Some notes on the formal properties of Bidirectional Optimality Theory

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Outline of talk

- OT and semantics: issues
- Blutner's Bidirectional OT
- Alternative definition and naive algorithm
- Finite state implementation

Optimality Theory: The basic picture

- Three components:
  1. **GEN**: (very general) relation between input and output
  2. **CON**: set of ranked violable constraints on input-output pairs
  3. **EVAL**: Choice function that identifies optimal input-output pairs among a set of candidates (depending on **CON**)

- **CON** induces a (well-founded) ordering of i/o pairs
- **EVAL** picks out the minimal members of its argument wrt. this ordering

$$\langle i, o\rangle \text{ is optimal iff } o \in \mathbf{EVAL_{CON}}(\{o' | \mathbf{GEN}(i, o')\})$$

- Two types of constraints:
  1. Markedness constraints ⇒ refer to output only
     - "syllables have onsets", "vowels are oral" ...
  2. Faithfulness constraints ⇒ refer to i/o pairing
     - "don't delete material", "don't add material" ...

Application to syntax/semantic

- In phonology/morphology, OT takes the speaker perspective
- applied to syntax/semantics, this means:
  1. **GEN** is given by compositional (underspecified) semantics
  2. Markedness constraints only apply to forms, not to meanings
  3. A form/meaning pair may be blocked by a better form for the same meaning, but not the other way round

Competition/Blocking in semantics and pragmatics

- Competition between forms
  - Scalar implicatures
  - Clausal implicatures
- But: also competition between meanings
  - Presupposition resolution (cf. van der Sandt (1992))
  - Bridging inference
  - ...

Reconciling the perspectives

- Tension between "speaker economy" and "hearer economy" is often discussed, for instance Horn (1984):

  Q-principle: Say as much as you can (given I).

  I-principle: Say no more than you must (given Q).

Blutner's formalization

Definition 1 (Blutner's Bidirectional Optimality)

1. $\langle f, m\rangle$ satisfies the Q-principle iff $\langle f, m\rangle \in \mathbf{GEN}$ and there is no other pair $\langle f', m\rangle$ satisfying the I-principle such that $\langle f', m\rangle < \langle f, m\rangle$.
2. $\langle f, m\rangle$ satisfies the I-principle iff $\langle f, m\rangle \in \mathbf{GEN}$ and there is no other pair $\langle f, m'\rangle$ satisfying the Q-principle such that $\langle f, m'\rangle < \langle f, m\rangle$.
3. $\langle f, m\rangle$ is z-optimal iff it satisfies both the Q-principle and the I-principle.

cf. Blutner (1998, 2000)

Application of Bidirectional OT to semantic/pragmatic issues include

- Iconicity effects: Blutner (2000)
- Syntax and semantics of German adverbs: Egg (1999); Jäger and Blutner (2000); von Stechow (2000)
- Anaphora resolution: Beaver (2000)
- Presupposition resolution: Zeevat (1999)
- ...

Alternative definition

Definition 2 (X-Optimality) A form-meaning pair $\langle f, m \rangle$ is x-optimal iff

1. $\langle f, m \rangle \in \mathbf{GEN}$,
2. there is no x-optimal $\langle f', m \rangle$ such that $\langle f', m \rangle < \langle f, m \rangle$.
3. there is no x-optimal $\langle f, m' \rangle$ such that $\langle f, m' \rangle < \langle f, m \rangle$.

Theorem 1 If "$<$" is well-founded, then there is a unique x-optimality relation and a unique z-optimality relation

Proof idea: Recursion theorem

Theorem 2 If "$<$" is transitive and well-founded, then x-optimality and z-optimality coincide.

Proof: see Jäger (2000)

