Population dynamic models in linguistic typology

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Evolution in biology and linguistics

- correspondence between biology and linguistics

language	≈	species
dialect	≈	race
idiolect	≈	individual traits

- concept of *evolution* can be applied to linguistic as well

genotype	≈	grammatical knowledge ("langue")
phenotype	≈	utterances ("parole")
replication	≈	learning

Mathematical models from evolutionary biology should be applicable to linguistics!

- Biological evolution is driven by variation and selection
- variation
  - Biology: mutations
  - Linguistics: errors, language contact, fashion...
- selection:
  - Biology: fitness = number of fertile offsprings
  - Linguistics: communicative functionality, efficiency, social prestige, learnability, ...

Overview of the talk

- empirical domain of study: case marking systems in the languages of the world
- functionality of case marking types
- case marking as a game
- Evolutionary Game Theory
- stability in the presence of noise
- conclusion

Ways of argument identification

- transitivity may lead to ambiguity

die Frau, die Maria kennt

the woman that Maria knows | the woman that knows Maria

- three ways out

1. word order
2. agreement
3. **case**

die Frau, die er kennt

the woman that he knows

die Frau, die ihn kennt

the woman that knows him

- Suppose one argument is a pronoun and one is a noun (or a phrase)

$$\{\text{I, BOOK, KNOW}\}$$

- both conversants have an interest in successful communication
- case marking (accusative or ergative) is usually more costly than zero-marking (nominative)
- speaker wants to avoid costs

speaker strategies	*hearer strategies*
always case mark the object (accusative)	ergative is agent and accusative object
always case mark the agent (ergative)	pronoun is agent
case mark the object if it is a pronoun	pronoun is object
	pronoun is agent unless it is accusative
⋮	⋮

Statistical patterns of language use

four possible clause types:

	O/p	O/n
A/p	he knows it	he knows the book
A/n	the man knows it	the man knows the book

statistical distribution (from a corpus of spoken English)

	O/p	O/n
A/p	pp $=$ 198	pn $=$ 716
A/n	np $=$ 16	nn $=$ 75

pn $\gg$ np

- functionality of speaker strategies and hearer strategies depends on various factors:
  - How often will the hearer get the message right?
  - How many case markers does the speaker need per clause — on average?

- speaker strategies that will be considered:

agent is pronoun	*agent is noun*	*object is pronoun*	*object is noun*
e(rgative)	e(rgative)	a(ccusative)	a(ccusative)
e	e	a	z(ero)
e	e	z	a
e	e	z	z
e	z	a	a
...	...	...	...
z	e	z	z
z	z	a	a
z	z	a	z
z	z	z	a
z	z	z	z

- hearer strategies:
  - strict rule: ergative means "agent", and accusative means "object"
  - elsewhere rules:

1. AA: "The first phrase is always the agent."
2. AO: "Pronouns are agents, and nouns are objects."
3. OA: "Pronouns are objects, and nouns are agents."
4. OO: "The first phrase is always the object."

- whether communication works depends both on speaker strategy S and hearer strategy H
- two factors for functionality of communication
  - communicative success ("hearer economy")

$$\delta_m(S, H) = \begin{cases} 1 & \text{iff } H(S(m)) = m \\ 0 & \text{else} \end{cases}$$

  - least effort ("speaker economy")

$$\textit{cost}(f) = \#\text{ of case markers in } f$$

Game Theory

- two (or more) “players”
- each has choice between several “strategies”
- each player receives “payoff” or “utility”
- payoff of each player depends on the strategies of all players
- communication:
  - **partnership game**
  - players have common interest — everybody gets the same payoff

The utility of communication

$$u(S,H) \;=\; \sum_m p_m \times (\delta_m(S,H) - k \times \textit{\textbf{cost}}(S(m)))$$

k... relative strength of speaker economy compared to hearer economy
p... probability distribution over meaning types

Nash Equilibria

- (classical) Game Theory studies how rational players ought to behave
- rational player:
  - logically omniscient
  - only goal is maximization of utility (neither competition nor altruism or fairness play a role in decision making)
- stable configuration: no player has an interest to change the *status quo*

