rational relations, then $R_1 \cup R_2, R_1 \circ R_2, R_1^{\cup}, R_1 R_2$, and R_1^* are also rational relations.
- If R is a rational relation, then $Dom(R)$ and $Rg(R)$ are regular languages.
- If L_1 and L_2 are regular languages, then $L_1 \times L_2$ and $\mathbf{I}_{L_1}$ are rational relations.

Conditional Intersection

Definition 3
Let R be a relation and L a language. The *conditional intersection* $R \uparrow L$ of R with L is defined as

$$R \uparrow L \doteq (R \circ \mathbf{I}_L) \cup (\mathbf{I}_{Dom(R) - Dom(R \circ \mathbf{I}_L)} \circ R)$$

Theorem 1 (Frank and Satta)
Let $\mathcal{O} = \langle \textbf{GEN}, \textbf{CON} \rangle$ with $\textbf{CON} = \langle c_1, \ldots, c_p \rangle$ be an OT-system such C solely consists of binary output markedness constraints. Then $\langle i, o \rangle$ is unidirectionally optimal iff $\langle i, o \rangle \in \textbf{GEN} \uparrow c_1 \cdots \uparrow c_p$.

Corollary 1
Let $\mathcal{O} = \langle \textbf{GEN}, \textbf{CON} \rangle$ with $\textbf{CON} = \langle c_1, \ldots, c_p \rangle$ be an OT-system such C solely consists of binary output markedness constraints. Then the optimal input-output pairing is a rational relation.

FS implementation of gradient constraints

Basic idea

- Consider the relation (inspired by Gerdemann and van Noord 2000)

$$\textbf{GEN} \circ <$$

- Range: set of sub-optimal outputs of **GEN**
- $Rg(\textbf{GEN}) - Rg(\textbf{GEN} \circ <)$ — set of optimal outputs
- Thus

$$\textbf{GEN} \circ \mathrm{I}_{Rg(\textbf{GEN}) - Rg(\textbf{GEN} \circ <)}$$

represents the optimal input-output pairing (it seems)

Formalization

- Constraint c is called **rational** with respect to R iff there is a rational relation S with

$$\{\langle x, y\rangle | c(x) < c(y)\} \cap (R^{\cup} \circ R) = S \cap (R^{\cup} \circ R)$$

- Notation: $S = rel_R(c)$
- Following operation generalizes conditional intersection to gradient constraints

Definition 4 (Generalized conditional intersection)

$$R \restriction S \doteq R \circ \mathbf{I}_{Rg(R)-Rg(R\circ S)}$$

⇒ applicable only under certain restrictions:

Definition 5 *Let R and S be relations.* **Optimality is global with respect to R and S** *iff*

$$\forall i, o(iRo \wedge \neg\exists o'(iRo' \wedge o'So) \rightarrow \neg\exists o'(o' \in Rg(R) \wedge o'So))$$

Fact 1 *Let R and S be relations such that optimality is global with respect to R and S. Then*

$$\langle i, o\rangle \in R \upharpoonright S \textit{ iff } iRo \wedge \neg\exists o'(iRo' \wedge o'So)$$

Theorem 2
Let $\mathcal{O} = \langle \textbf{GEN}, \textbf{CON} \rangle$ with **CON** $= \langle c_1, \cdots, c_p \rangle$ be an OT system such that all elements of **CON** are rational markedness constraints and for all i, optimalitity is global with respect to **GEN** $\upharpoonright rel(c_1) \cdots \upharpoonright rel(c_{i-1})$. Then $\langle i, o \rangle$ is unidirectionally optimal iff

$$\langle i, o \rangle \in \textbf{GEN} \upharpoonright rel(c_1) \cdots \upharpoonright rel(c_p)$$

.

Corollary 2
Let $\mathcal{O} = \langle \textbf{GEN}, \textbf{CON} \rangle$ with **CON** $= \langle c_1, \cdots, c_p \rangle$ be an OT system such that **GEN** is a rational relation, all elements of **CON** are rational markedness constraints, and for all i, optimalitity is global with respect to **GEN** $\upharpoonright c_1 \cdots \upharpoonright c_{i-1}$. Then the optimal input-output pairing is a rational relation.