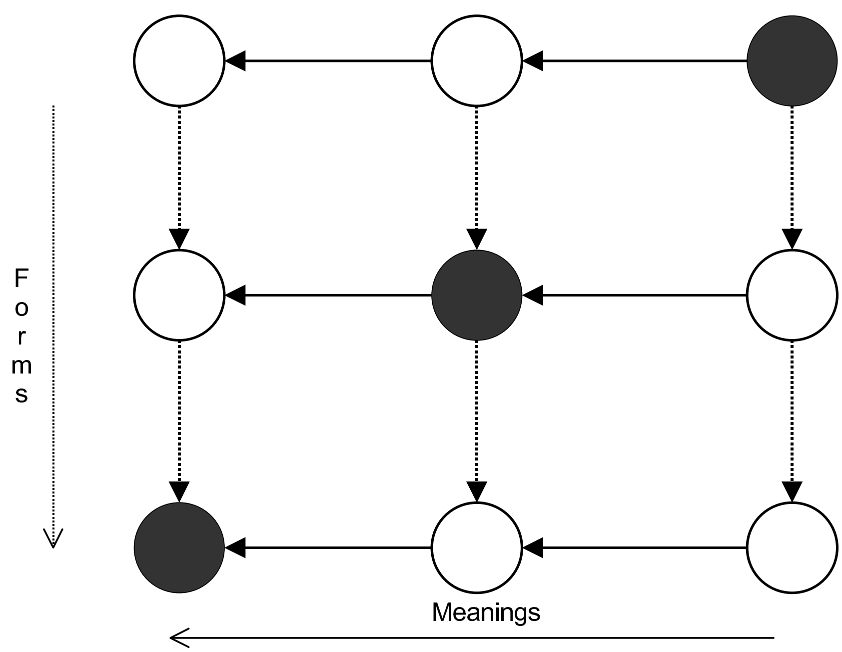
Algorithm

$OPT = \emptyset$;
$BLCKD = \emptyset$;

while $(OPT \cup BLCKD \neq \mathbf{GEN})$ {
$OPT = OPT \cup \{x \in \mathbf{GEN} - BLCKD | \forall y < x : y \in OPT \cup BLCKD\}$;
$BLCKD = BLCKD \cup \{\langle f, m\rangle \in \mathbf{GEN} - OPT |$
$\langle f', m\rangle \in OPT \vee \langle f, m'\rangle \in OPT\}$;
}

return (OPT);

Iconicity



OT and finite state techniques: Frank and Satta 1998

- Naive algorithms only work with finite candidate sets
- Bad news: Set of optimal candidates might be undecidable if candidate set is infinite
- Good news: Large subclass of OT systems can even be implemented by finite state techniques

Computational issues

- Set of optimal outputs might be undecidable, even if **GEN** and all constraints are decidable
  - Let T be a Turing machine
  - **GEN** $= \mathbb{N} \times \mathbb{N}$
  - $c_1 = \{n | T \text{ halts after less than } n \text{ steps}\}$
  - $c_2 = \{0\}$

  $\Rightarrow$ 0 is an optimal output iff T halts $\Rightarrow$ undecidable in the general case

Finite State techniques

- FSA (Finite State Automaton): standard definition, each FSA defines a **regular language**
- FST (Finite State Transducer):
  - FSA that produces output
  - every state transition consumes one input sign or the empty string and produces an output sign or the empty string
  - every FST defines a **rational relation**

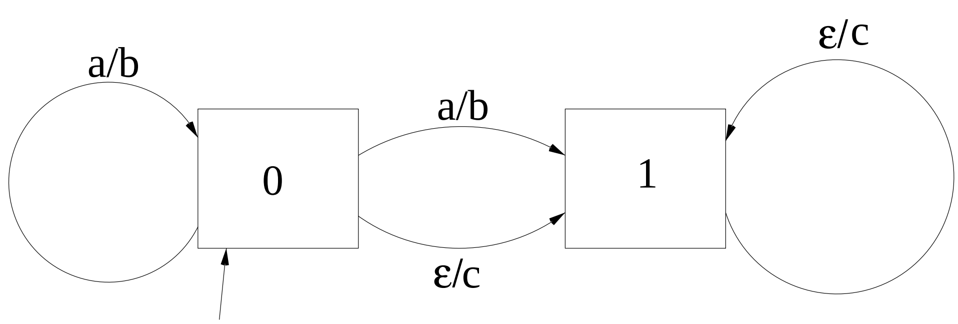


Figure 1: FST implementing the rational relation $\{\langle a^n, b^n c^* \rangle | n \in \mathbb{N}\}$

Some closure properties of regular languages and rational relations

- Every finite language is regular.
- If L_1 and L_2 are regular languages, then $L_1 \cap L_2, L_1 \cup L_2, L_1 - L_2$ are also regular languages.
- If R_1 and R_2 are rational relations, then $R_1 \cup R_2, R_1 \circ R_2$ and $R_1^{\cup}$ are also rational relations.
- If R is a rational relation, then $Dom(R)$ and $Rg(R)$ (the domain $\{x|\exists y.xRy\}$ and the range $\{y|\exists x.xRy\}$ of R) are regular languages.
- If L_1 and L_2 are regular languages, then $L_1 \times L_2$ and $\mathbf{I}_{L_1}$ are rational relations.

- Frank and Satta: If
  - **GEN** is a rational relation,
  - there are no faithfulness constraints
  - all constraints are binary (i.e. they don't count violations) and
  - all constraints can be represented by a regular language,

  then the set of optimal input-output pairs is a rational relation

Conditional Intersection

Definition 3
Let R be a relation and $L \subseteq Rg(R)$. The *conditional intersection* $R \uparrow L$ of R with L is defined as

$$R \uparrow L \doteq (R \circ \mathbf{I}_L) \cup (\mathbf{I}_{Dom(R) - Dom(R \circ \mathbf{I}_L)} \circ R)$$

Theorem 3 (Frank and Satta)
Let $\mathcal{O} = \langle \mathbf{GEN}, C \rangle$ with $C = \langle c_1, \ldots, c_p \rangle$ be an OT-system such C solely consists of binary output markedness constraints. Then $\langle i, o \rangle$ is unidirectionally optimal iff $\langle i, o \rangle \in \mathbf{GEN} \uparrow c_1 \cdots \uparrow c_p$.