Definition 1 (Nash Equilibrium) *A pair of strategies* (S, H) *is a* Nash Equilibrium *iff*

$$\forall S'(S' \neq S \rightarrow \neq S \rightarrow u(S, H) > u(S', H))$$

and

$$\forall H'(H' \neq H \rightarrow H' \neq H \rightarrow u(S, H) > u(S, H'))$$

- a cell is a NE iff it has the maximal value in its row and its column

	hearer strategies	
speaker	100	50
strategies	50	0

The game of case

- strategy space and utility function are known
- probability of meaning types can be estimated from corpus study
- coefficient k is hard to estimate though

- $k = 0.1$

Speaker strategies	Hearer strategies			
	AA	AO	OA	OO
$eezz$	0.90	0.90	0.90	0.90
$zzaa$	0.90	0.90	0.90	0.90
$ezaz$	0.85	0.85	0.85	0.85
$zeza$	0.81	0.81	0.81	0.81
$zeaz$	0.61	0.97	0.26	0.61
$ezzz$	0.86	0.86	0.87	0.86
$zezz$	0.54	0.89	0.54	0.54
$zzaz$	0.59	0.94	0.59	0.59
$zzza$	0.81	0.81	0.82	0.81
$zzzz$	0.50	0.85	0.15	0.50

- $k = 0.1$

Speaker strategies	Hearer strategies			
	AA	AO	OA	OO
$eezz$	0.90	0.90	0.90	0.90
$zzaa$	0.90	0.90	0.90	0.90
$ezaz$	0.85	0.85	0.85	0.85
$zeza$	0.81	0.81	0.81	0.81
$zeaz$	0.61	0.97	0.26	0.61
$ezzz$	0.86	0.86	0.87	0.86
$zezz$	0.54	0.89	0.54	0.54
$zzaz$	0.59	0.94	0.59	0.59
$zzza$	0.81	0.81	0.82	0.81
$zzzz$	0.50	0.85	0.15	0.50

- Problems for classical GT
  - multiple equilibria $\Rightarrow$ no predictions possible
  - "perfectly rational player" is too strong an idealization

Evolutionary Game Theory

- populations of players
- individuals are (genetically) programmed for certain strategy
- individuals replicate and thereby pass on their strategy
- number of offsprings is monotonically related to average utility of a player

Replicator dynamics

$$\frac{d}{dt}s_i = s_i(\sum_j h_j u(S_i, H_j) - \sum_k s_k \sum_j h_j u(S_k, H_j))$$

$$\frac{d}{dt}h_i = h_i(\sum_j s_j u(S_j, H_i) - \sum_k h_k \sum_j s_j u(S_j, H_k))$$

Replicator dynamics

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proportion of the population

Replicator dynamics

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proportion of the population
velocity of change

Replicator dynamics

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proportion of the population
velocity of change
average utility of strategy j

Replicator dynamics

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proportion of the population
velocity of change
average utility of strategy j
population average

Evolutionary stable states

- A state is evolutionary stable iff
  - it is stationary under the replicator dynamics
  - it is robust against small amounts of mutations

Definition 2 (Strict Nash Equilibrium) *A pair of strategies* (S, H) *is a* Strict Nash Equilibrium *iff*

$$\forall S'(S \neq S \rightarrow u(S, H) > u(S', H))$$

and

$$\forall H'(H' \neq H \rightarrow u(S, H) > u(S, H'))$$

Theorem 1 (Selten 1980) (S, H) *is evolutionary stable if and only if it is a Strict Nash Equilibrium.*