Bidirectionality

Application to syntax/semantic

- In phonology/morphology, OT takes the speaker perspective
- applied to syntax/semantics, this means:
  1. **GEN** is given by compositional (underspecified) semantics
  2. Markedness constraints only apply to forms, not to meanings
  3. A form/meaning pair may be blocked by a better form for the same meaning, but not the other way round

Competition/Blocking in semantics and pragmatics

- Competition between forms
  - Scalar implicatures
  - Lexical blocking
- But: also competition between meanings
  - Presupposition resolution (cf. van der Sandt 1992)
  - Bridging inference
  - ...

Blutner's OT-formalization

Definition 6 (Bidirectional optimality) *Let $\mathcal{O} = \langle \mathbf{GEN}, \mathbf{CON} \rangle$ be an OT-system. Then $\langle i, o \rangle$ is bidirectionally optimal iff*

1. $\langle i, o \rangle \in \mathbf{GEN}$,
2. *there is no bidirectionally optimal* $\langle i', o \rangle \in \mathbf{GEN}$ *such that* $\langle i', o \rangle <_{\mathcal{O}} \langle i, o \rangle$, *and*
3. *there is no bidirectionally optimal* $\langle i, o' \rangle \in \mathbf{GEN}$ *such that* $\langle i, o' \rangle <_{\mathcal{O}} \langle i, o \rangle$.

Algorithm

$OPT = \emptyset$;
$BLCKD = \emptyset$;

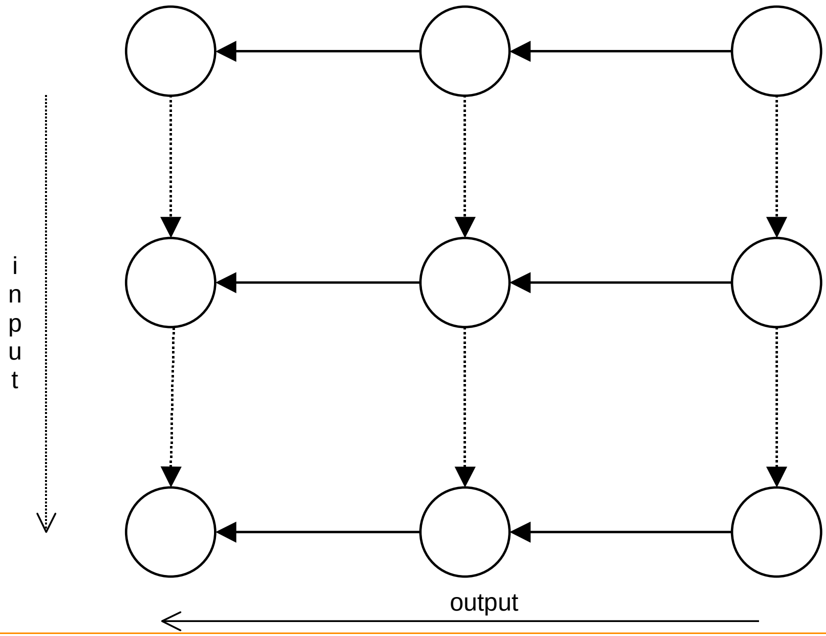
while $(OPT \cup BLCKD \neq \mathbf{GEN})$ {

$$OPT = OPT \cup \{x \in \mathbf{GEN} - BLCKD \mid \forall y < x : y \in OPT \cup BLCKD\};$$

$$BLCKD = BLCKD \cup \{\langle i, o\rangle \in \mathbf{GEN} - OPT \mid \langle i', o\rangle \in OPT \vee \langle i, o'\rangle \in OPT\};$$