Extension to Bidirectionality

- Bidirectional OT: competition both between different inputs and different outputs
- Thus both **input markedness constraints** and **output markedness constraints**
- So we also need **backward conditional intersection:**

$$R \downarrow L \doteq (R^{\cup} \uparrow L)^{\cup}$$

Definition 4 (Bidirectional Conditional Intersection)
Let $\mathcal{O} = \langle \mathbf{GEN}, C \rangle$ be an OT-system and c_i be a binary markedness constraint.

$$R \Uparrow c_i \doteq \begin{cases} R \circ \mathbf{I}_{Rg((\{\varepsilon\} \times Rg(R)) \uparrow c_i)} \\ \text{if } c_i \text{ is an output markedness constraint} \\ \\ \mathbf{I}_{Dom((Dom(R) \times \{\varepsilon\}) \downarrow c_i)} \circ R \\ \text{else} \end{cases}$$

Lemma 1
Let $\mathcal{O} = \langle \mathbf{GEN}, C \rangle$ be an OT-system (with binary markedness constraints only), where $C = \langle c_1, \ldots, c_p \rangle$. Then

$$\langle i, o \rangle \in \mathbf{GEN} \Uparrow c_1 \cdots \Uparrow c_p$$

iff $\langle i, o \rangle \in \mathbf{GEN}$, and there are no i', o' with $\langle i', o' \rangle \in \mathbf{GEN}$ and $\langle i', o' \rangle < \langle i, o \rangle$.

- **Notation:** $R^C \doteq R \Uparrow c_1 \cdots \Uparrow c_n$ (where $C = c_1, \ldots, c_p$)

Definition 5
Let $\mathcal{O} = \langle \mathbf{GEN}, C \rangle$ be an OT-system.

$$\begin{aligned}
OPT_0 &= \emptyset \\
OPT_{\alpha+1} &= OPT_\alpha \cup \\
&\quad (\mathbf{I}_{Dom(\mathbf{GEN})-Dom(OPT_\alpha)} \circ \mathbf{GEN} \circ \mathbf{I}_{Rg(\mathbf{GEN})-Rg(OPT_\alpha)})^C \\
OPT_\beta &= \textstyle\bigcup_{\alpha<\beta} OPT_\alpha \text{ (β a limit ordinal)} \\
OPT &= \textstyle\bigcup OPT_\alpha
\end{aligned}$$

Lemma 2
Let $\mathcal{O} = \langle \mathbf{GEN}, C \rangle$ be an OT-system. Then $\langle i, o \rangle \in OPT$ iff $\langle i, o \rangle$ is x-optimal.

Lemma 3
Let $\mathcal{O} = \langle \mathbf{GEN}, C \rangle$ be an OT-system with $C = c_1, \ldots, c_p$, where all c_i are binary markedness constraints. Then $OPT = OPT_{2^p}$.

Theorem 4
Let $\mathcal{O} = \langle \mathbf{GEN}, C \rangle$ be an OT-system with $C = \langle c_1, \ldots, c_p \rangle$, where all c_i are binary markedness constraints. Furthermore, let $\mathbf{GEN}$ be a rational relation and let all c_i be regular languages. Then the set of x-optimal elements of **GEN** is a rational relation.

Outlook: Extension to faithfulness constraints

- Generalization to faithfulness constraints requires closure of relations under intersection:

$$R \uparrow S \doteq (R \cap S) \cup (R \circ \mathbf{I}_{Rg(R) - Rg(Dom(R \cap S) \times Rg(R))})$$

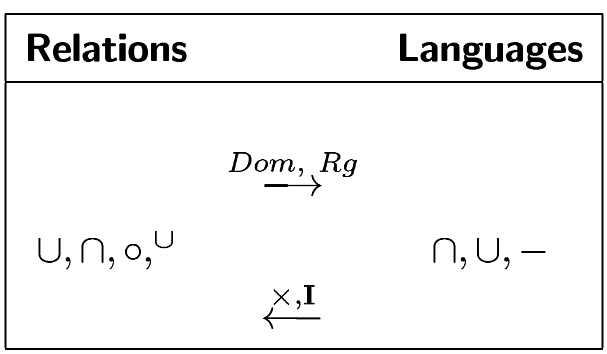


Figure 2: Closure conditions needed for x-optimality

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