- applied to The Game of Case

Speaker strategies	Hearer strategies			
	AA	AO	OA	OO
$eezz$	0.90	0.90	0.90	0.90
$zzaa$	0.90	0.90	0.90	0.90
$ezaz$	0.85	0.85	0.85	0.85
$zeza$	0.81	0.81	0.81	0.81
$zeaz$	0.61	0.97	0.26	0.61
$ezzz$	0.86	0.86	0.87	0.86
$zezz$	0.54	0.89	0.54	0.54
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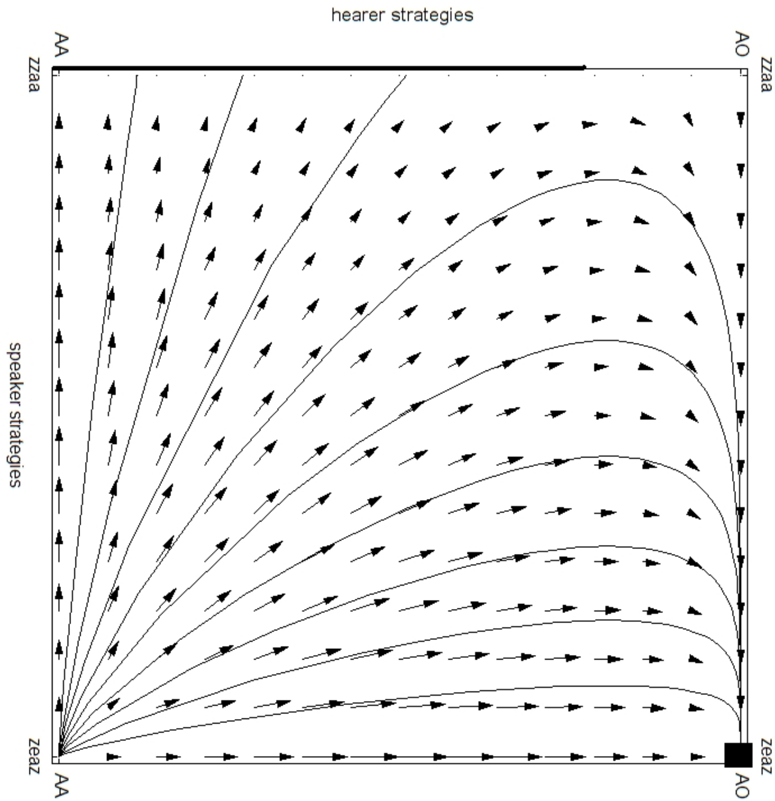
$ezzz$	0.86	0.86	0.87	0.86
$zezz$	0.54	0.89	0.54	0.54
$zzaz$	0.59	0.94	0.59	0.59
$zzza$	0.81	0.81	0.82	0.81
$zzzz$	0.50	0.85	0.15	0.50

- only one evolutionary stable state: $zeaz/AO$
- called *split ergative* by typologists
- very common among Australian aborigines languages

Why are non-strict Nash Equilibria unstable?

- Dynamics without mutation



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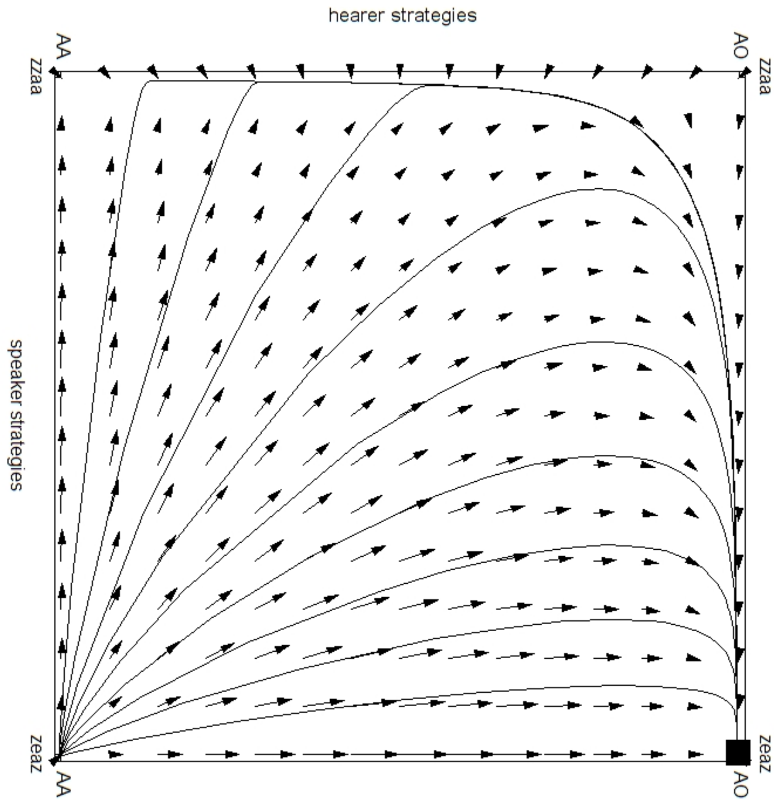