}

return (OPT);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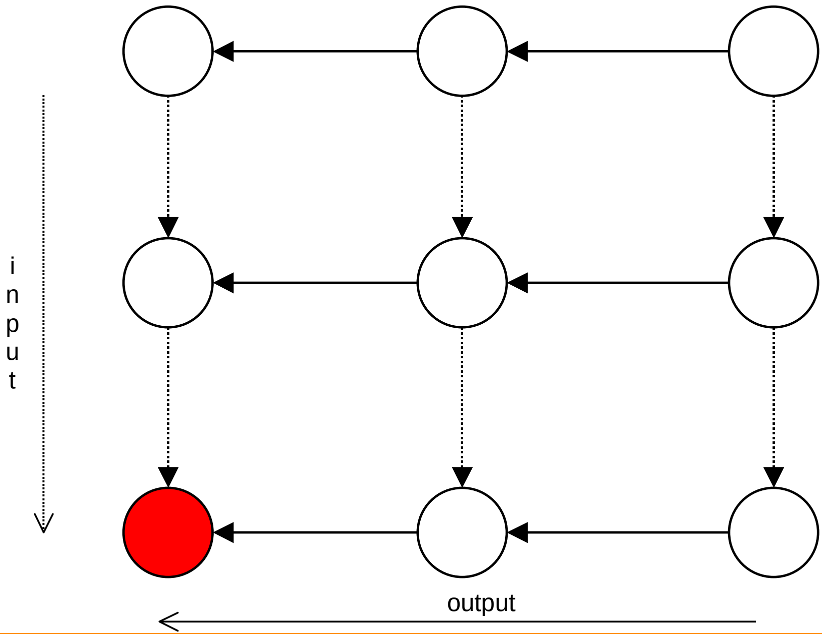
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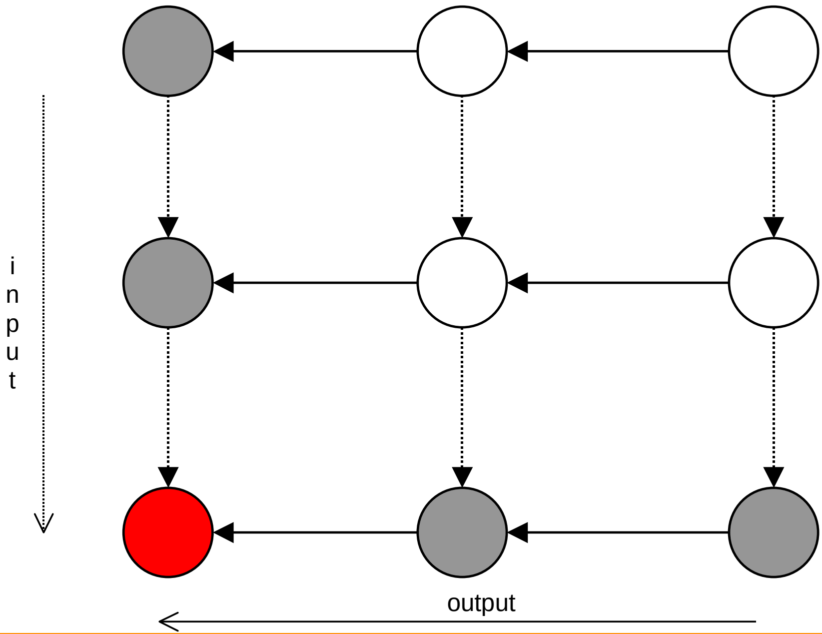
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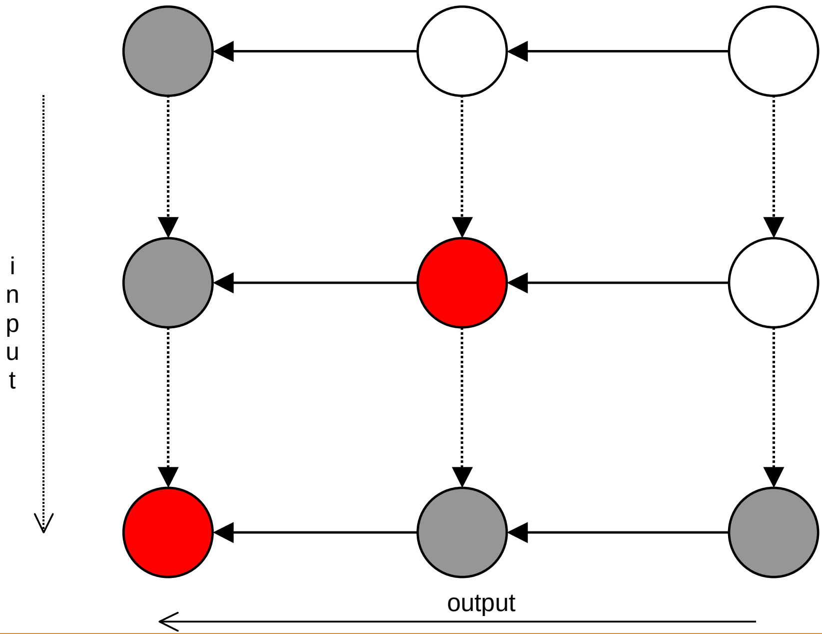