If speakers get lazier...

- $k = 0.45$

Speaker strategies	Hearer strategies			
	AA	AO	OA	OO
$eezz$	0.550	0.550	0.550	0.550
$zzaa$	0.550	0.550	0.550	0.550
$ezaz$	0.458	0.458	0.458	0.458
$zeza$	0.507	0.507	0.507	0.507
$zeaz$	0.507	0.863	0.151	0.507
$ezzz$	0.545	0.538	0.553	0.545
$zezz$	0.505	0.861	0.148	0.505
$zzaz$	0.510	0.867	0.154	0.510
$zzza$	0.539	0.531	0.547	0.539
$zzzz$	0.500	0.849	0.152	0.500

If speakers get lazier...

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Speaker strategies	Hearer strategies			
	AA	AO	OA	OO
$eezz$	0.550	0.550	0.550	0.550
$zzaa$	0.550	0.550	0.550	0.550
$ezaz$	0.458	0.458	0.458	0.458
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$zzza$	0.539	0.531	0.547	0.539
$zzzz$	0.500	0.849	0.152	0.500

... and lazier ...

- $k = 0.53$

Speaker strategies	Hearer strategies			
	AA	AO	OA	OO
$eezz$	0.470	0.470	0.470	0.470
$zzaa$	0.470	0.470	0.470	0.470
$ezaz$	0.368	0.368	0.368	0.368
$zeza$	0.436	0.436	0.436	0.436
$zeaz$	0.483	0.839	0.127	0.483
$ezzz$	0.473	0.465	0.480	0.473
$zezz$	0.497	0.854	0.141	0.497
$zzaz$	0.494	0.850	0.137	0.494
$zzza$	0.476	0.468	0.484	0.476
$zzzz$	0.500	0.848	0.152	0.500

… and lazier …

- $k = 0.53$

Speaker strategies	Hearer strategies			
	AA	AO	OA	OO
$eezz$	0.470	0.470	0.470	0.470
$zzaa$	0.470	0.470	0.470	0.470
$ezaz$	0.368	0.368	0.368	0.368
$zeza$	0.436	0.436	0.436	0.436
$zeaz$	0.483	0.839	0.127	0.483
$ezzz$	0.473	0.465	0.480	0.473
$zezz$	0.497	0.854	0.141	0.497
$zzaz$	0.494	0.850	0.137	0.494
$zzza$	0.476	0.468	0.484	0.476
$zzzz$	0.500	0.848	0.152	0.500

… and lazier…

- $k = 0.7$

Speaker strategies	Hearer strategies			
	AA	AO	OA	OO
$eezz$	0.300	0.300	0.300	0.300
$zzaa$	0.300	0.300	0.300	0.300
$ezaz$	0.177	0.177	0.177	0.177
$zeza$	0.287	0.287	0.287	0.287
$zeaz$	0.431	0.788	0.075	0.431
$ezzz$	0.318	0.310	0.326	0.318
$zezz$	0.482	0.838	0.126	0.482
$zzaz$	0.457	0.814	0.101	0.457
$zzza$	0.343	0.335	0.350	0.343
$zzzz$	0.500	0.848	0.152	0.500

... and lazier...

- $k = 0.7$

Speaker strategies	Hearer strategies			
	AA	AO	OA	OO
$eezz$	0.300	0.300	0.300	0.300
$zzaa$	0.300	0.300	0.300	0.300
$ezaz$	0.177	0.177	0.177	0.177
$zeza$	0.287	0.287	0.287	0.287
$zeaz$	0.431	0.788	0.075	0.431
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...

- $k = 1$

Speaker strategies	Hearer strategies			
	AA	AO	OA	OO
$eezz$	0.000	0.000	0.000	0.000
$zzaa$	0.000	0.000	0.000	0.000
$ezaz$	−0.160	−0.160	−0.160	−0.160
$zeza$	0.024	0.024	0.024	0.024
$zeaz$	0.340	0.697	−0.016	0.340
$ezzz$	0.045	0.037	0.053	0.045
$zezz$	0.455	0.811	0.099	0.455
$zzaz$	0.394	0.750	0.037	0.394
$zzza$	0.106	0.098	0.144	0.106
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...

- $k = 1$

Speaker strategies	Hearer strategies			
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$eezz$	0.000	0.000	0.000	0.000
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$ezaz$	−0.160	−0.160	−0.160	−0.160
$zeza$	0.024	0.024	0.024	0.024
$zeaz$	0.340	0.697	−0.016	0.340
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taking stock

$zeaz/AO$ split ergative Australian languages	
$zzaz/AO$ differential object marking English, Dutch, ...	$ezzz/OA$??? Wakhi
$zezz/AO$ differential subject marking several caucasian languages	$zzza/OA$??? Nganasan
$zzzz/AO$ no case marking Bantu languages	$zzza/OA$
$zzzz/AO$	

- only very few languages are not evolutionary stable in this sense
  $zzaa$: *Hungarian*, $ezza$: *Arrernte*, $eeaa$: *Wangkumara*
- curious asymmetry: if there are two competing stable states, one is common and the other one rare

Random mutation and its consequences for evolutionary stability

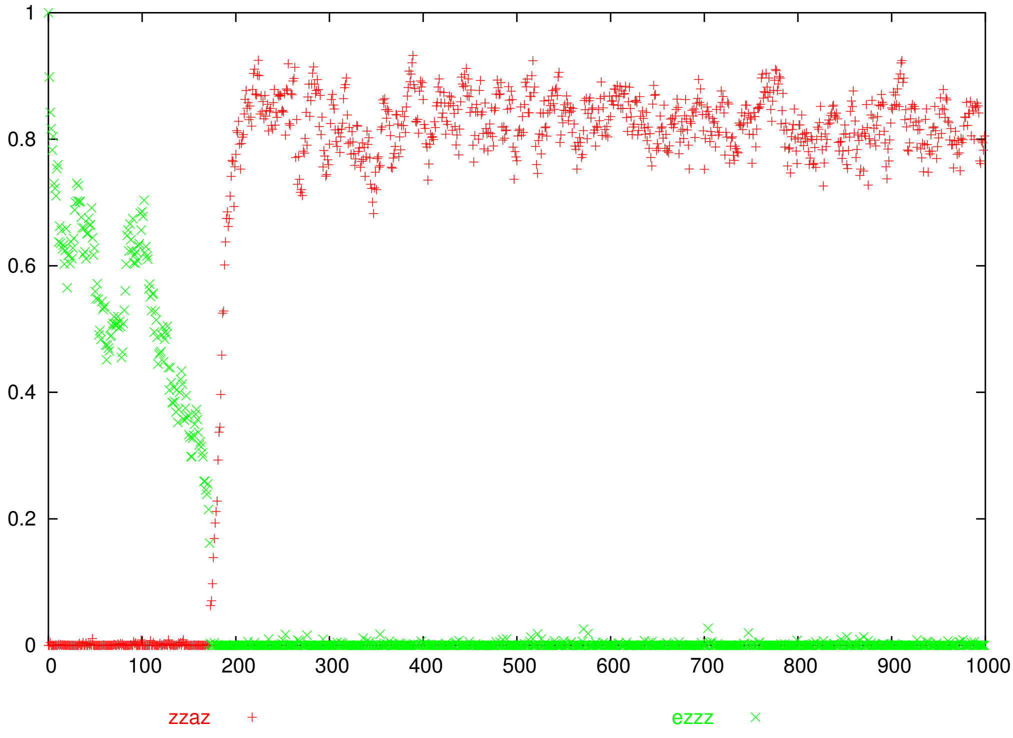
- idealizations of standard Evolutionary Game Theory
  - populations are (practically) infinite
  - mutations rate is constant and low
- better model (Young 1993; Kandori, Mailath and Rob 1993)
  - finite population
  - mutation is noisy