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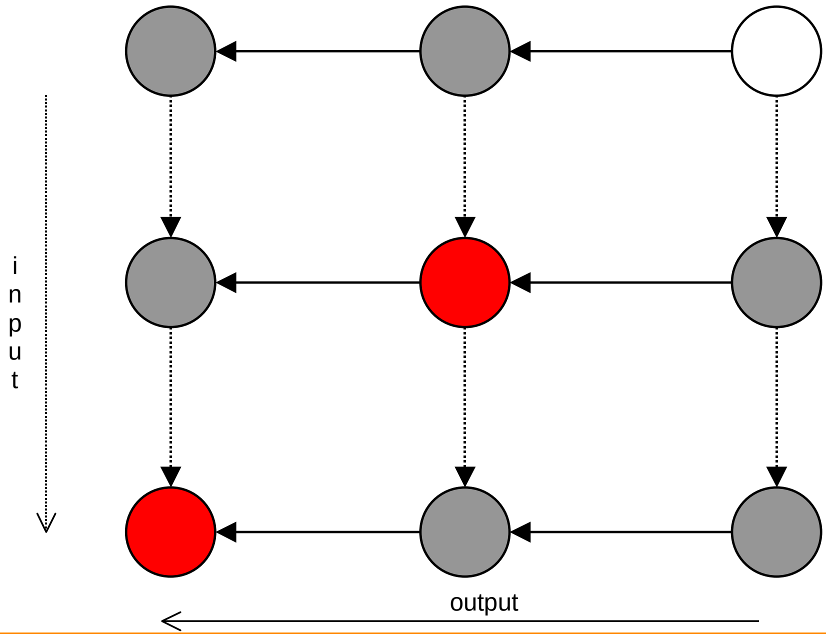
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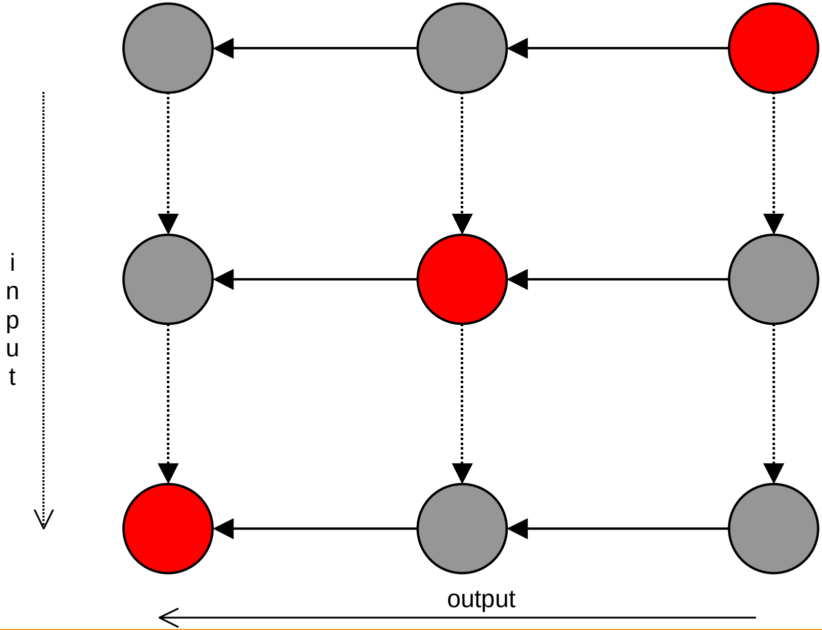
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Extension of Frank and Satta's construction to Bidirectionality