Consequences of finite population model

- every mutation barrier will occasionally be taken
- no absolute stability
- if multiple Strict Nash Equilibria coexist, system will oscillate between them
- some equilibria are more stable than others
- system will spend most of the time in most robustly stable state
- stochastically stable states

Stochastic stability of case systems

- $k = 0.45$
- competition between $zzaz/AO$ and $ezzz/OA$



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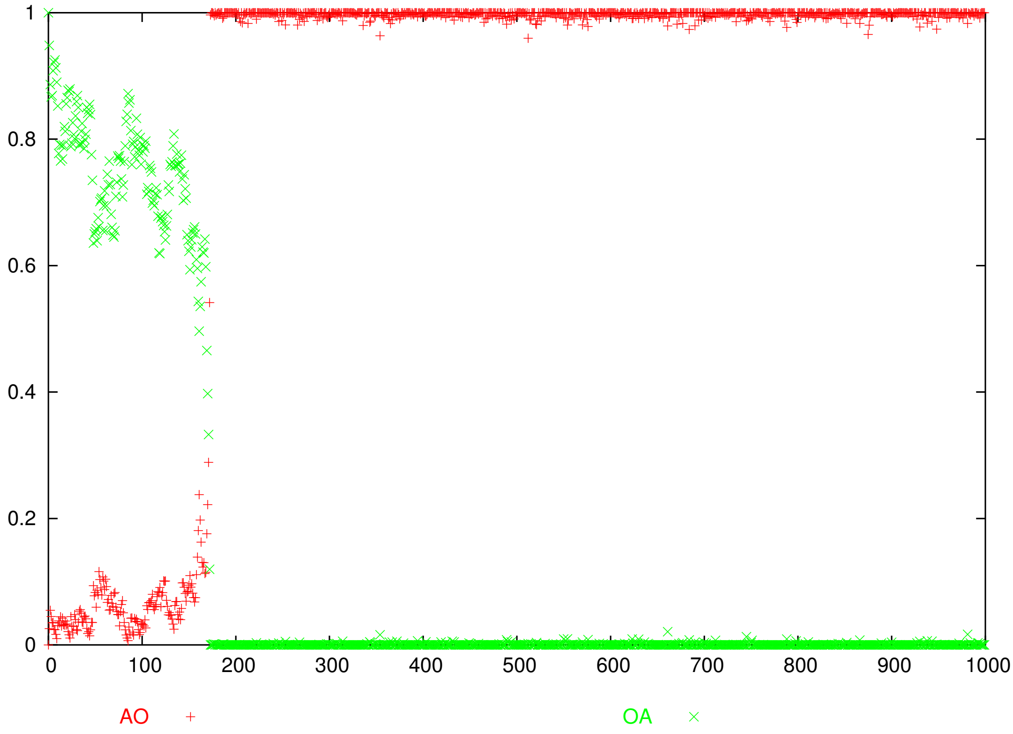

Stochastically stable case marking systems

$zeaz/AO$
split ergative
Australian languages

$zzaz/AO$
differential object marking
English, Dutch, ...

$zezz/AO$
differential subject marking
several caucasian languages

$zzzz/AO$
no case marking
Bantu languages

Conclusion

- out of $4 \times 16 = 64$ possible case marking patterns only four are stochastically stable
- estimate: at least 95 % of all languages that fit into this categorization are stochastically stable
- precise numbers are hard to come by though
- linguistic universals need not be based on innate “language instinct” but can be result of evolutionary pressure in the sense of cultural evolution