- Basic idea: implement the algorithm above by means of a cascade of Frank/Satta constructions!
- Bidirectional OT: competition both between different inputs and different outputs
- Thus both **input markedness constraints** and **output markedness constraints**
- So we also need **backward conditional intersection:**

$$R \downarrow L \doteq (R^{\cup} \uparrow L)^{\cup}$$

Definition 7 (Bidirectional Conditional Intersection)
Let $\mathcal{O} = \langle \textbf{GEN}, C \rangle$ be an OT-system and c_i be a binary markedness constraint.

$$R \Uparrow c_i \doteq \begin{cases} R \circ \mathbf{I}_{Rg((\{\varepsilon\} \times Rg(R)) \uparrow c_i)} \\ \text{if } c_i \text{ is an output markedness constraint} \\ \\ \mathbf{I}_{Dom((Dom(R) \times \{\varepsilon\}) \downarrow c_i)} \circ R \\ \text{else} \end{cases}$$

Lemma 1

Let $\mathcal{O} = \langle \textbf{GEN}, \textbf{CON} \rangle$ be an OT-system (with binary markedness constraints only), where **CON** $= \langle c_1, \ldots, c_p \rangle$. Then

$$\langle i, o \rangle \in \textbf{GEN} \Uparrow c_1 \cdots \Uparrow c_p$$

iff $\langle i, o \rangle \in$ **GEN**, and there are no i', o' with $\langle i', o' \rangle \in$ **GEN** and $\langle i', o' \rangle < \langle i, o \rangle$.

- **Notation:** $R^{\mathbf{CON}} \doteq R \Uparrow c_1 \cdots \Uparrow c_n$ (where **CON** $= c_1, \ldots, c_p$)

Definition 8
Let $\mathcal{O} = \langle \textsf{\textbf{GEN}}, \textsf{\textbf{CON}} \rangle$ be an OT-system.

$$
\begin{aligned}
OPT_0 &= \emptyset \\
OPT_{\alpha+1} &= OPT_\alpha \cup \\
&\quad (\mathbf{I}_{Dom(\textsf{\textbf{GEN}})-Dom(OPT_\alpha)} \circ \textsf{\textbf{GEN}} \circ \mathbf{I}_{Rg(\textsf{\textbf{GEN}})-Rg(OPT_\alpha)})^{\mathbf{CON}} \\
OPT_\beta &= \bigcup_{\alpha<\beta} OPT_\alpha \text{ (β a limit ordinal)} \\
OPT &= \bigcup OPT_\alpha
\end{aligned}
$$

Lemma 2
Let $\mathcal{O} = \langle \textsf{\textbf{GEN}}, \textsf{\textbf{CON}} \rangle$ be an OT-system. Then $\langle i, o \rangle \in OPT$ iff $\langle i, o \rangle$ is bidirectionally optimal.

Lemma 3

Let $\mathcal{O} = \langle \textbf{GEN}, \textbf{CON} \rangle$ be an OT-system with **CON** $= c_1, \ldots, c_p$, where all c_i are binary markedness constraints. Then $OPT = OPT_{2^p}$.

Corollary 3

Let $\mathcal{O} = \langle \textbf{GEN}, \textbf{CON} \rangle$ be an OT-system with **CON** $= \langle c_1, \ldots, c_p \rangle$, where all c_i are binary markedness constraints. Furthermore, let **GEN** be a rational relation and let all c_i be regular languages. Then the set of bidirectionally optimal elements of **GEN** is a rational relation.

Gradient constraints and bidirectionality

- FS construction for gradient constraints does not carry over to bidirectionality
- consider the following OT system:
    - $\mathcal{O} = \langle \textbf{GEN}, C \rangle$
    - $\textbf{GEN} = \{\langle a^i b^j, a^k b^l \rangle | i = k \vee j = l\}$
    - $C = \langle c_1, c_2 \rangle$
    - $c_1(\langle i, o \rangle) = \#_a(i)$
    - $c_2(\langle i, o \rangle) = \#_b(o)$
- **GEN** is rational relation
- both constraints are rational

- Crucial observation:

$$OPT_n = \{\langle a^i b^y, a^z b^i \rangle | i < n \wedge (y = i \vee z = i)\}$$

- Hence

$$OPT = \bigcup_{n \in \mathbb{N}} OPT_n = \{\langle a^n b^y, a^z b^n \rangle | y = n \vee z = n\}$$

- Now: $Rg(\mathbf{I}_{aa^*} \circ OPT) = \{a^n b^n | n > 0\}$
- Thus OPT is not regular

Conclusion

- Main results:
  - Frank and Satta style FS construction carries over to bidirectionality
  - Gerdemann and van Noord type FS construction does not carry over to bidirectionality

$\Rightarrow$ Bidirectional OT intrinsically more complex than unidirectional OT

- Possible applications:
  - Bidirectionality adds recursive structure that is absent in generator $\rightsquigarrow$ repercussions for architecture of grammar?
  - Extrapolation to complexity classes beyond the power of FSAs?
  - Bidirectionality arguably useful also for morphology (Wunderlich 2001), practical application in computational morphology?

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